



# Spectral element for vibration analysis of a honeycomb plate

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*Abstract: In many industrial applications, the choice of composite over metallic material structures is due to reducing the weight of a system without compromising its strength and stiffness, which is considered one of the most important design criteria. In this article, an analysis of the dynamic response of a honeycomb plate in the XY plane composed of spectral beam elements in each hexagonal cell is performed. For the numerical implementation, it is used the Spectral Element Method (SEM) is a method that does not need a large number of elements in the mesh. A spectrally formulated can be more accurate and computational low-cost than the conventional FEM, generally used to analysis composite structures. The results are validated with the one obtained by ANSYS.*

**Keywords:** Beam, Honeycomb, Hexagonal, FRF, Spectral Element Method

## 1 INTRODUCTION

Honeycomb structures have been increasingly gaining more relevance in the industry due to their lightweight, high-specific bending stiffness and strength under distributed loads and their good energy-absorbing capacity [1]. Determination and control of the elastic properties of these elements are essential for their adoption as structural elements of various devices and systems [2]. Works in this area usually use the finite element method to perform the discretisation and analysis of the response of a honeycomb structure [1,3]. The finite element method is popular. However, implying mesh elements, together with complex structures, demands computational time.

The Spectral Element Method (SEM) is an alternative to other methods of dynamic analysis, such as Modal Analysis (MA), Finite Element Method (FEM) or Wave Finite Element (WFE), since its formulation deals with a model that relates forces and spectral displacements nodal based on the analytical solution of the wave [4]. Therefore, SEM does not need a discretisation with large numbers of elements turning. This paper analyses the dynamic behaviour of a honeycomb plate structure composed of the Euler-Bernoulli beam's spectral elements. Since this method deals with the exact solution of the dynamic system in the frequency domain, the dynamic response is accurate, and the model is easy and efficient [4, 6].

## 2 Spectral Element Method

The beam element model shown in Fig. 1, contains two nodes with a uniform rectangular cross-section, where the properties are assumed to be deterministic variables. Internal structural damping is introduced into the beam formulation by adding into Young's modulus weighted by a complex damping factor  $i\eta$ ,  $i = \sqrt{-1}$ ,  $\eta$  is the hysteretic structural loss factor. To obtain  $E = E(1 + i\eta)$ .

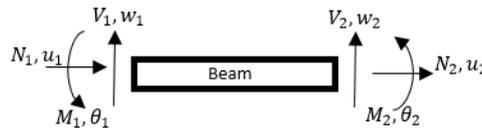


Figure 1 – Degree of freedoms for a single cell wall (i.e., beam) [3].

The beam represented in Figure 1 is composed of joints of elements of the type beam. The structure presents homogeneous density and thickness, perfect continuity at the interfaces, small vibration amplitudes and linear elasticity. The variables of transverse displacement, axial displacement and rotation are represented by the terms  $w(x, t)$ ,  $u(x, t)$  and  $\theta(x, t)$ . The relations of bending moments, shear force and axial force, are represented by  $M$ ,  $V$  and  $N$ . The mathematical model for the Euler-Bernoulli beam considering the dimensions uniform prismatic, has the following equation of motion,



$$EI \frac{\partial^4 w(x, t)}{\partial x^4} = \rho A \frac{\partial^2 \theta(x, t)}{\partial t^2} + f(x, t) \quad (1)$$

where  $E$ ,  $\rho$ ,  $A$  and  $I$  are Young's modulus, mass density, transverse area and moment of inertia, respectively. Assuming that the transverse displacement and rotation can be expressed as

$$\begin{Bmatrix} w(x, t) \\ \theta(x, t) \end{Bmatrix} = \frac{1}{N} \sum_{n=0}^{N-1} \begin{Bmatrix} W(x, \omega_n) \\ \Theta(x, \omega_n) \end{Bmatrix} e^{i\omega_n t} \quad (2)$$

By considering the external force  $f(x, t) = 0$  and applying the spectral terms of Eq. (2) in Eq. (1), we obtained the equation of motion of the beam in the frequency domain, expressed by

$$EI \frac{\partial^4 W(x, \omega_n)}{\partial x^4} - \omega^2 \rho A W(x, \omega_n) = 0 \quad (3)$$

The general solution is assumed to type  $W = ae^{-ik(\omega)x}$  and its solution represents the following dispersion relation  $k^4 - k_F^4 = 0$ , where  $k_F$  does the following relation define the wavenumber as

$$k_F = \sqrt{\omega} \left( \frac{\rho A}{EI} \right)^{1/4} \quad (4)$$

Therefore, the general solution under wave propagation base is

$$w = a_1 e^{-ik_F x} + a_2 e^{-k_F x} + a_3 e^{ik_F x} + a_4 e^{k_F x} = \mathbf{e}(x, \omega) \mathbf{a} \quad (5)$$

where  $\mathbf{e}(x, \omega) = [e^{-ik_F x}, e^{-k_F x}, e^{ik_F x}, e^{k_F x}]$ , and  $\mathbf{a} = \{a_1, a_2, a_3, a_4\}^T$ . For finite beam element of length  $L$  with defined nodes on the contours  $x = 0$  and  $x = L$ , we have that the transverse displacement and rotation can be related to the wave equation,

$$\mathbf{d} = \begin{Bmatrix} W_1 \\ \theta_1 \\ W_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{e}(0, \omega) \\ \mathbf{e}'(0, \omega) \\ \mathbf{e}(L, \omega) \\ \mathbf{e}'(L, \omega) \end{Bmatrix} = \mathbf{H}_B(\omega) \mathbf{a}, \quad (6)$$

$$\text{where } \mathbf{H}_B(\omega) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -ik_F & -k_F & ik_F & k_F \\ e^{-ik_F L} & e^{-k_F L} & e^{ik_F L} & e^{k_F L} \\ -ik_F e^{-ik_F L} & -k_F e^{-k_F L} & ik_F e^{ik_F L} & k_F e^{k_F L} \end{bmatrix},$$

One can express the components of force and moment as

$$\mathbf{f}_c = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} -Q(0) \\ -M(0) \\ Q(L) \\ M(L) \end{Bmatrix} = \mathbf{G}_B(\omega) \mathbf{a}, \quad (7)$$

where  $(x) = -EIW'''(x)$ ,  $M(x) = EIW''(x)$ , and

$$\mathbf{G}_B(\omega) = \begin{bmatrix} -ik_F^3 & ik_F^3 & k_F^3 & -k_F^3 \\ -ik_F^2 & ik_F^2 & k_F^2 & -k_F^2 \\ ik_F^3 e^{-ik_F L} & -ik_F^3 e^{ik_F L} & -k_F^3 e^{k_F L} & k_F^3 e^{-k_F L} \\ k_F^2 e^{-ik_F L} & -k_F^2 e^{ik_F L} & ik_F^2 e^{k_F L} & -ik_F^2 e^{-k_F L} \end{bmatrix},$$

By relating Eq. (6) with Eq. (7), it is possible to establish the relations between displacement and force,

$$\mathbf{S}_B(\omega) = \mathbf{G}_B(\omega) \mathbf{H}_B^{-1}(\omega), \quad (8)$$

where  $S_B(\omega)$  is the dynamic stiffness matrix, also known as the spectral stiffness matrix for the Euler-Bernoulli beam.

### 3 RESULTS

The analysis of dynamic characteristics and behaviour of the honeycomb plate is investigated. The plate is composed of nine hexagonal cells formed by beam spectral elements, as shown in Fig. 1, computing a set of 38 equal rectangular beams of base equals 0.0127 m, height of 0.002286 m, and moment of inertia  $I_z = 1.26430295526 \times 10^{11} \text{ m}^3$ . Mechanical properties are assumed by Young's Modulus of 71 GPA and density  $\rho = 2700 \text{ kg/m}^3$ . Vibration features as the modal parameters and the honeycomb plate frequency response function (FRF) are calculated using SEM and FEM. The SEM honeycomb plate is validated by comparison with the results obtained using the ANSYS APDL software. Figure 1 exhibits the honeycomb plate, the blue dot (Coordinates 3919, 3394) shows the measurement point where FRF were calculated via FEM and SEM. The first fifteen resonance frequencies were extracted and compared, as shown in Table 1.

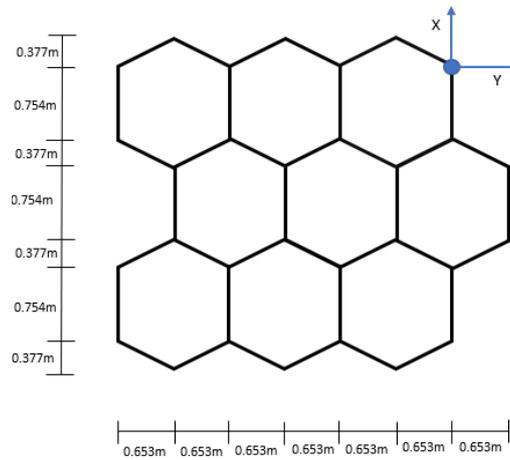


Figure 2 – Honeycomb plate with nine hexagonal cells.

Table 1: Comparison between the first fifteen natural frequencies and the resonance frequencies obtained with the SEM and FEM model.

Mode	Resonance Frequency (Hz)	
	FEM	SEM
1°	0.37	-
2°	0.43	0.42
3°	0.57	0.52
4°	0.87	0.91
5°	0.97	0.97
6°	-	1.25
7°	-	1.48
8°	-	1.53
9°	1.62	1.74
10°	1.79	1.80
11°	2.01	-
12°	-	2.35
13°	2.48	2.43
14°	-	2.64
15°	2.97	-



The resonant frequencies obtained by the FEM and SEM showed proximity between them. It can be seen that the FEM was unable to identify the 6th, 7th, 8th, 12th and 14th resonance frequencies, while the SEM was unable to identify the 1st, 11th and 15th resonance frequencies. These differences in each method in determining the honeycomb plate's resonance frequencies may result from the excitation and measurement point chosen for the analysis, which may cause the non-excitation of some modes. However, it is noticeable that the SEM performed better in this honeycomb plate analysis with a high modal density than the FEM because it calculates the dynamic response more accurately. The FEM used 200 elements in the mesh for modelling the honeycomb plate, and SEM needed 38 elements. Figure 2 displays the FRFs of the honeycomb plate obtained using FEM (Orange line) and obtained using SEM (blue line), it is possible to observe the proximity in the results in obtaining the natural frequencies of the structure. However, it is observed that when it comes to high modal density structures, such as the honeycomb plate, the SEM is a more efficient and accurate method compared to FEM. This can be because SEM uses high-order polynomial approximations, which results in a more compact and accurate solution.

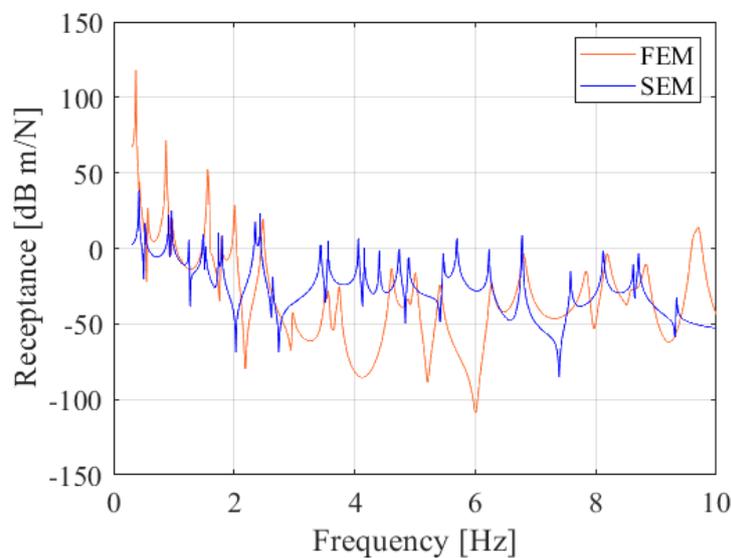


Figure 2: Comparison between resonance frequencies of the honeycomb plate identified using FEM and SEM.

## 4 CONCLUSION

Both the finite element method (FEM) and the spectral element method (SEM) are tried and tested. Although they share similarities in their general approach and mathematical formulation, their implementation and solution accuracy differ. SEM has been demonstrated to be a better method for FEM in terms of efficiency and accuracy for high modal density structures, like the honeycomb plate. There can be many reasons for this, such as that SEM incorporates high-order polynomial approximations, leading to a more precise and concise solution. Furthermore, SEM leverages the spectral properties of the mesh to provide a more efficient representation of the solution. Structures. In conclusion, while both FEM and SEM have their strengths and weaknesses, SEM is better suited for high modal density structures such as honeycomb plates due to its higher accuracy and efficiency. This makes it an ideal tool for engineers and researchers needing a reliable method for modelling these types of structures.

## 5 REFERENCES

- [1] A. Boudjemai, R. Amri, A. Mankour, H. Salem, M.H. Bouanane, D. Boutchicha, 2011, "Modal analysis and testing of hexagonal honeycomb plates used for satellite structural design", *Materials & Design*, 35, 2012, Pages 266- 275.
- [2] A. Singh, T. Mukhopadhyay, S. Adhikari, B. Bhattacharya, "Voltage-dependent modulation of elastic moduli in lattice metamaterials: Emergence of a programmable state-transition capability", *International Journal of Solids and Structures*, Volumes 208–209, 2021, Pages 31-48.

- [3] Mukherjee , S ., Scarpa , F ., & Gopalakrishnan , S . ( 2016 ). Phononic band gap design in honeycomb lattice with combinations of auxetic and conventional University of Bristol - Explore Bristol Research auxetic and conventional core. (2016). 25.
- [4] Lee U., 2004. Spectral Element Method in Structural Dynamics, Binha University Press.
- [5] Leo D. J., 2007. Engineering analysis of smart material systems. John Wiley and Sons, New Jersey, p. 1–7.
- [6] Moura B. B., Machado M. R., Borges M. C. R., 2020. "Vibration and wave propagation control in a smart metamaterial beam with periodic arrays of shunted piezoelectric patches". 49th International Congress and Exposition on Noise Control Engineering, INTER-NOISE, Seoul, Korea.
- [7] Boudjemai, A., Amri, R., Mankour, A., Salem, H., Bouanane, M. H., & Boutchicha, D. (2012). Modal analysis and testing of hexagonal honeycomb plates used for satellite structural design. *Materials and Design*, 35, 266–275. <https://doi.org/10.1016/j.matdes.2011.09.012>

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