



# OPTIMIZATION AND CONTROL OF A 6X6 MILITARY VEHICLE'S ACTIVE SUSPENSION SYSTEM

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*Abstract: In military vehicles, the suspension system performs an essential role in stability, driveability and comfort, being responsible to reduce vibrations induced by ground irregularities, which provides the increasement of suspension and vehicle components life cycle. In this context, the objective of this work is to analyze the dynamic time domain response of an active front suspension model in a space-state formulation, obtaining through multibody modeling the optimized response for the system, taking into consideration the main variables of interest: control force and reduction of the speed of the suspended mass. The first closed-loop control system designed is the Poles Placement (FSFB), designed by some project restrictions and parameters, based exclusively on the closed-loop eigenvalues. In order to find the best fit for the controller closed-loop transfer function parameters, a Genetic Algorithm (GA) associated with LQR model is developed, intended to produce an optimal model for the controller. In order to simplify the controller design, the plant parameters are, at first, equivalent to a quarter-car model of a 6X6 Military Vehicle, and the results obtained are simulated through MATLAB/Simulink® for an independent front suspension as first step, and finally for a half-car model, aimed to understand the vertical dynamics phenomenon including variables as pitch and gravity center speed.*

**Keywords:** Vertical Dynamics, Multi-Objective Optimization, Genetic Algorithms, Full State Feedback, Control Systems

## INTRODUCTION

Vehicle suspension systems are essential for the management of the driveability and vertical dynamics of a vehicle. In terms of vertical dynamics, the suspension systems must be able to support the vehicle's chassis, guarantee the contact between the tires and the ground, and mainly, attenuate the vibrations induced by road irregularities.

In the Dynamics field, mechanical vibrations are described as oscillations capable to deviate a body from its static state, which can be characterized as free or forced. Forced vibrations, in turn, are classified as physical phenomena caused by disturbing external forces, leading the mechanical system to forced oscillations (MCCALLION, 1973).

In this context, the design of a suspension system must consider parameters such as stiffness and damping coefficients, in order to obtain the behavior of the vehicle's suspended mass within acceptable and desirable levels.

Suspension systems can be characterized as passive, semi-active and active. Passive systems do not rely on external energy sources, while active systems may have sensors and actuators, in order to optimize the suspension behavior (SILVEIRA, 2014). In case of active systems, they may have sensors and actuators to apply an external control force to the system. For instance, the electromechanical and electropneumatic suspension models are recognized as active suspension systems. These kinds of engineering resources can neutralize with much higher efficiency the effects of forced vibrations.

This paper proposes a truck front active suspension system mechanism modeled by the action of a closed-loop electro-hydraulic system for a typical  $\frac{1}{4}$  vehicle model, using the Euler-Lagrange formulation and two different control techniques: First a Poles-placement (FSFB) controller is developed based on the design specifications (maximum overshoot and settling time). Then, a sensitivity analysis is provided aimed to understand the impact produced by changes imposed to the plant and controller parameters. In a second step, a Linear Quadratic Regulator (LQR) associated with the Genetic Algorithm (GA) is developed, in order to produce an optimal controller.

The control systems are designed and compared in a quarter-car vehicle model, whose parameters are equivalent to those of a SCANIA 6x6 truck, using both techniques: FSFB and GA-LQR. Finally, a half-car model is developed and analyzed in order to compare the outputs for both suspension systems against the passive system, considering variables such as the vehicle pitch and the center of gravity speed during a sequence of bumps (sinusoidal input).

## MULTIBODY MODELLING AND STEADY-SPACE FORMULATION

For the preliminary study of the vehicular dynamics, the truck suspension system can be represented by a  $\frac{1}{4}$  vehicle.

$$M_1 \ddot{\mathbf{Z}}_1(t) = C_2 [\dot{\mathbf{Z}}_2(t) - \dot{\mathbf{Z}}_1(t)] - C_1 [\mathbf{Z}_1(t) - \mathbf{Z}_r(t)] + K_2 [\mathbf{Z}_2(t) - \mathbf{Z}_1(t)] - K_1 [\mathbf{Z}_1(t) - \mathbf{Z}_r(t)] - \mathbf{f}a(t) \quad (1)$$

$$M_2 \ddot{\mathbf{Z}}_2(t) = -C_2 [\dot{\mathbf{Z}}_2(t) - \dot{\mathbf{Z}}_1(t)] - K_2 [\mathbf{Z}_2(t) - \mathbf{Z}_1(t)] + \mathbf{f}a(t) \quad (2)$$

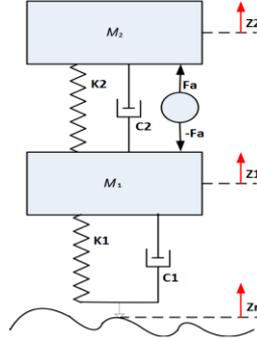


Figure 1. 1/4 Car suspension model. Available from: QUANSER MANUAL, 2010

Where the spring stiffness and suspension damping coefficient are  $K_1$  and  $C_1$ , respectively. The tire is modeled as a spring-damp, also characterized by  $K_1$  and  $C_1$  coefficients. The displacement of mass  $M_1$  is represented by  $Z_1$  while the displacement of mass  $M_2$  is represented by  $Z_2$ , and  $f_a(t)$  represents an active control force.

The front passive suspension is considered a continuous, linear and time invariant dynamic system, so it is convenient for the study to describe it in terms of its state variables:

$$x_1(t) = Z_2(t); \quad x_2(t) = \dot{Z}_2(t); \quad x_3(t) = Z_2(t) - Z_1(t); \quad x_4(t) = \dot{Z}_2(t) - \dot{Z}_1(t) \quad (3)$$

Considering a unit step test, the system above can be written in the state space form as follows, where  $\frac{f_a(t)}{Z_r(t)} = u(t)$ :

$$\dot{x} = Ax(t) + Bu(t), \quad (4)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{C_2}{M_2} & -\frac{C_1}{M_1} & \frac{C_2}{M_2} & \frac{C_1}{M_1} \\ \frac{C_1}{M_1} & 0 & -\left(\frac{C_2}{M_2} + \frac{C_2}{M_1} + \frac{C_1}{M_1}\right) & 0 \\ \frac{K_1}{M_1} & 0 & -\left(\frac{K_2}{M_2} + \frac{K_2}{M_1} + \frac{K_1}{M_1}\right) & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{M_2} & \frac{C_2}{M_2} \frac{C_1}{M_1} \\ 0 & -\frac{C_1}{M_1} \\ \frac{1}{M_1} + \frac{1}{M_2} & -\frac{K_1}{M_1} \end{bmatrix} \quad (5)$$

Before the action of any road input or active control force, the system is in stationary state, so zero initial conditions can be assumed:

$$x_1 = 0, \quad x_2 = 0, \quad x_3 = 0, \quad x_4 = 0. \quad (6)$$

In vehicle active suspension optimization projects, the following indices are generally considered: the passenger comfort and suspension deformation (driveability). The use of this criteria imposes some restrictions to  $x_2 = \dot{Z}_2(t)$  and  $x_1 = Z_2(t)$ , respectively, and for sure, to  $f_a(t)$  which is the control force to be developed by the controller.

## CONTROLLERS PROJECT

The controller projects are developed applying two different strategies, in order to evaluate the dynamic response and to compare both techniques. The first one is the FSFB, which consists of a poles placement controller. The second method is the (GA-LQR), which consists of an association of the LQR method and the Genetic Algorithm, where the main goal is to achieve the LQR optimal parameters in a iterative manner, in order to minimize the objective function  $J$  considering the project restrictions.

### Full State Feedback Controller (Poles-placement)

The application of the poles-placement starts with choosing the desired poles, based on certain natural response or frequency specifications. These specifications can be, for example, the maximum overshoot  $M_p$ , damping ratio  $\zeta$  and

settling time  $T_s$ . Assuming that exist a closed-loop desired poles to match the project specifications, there is a gain matrix  $K$  capable, trough the feedback of states, to force the closed-loop poles of the system to allocate themselves in the desired positions, as long as the system is fully controllable and observable. In usual controllers project, it is possible to design a system to meet the specification of maximum overshoot and settling time, based on the natural frequency and the damping ratio of the dominant pair of complex poles, as follows:

$$Mp_{\%} = \exp(-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}) \quad (7)$$

$$T_s = \frac{4}{\zeta \omega_n} \quad (8)$$

### Linear Quadratic Regulator (LQR) and quadratic performance index

Optimal controller designs for linear systems using LQR are easily found in the literature (KAILATH, 1979). For a continuous system described as (5), the LQR problem is intended to determine a full state feedback (FSFB) controller that minimizes the Cost Function  $J$ . In order to find the minimum, both matrices  $Q$  and  $R$  are considered the weight matrices, such that  $Q = Q' \geq 0$  and  $R = R' \geq 0$ .

The optimal controller LQR that minimize the Cost Function (12) is given by:

$$u(t) = Kx(t), \text{ when} \quad (9)$$

$K = -R^{-1}B'P$ , where  $P = P' \geq 0$ , is the solution for the algebraic Ricatti's equation:

$$A'P + PA - PBR^{-1}B'P + Q = 0 \quad (10)$$

At the optimization process, a performance index widely used in the literature is the quadratic performance index (THOMPSON, 1976), which is presented in (10) below.

$$J = \int_0^{\infty} q_1 \cdot fa^2(t) + q_2 \cdot Z2^2(t) + q_3 \cdot \dot{Z}2^2(t) \quad (11)$$

Where  $q_1$ ,  $q_2$  and  $q_3$  are the penalty constants. With the criteria described above, which considers  $x_2 = \dot{Z}2(t)$  and  $x_1 = Z2(t)$  to be optimized, we can obtain the Cost Function  $J$  in terms of the state variables  $x(t)$  and the control force  $fa(t) = u(t)$  on a matrix form.

$$J = \int_0^{\infty} x(t)^T Qx(t) + u(t)^T Ru(t) \quad (12)$$

Where the penalty matrices  $Q$  and  $R$  are taken to be positive semidefinite as follows:

$$Q = \begin{bmatrix} q_2 & 0 & 0 & 0 \\ 0 & q_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, R = [q_1] \quad (13)$$

### Genetic Algorithm associated with lqr (GA-LQR model)

Genetic algorithms are a parallel search and optimization technique, inspired by the Darwinian principle of natural selection and genetic reproduction (GOLDBERG). According to C. Darwin's theory, the principle of selection favors the fittest individuals, with greater longevity, and therefore, greater probability of reproduction. In this context, GA looks for a better solution for the optimization problems, through an iterative search process, initiated by generating an initial population, which combined with the best representatives, generates a new one, replacing the previous one. At each new iteration, a new population is generated with individuals that generate the best solution to the optimization problem, culminating in their convergence.

In the GA structure, some terms are used and their definition becomes necessary:

- 1) Gene: Optimization variable;
- 2) Chromosome: Set of genes;
- 3) Initial Population: Randomly generated set of chromosomes;
- 4) Generations: Genetically modified populations from previous generations through recombination, selection and/or mutation;
- 5) Recombination: Process of modifying and creating a new chromosome from the combination of 2 or more chromosomes;
- 6) Mutation: Process of changing an chromosome at random;
- 7) Fitness Function: Solution function evaluated (Objective Function);
- 8) Stopping Criteria: End of the iterations, which can be the number of generations and execution time.

### GA-LQR Methodology

As a solution to the problem presented in equation (12), the GA-LQR appears as an alternative, delivering controller designs with good performance and stability as a result. The GA-LQR model presented in figure 2 performs the search for the state and control weighting matrices  $Q$  and  $R$ , in order to design a controller that satisfies the physical and design constraints for the dynamic system.

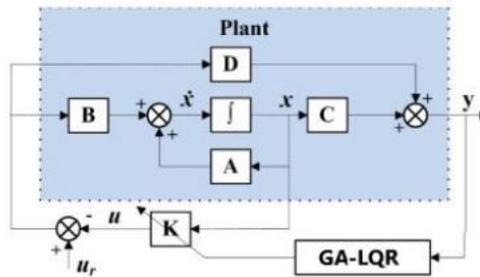


Figure 2: GA-LQR Structure (MORAES, 2007)

In order to be able to start the iteration process, it is necessary to establish the inputs to the optimization problem:

#### I. QR Population Modeling

A chromosome  $QR_{n \times g}$  is composed of  $g$  genes, depending on the dimension  $n$  and the  $m$  inputs of the system. Considering the active suspension optimization problem, subject to an input (step/bumps), the chromosome has the following structure:

$$QR_{n \times g} = [q1; q2; q3; q4; r1]$$

#### II. Genetic Operations

- Elitist Selection: At this stage of the process, the  $n_{ie}$  best chromosomes are stored for the next generation;
- Crossover Model: The crossover is the recombination operation, which in this model was responsible for combining 2 chromosomes from an initial (previous) population  $G$ . The chromosomes  $QR_{l1}$  e  $QR_{l2}$  from a population  $QR_{n \times g}$  share genetic information, creating a new one  $QR_{G+1, l1}$  as follows:

$$QR_{G+1, l} = [q1_{G, l1}, q2_{G, l1}, q3_{G, l1}, q4_{G, l2}, r1_{G, l2}]$$

- Mutation Model: In the GA-LQR project of the active suspension, the alteration of a gene of the mutated chromosomes was considered for the generation of the population  $G + 1$  randomic, using the function  $\text{randi}(5, 1, n_{im})$  of MATLAB/Simulink, where  $n_{im}$  is the number of chromosomes mutated at the new generation, and 5 is the number of genes of one chromosome  $QR_{n \times g}$ .

#### III. Objective Function and Constraints

In the controller design, the objective function to be minimized is characterized by the cost functional  $J$ , however, for realistic modeling of the Fitness Function of the optimization problem, the constraints must be considered, so that the

controller design proposed by the iterative method GA meets not only the stability and design characteristics of the control system, but also the physical constraints of the dynamic system, which are described below:

- According to the ISO 2631 standards (ISO 2631, 1997), there is a perception of comfort when the RMS acceleration of the suspended mass does not exceed  $0.315\text{m/s}^2$ :

$$F1 = \ddot{Z}_2 - 0.315 \leq 0 \quad (14)$$

- In order to meet the physical requirements of the suspension, the working space must not exceed 0.127m:

$$F2 = Z_2 - Z_1 - 0.127 \leq 0 \quad (15)$$

- To introduce design requirements, a maximum overshoot  $M_p = 25\%$ , equivalent to a 15% reduction in relation to the passive system, and a settling time  $T_s = 1,8 \text{ sec}$ , equivalent to a 10% of reduction in relation to the passive system. Finally, an steady state error of  $e_{ss} = 2\%$ , ensuring system stability and imposing the constraints F3 e F4 below:

$$F3 = \zeta\omega_n + 2.31 \leq 0 \quad (16)$$

$$F4 = \frac{y_p}{y_a} - 0.02 \leq 0 \quad (17)$$

#### IV. Fitness Function

To define the fitness function, the constraints of the problem must be considered, so that they must be incorporated into the objective function through the approach of penalty functions. Modeling the problem with constraints through the introduction of penalty constants makes its application possible, while making the problem unconstrained, and therefore, easier to solve. The choice of penalty parameters must guarantee proportionality to the unconstrained fitness function, so that the algorithm moves in the search direction in the feasible region. In the GA-LQR controller design, constraints F1, F2, F3 and F4 assume the role of auxiliary function, being introduced in the fitness function with their respective penalty parameters  $a_1 = 1$ ,  $a_2 = 10^2$ ,  $a_3 = 10$  e  $a_4 = 1$ . Thus, failures to meet the constraints of the problem incurs an increase of the fitness function value, which diverges from the optimal model. In this way, the optimization problem can be summarized as:

$$\mathbf{Min} F_{obj}(Q, R, X, u) = \int_0^{\infty} x(t)^T Qx(t) + u(t)^T Ru(t) \quad (18)$$

Subject to

$$\begin{aligned} \ddot{Z}_2 - 0.315 &\leq 0 \\ Z_2 - Z_1 - 0.127 &\leq 0 \\ \zeta\omega_n + 2.31 &\leq 0 \\ \frac{y_p}{y_a} - 0.02 &\leq 0 \\ 0 &\leq QR_{nxG} \leq 100 \end{aligned} \quad (19)$$

It is worth mentioning that the management of the penalty constants in the objective function can lead to prioritization of one variable to the detriment of another. For example, the association of a penalty constant in one of the constraints that is much higher than the valuation of another, will make the GA look for solutions that are increasingly suitable for the viable region of this constraint, so that the fitness function is minimized. Therefore, in order to meet all requirements

and minimize the fitness function satisfying all constraints, it is important to guarantee its proportionality, considering the dimensions of the variables of interest.

V. GA-LQR Model Parameters

Finally, to perform the optimization process satisfactorily and find the convergence, it is essential that the choice of parameters of the genetic algorithm is adequate. The table 2 lists the GA parameters that structurize the iterative process:

Table 1: GA Procedure

GA Parameters	Value
Number of variables (Genes)	5
Population Size	100
Selection Model	Elitist – 20%
Mutation	1 Gene – 30%
Crossover (Recombination)	50%
Stopping Criteria	30 Generations

NUMERICAL ANALYSIS

Numerical simulations were performed using MATLAB/Simulink, where a full 6x6 SCANIA P 410 CB truck model was simplified to a ¼ vehicle model for analysis of the active front suspension (SCANIA SPECS, 2020). The vehicle is shown in figure 3, and its simplified parameters are summarized in table 2.

Table 2: Simplified ¼ Car Parameters

Suspension Parameters	Value
Sprunged mass (M2)	4500 kg
Unsprunged mass (M1)	490 kg
Suspension stiffness (K2)	59.600 N/m
Tire stiffness (K1)	770.700 N/m
Damping coefficient - Suspension (C2)	20.000 N s/m
Damping coefficient - Tire (C1)	2.660 N s/m



Figure 3: SCANIA P 410. Adapted from: ACUÑA, 2020

The controller project for the active suspension system was made based on an input from the control action  $f_a(t)$  since the system presents itself as a regulator, keeping its current state and, once subject to disturbances, it performs a control action in order to return to your previous position. Through MATLAB functions “crtb” and “obsv”, it is possible to assure that the system is controllable and observable.

The first active suspension system developed is the Poles Placement/FSFB model. In order to choose the poles-placement for the FSFB, a project specification for the closed loop performance is needed. With the state space model designed, a maximum overshoot  $Mp = 25\%$  and the settling time  $Ts = 1.8 \text{ sec}$  for  $x_1 = Z_2(t)$  response will be assumed. With MATLAB, by setting the quarter-truck mass (M2) position ( $x_2$ ) as the output, the open-loop Laplace transform transfer function described below is found:

$$\frac{24.13 S^2 + 7062 S + 20830}{S^4 + 50.69 S^3 + 1732 S^2 + 7062 S + 20830} \tag{20}$$

In order to meet the specifications and considering equation (20), let’s consider there is a gain matrix K which places the closed-loop dominant poles as follows:  $[-2.31 \pm 2.938j]$ , as per the equations (7) and (8) solution, The second pair of poles were fixed in order to keep the distance of the dominant poles as follows:  $[-23.233 \pm j31.343j]$ .

Intended to achieve the project requirements mentioned on the constraints imposed, at the ending of the 30th generation, as per figure 4, the GA-LQR model reached the values of  $QR = [0.72, 0.039, 0.17, 0.016, 76]$ , with its dominant poles pair at  $[-2.028 \pm 2.69i]$ , culminating in the closed-loop transfer function described in (18) equation.

$$\frac{24.13 S^2 + 7062 S + 20830}{S^4 + 61.67 S^3 + 2088 S^2 + 2118 S + 20930} \tag{21}$$

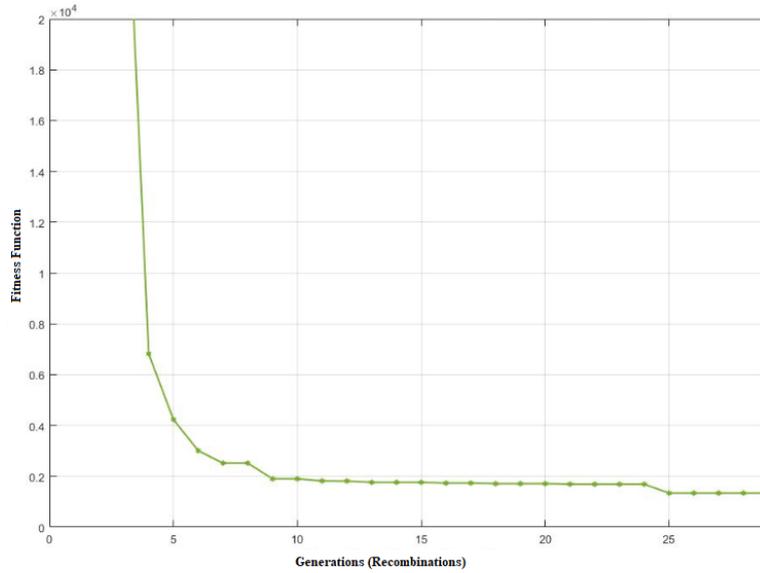


Figure 4: GA- LQR convergence curve [Author]

By submitting both quarter-car controllers with 2 DOFs to a unit step-type test input, the values of maximum overshoot  $M_p$ , settling time  $T_s$ , and the response for  $x_1$  and  $x_2$  for the system are obtained and shown in figure 5. It is possible to note that the GA-LQR model is able to reduce in 15% the value of the performance indicator  $J$  compared to the FSFB model, combined with a reduction of e 2% of overshoot  $M_p$ .

Table 3: ¼ Suspension Parameters (2 DOFs)

Suspension Model	(J)	% Mp	Ts(sec)	X1 (m)	X2 (m/s <sup>2</sup> )
Open-loop	-	29.429	1.97	0.404	2.01
FSFB	204	24.74	1.79	0.372	1.87
GA-LQR	161	22.69	1.69	0.321	1.64

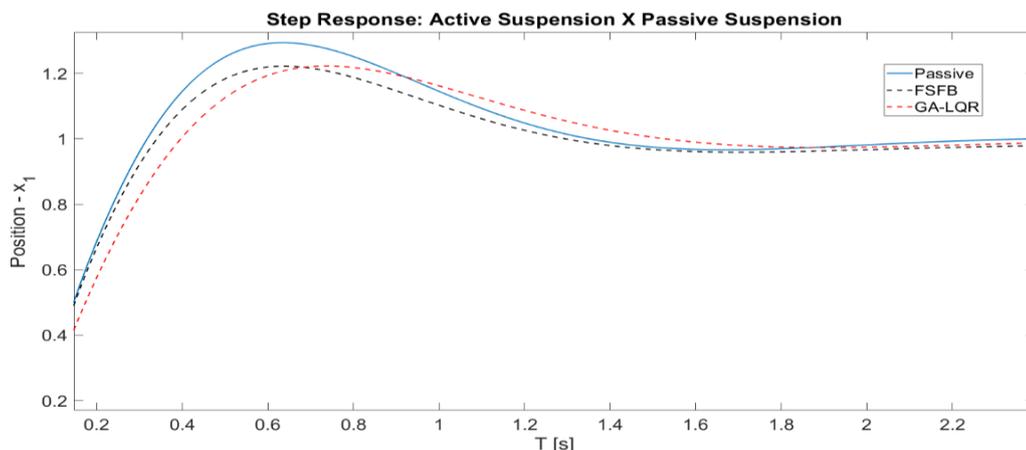


Figure 5: ¼ car step-response analysis. [Author]

In order to extend the  $\frac{1}{4}$  car model to a half-car model, all geometric truck parameters and also the back suspension (bogie type) are listed (table 4) and introduced to the model. It is essential to mention that the quarter-car sprung mass value is calculated from the whole model, taking in consideration the dynamic weight distribution (GILLESPIE, 1992).

For the half-car model dynamic analysis (7 DOFs), the longitudinal displacement was performed with constant speed  $V_x$  along the sinusoidal test track described at table 5 and figure 7, where the results of the speed of the center of gravity  $V_{CG}$  and pitch  $\theta$  are shown at figure 8 and resumed at table 6.

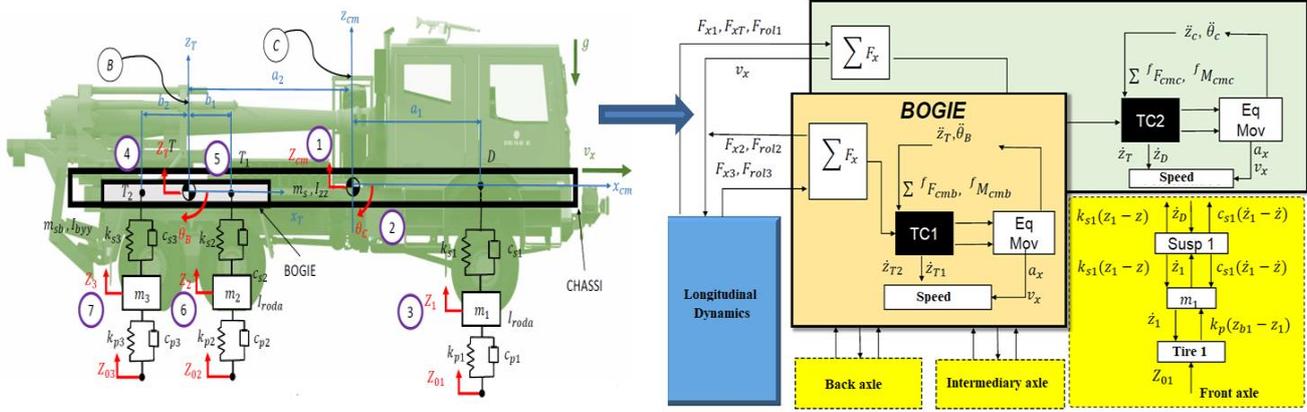


Figure 6: Half-car model. [Author]

Table 4:  $\frac{1}{2}$  Car model parameters

Parameters		
Distance from CG to front axle	$a_1$	2,990 m
Distance from back suspension (bogie) center to CG	$a_2$	1,988 m
Distance from back suspension (bogie) center to intermediary axle	$b_1$	0,675 m
Distance from back suspension (bogie) center to back axle	$b_2$	0,675 m
CG height	$h_{CM}$	1,359 m
Half-car mass	$m_v$	22.225 Kg
Chassis mass	$m_s$	18.135 Kg
Bogie mass	$m_{sb}$	200 Kg
Front axle unsprung mass	$m_1$	490 Kg
Intermediate axle unsprung mass	$m_2$	1.700 Kg
Back axle unsprung mass	$m_3$	1.700 Kg
Inertia momentum - Chassis x CG (y-axle)	$I_{yy}$	$1,040 \times 10^5$ Kg m <sup>2</sup>
Inertia momentum - Bogie x CG (y-axle)	$I_{byy}$	66,080 Kg m <sup>2</sup>
Inertia momentum - Vehicle x CG (z-axle)	$I_{zz}$	727,390 Kg m <sup>2</sup>
Intermediary suspension damping coefficient	$c_{s2}$	10.000
Intermediary suspension stiffness	$k_{s2}$	3.220.000
Back Suspension damping coefficient	$c_{s3}$	8.000
Back Suspension stiffness	$k_{s3}$	3.220.000

For a realistic simulation, the equations (22), (23) and (24) will be tested to mathematically model the displacement of the suspension in contact with a sinusoidal road, along the path of the  $\frac{1}{2}$  vehicle model (ACUNA, 2020):

$$Z_{01}(t) = h \operatorname{sen} \left\{ \frac{2\pi}{L} [v_x t - (d - a_1)] \right\} \quad (22)$$

$$Z_{02}(t) = h \operatorname{sen} \left\{ \frac{2\pi}{L} [v_x t - (d + a_2 - b_1)] \right\} \quad (23)$$

$$Z_{02}(t) = h \operatorname{sen} \left\{ \frac{2\pi}{L} [v_x t - (d + a_2 + b_2)] \right\} \quad (24)$$

**Table 5:** Road and truck geometric parameters

Road Parameters	
$h = 0,3$	bumps height in meters (m);
$L = 7$	road length in meters (m);
$Vx = 10$	longitudinal speed (m/s);
$t = 60$	simulation time in seconds (s);
$d = 50$	road excitation position (m);
$a_1$	distance from the center of gravity to front axle (presented in table 4);
$a_2$	distance from the center of gravity to the center of intermediate and back axle (presented in table 4);
$b_1$	distance from the back suspension (bogie type) anchor point (chassis) to the intermediate axle
$b_2$	distance from the back suspension (bogie type) anchor point (chassis) to the back axle

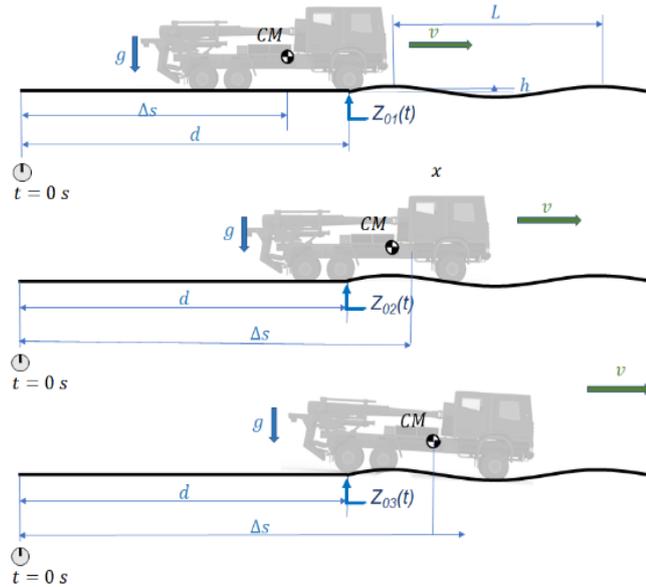


Figure 7: Bumps excitation. Available from: ACUÑA, 2020

Once applied to a half-car model, the results found are shown and compared in table 6 and figure 8, reinforcing the feasibility and the optimal characteristic of the active suspension for the truck model.

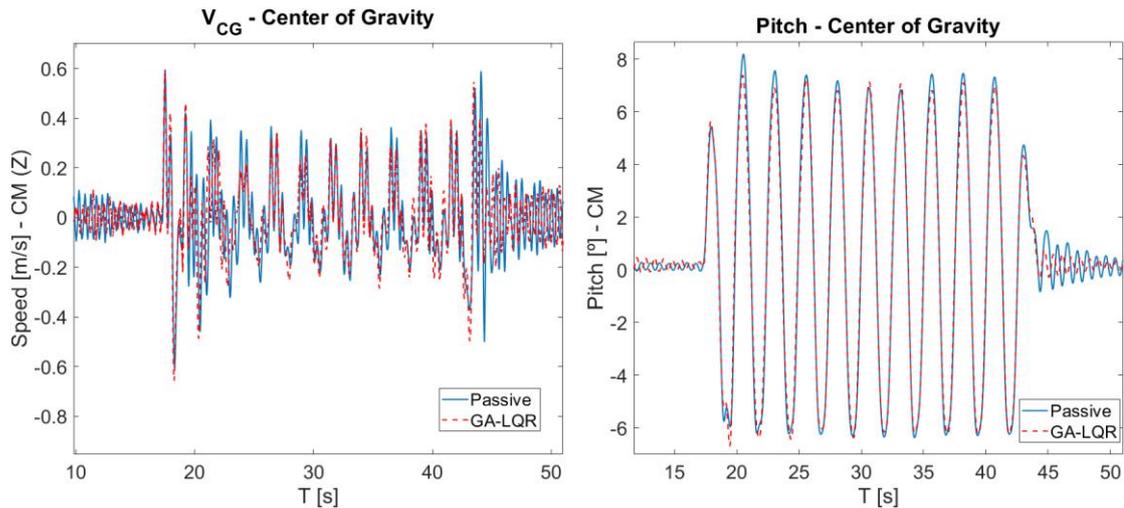


Figure 8: **Half-Car** Analysis. [Author]

**Table 6:** Numerical Analysis for the **half-car** model

Outputs	Passive	GA-LQR
$V_{CG}$	0,61 m/s	<b>0,59 m/s</b>
$\theta$	8,2°	<b>7,3°</b>

## CONCLUSION

This paper was intended to apply two different techniques of an active suspension model, in order to compare which strategy should present the best performance for the suspension system, optimizing the cost function.

First, the passive suspension was described in the state space model and the initial conditions were established. Then, considering an speed-bump input, the controllers were designed to represent a control force applied to the active systems, in order to recognize how the system should behave for the same plant parameters. Finally, for the unit-step response,

both active suspension systems designed, the closed-loops LQR and FSFB models were simulated and compared, in order to find the gain matrix  $K$  needed to match the project specifications.

The numerical results show that the management of the gain matrix  $K$  together with the penalty matrices  $R$  and  $Q$  for the GA-LQR model, the dynamic system project specifications can be assured even obtaining considerable reductions of the overshoot ( $M_p$ ) and the settling time ( $T_s$ ) for the unit step response while also reducing the Cost Function ( $J$ ). The simulations also show better behavior of the LQR model for the road bump profile considering driveability and quarter-truck mass speed variables.

Trough MATLAB/Simulink tools, it was verified that the GA-LQR was able to optimize the cost function  $J$ , while stabilizes the system much faster than the FSFB does. The reductions observed were around 20% for the cost function, 9% for the overshoot and 5% for the system settling time. From the passive system, these values achieves 33% and 15% of reduction for the overshoot and settling time, respectively. Considering the half-car model trafegability, reductions of the pitch and the center of gravity speed were also verified, reinforcing the better behavior of the active suspension model.

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