



# The influence of proper orthogonal decomposition and smooth orthogonal decomposition on rotor unbalance fault classification

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*Abstract:* This work aims to classify different unbalance intensities located at the tip of gas turbine blades. The objective is to verify how the inclusion of proper (POD) and smooth orthogonal decomposition (SOD), as features of the machine learning problem, impact the accuracy of classifying the intensity of unbalance presented at the gas turbine. First, a full gas turbine model with previously identified bearings is set as a reference for test and training data set data generation. These rotordynamic models provided features related to displacement and velocity measured in both bearing nodes, which compose the base for the classification model under different unbalance conditions. Then, different groups of features were proposed to compare the influence of POD and SOD in classification results, which presented a gain of 3% compared with the models without these pair of features. Results demonstrated that both orthogonal decompositions provide different aspects of the physical phenomena, thus improving the classification of rotordynamic severity unbalance.

**Keywords:** rotordynamics, fault classification, POD, SOD

## INTRODUCTION

The use of machine vibration data as a source for fault detection and diagnosis is widely used in the industry and has become a fundamental tool for good engineering practices (Bently and Hatch, 2002). However, although the data is easily accessible, fault diagnosis challenges still exist, especially on large machines (Nembhard, Sinha, and Yunusa-Kaltungo, 2014). In this sense, the diagnoses of faults that could be catastrophic or cause great unavailability must be obtained in advance so that there is due treatment through planned interventions or operational measures to mitigate severe damage or loss of revenue arising from unavailability.

Today, a large part of the industry adopts predictive maintenance practices in its work routines, and based on technological innovations, which allow the continuous monitoring of several variables, the technician in charge can infer the condition of the equipment under analysis (condition based maintenance - CBM). CBM is divided into three stages: detection, diagnosis, and prediction of the fault event (Randall, 2011).

In this sense, computational advances allow simulations of physical models for analyzes of interest to the industry, with the advantage that their results maintain a link with the physical meaning of the phenomenon studied (Zhong, Xue, and Ding, 2018). However, complex models' computational cost still conditions the analyses to simplifications that limit their conclusions, except for specific conditions and regimes of interest. At the same time, advances in data mining techniques are already present in engineering, aggregating information from physical phenomena or enabling the construction of data-driven models. In this way, physical models have continuously been developed under data guidance, such as coefficients determined empirically by comparing the model response with available data (Parish and Duraisamy, 2016).

These advances have also raised expectations for the industry regarding the possibility of creating a digital twin of a given device, allowing fast and highly accurate decision-making. The digital twin permeates the advances of recent research in data-driven, using computational models, continually updated with new data from the physical system to replicate the performance and degradation of equipment (Ritto and Rochinha, 2020). Thus, the digital twin presents itself as an integrator of the approaches of physical models and models based on data by creating a virtual tool for prediction, which is updated over time (Wagg, et al. 2020).

Under this context, a virtual tool for prediction depends on data generated for machine learning algorithms, and the classification results are strictly related to feature selection. The use of orthogonal decompositions, proper (POD) and smooth orthogonal decomposition (SOD), demonstrated relevance to oceanographic data sets, as discussed by Kuehl et al. (2014) and in rotordynamic faults by Pereira and Ritto (2021).

The present work discusses the importance of adding POD and SOD as features to enhance data-driven models that classify gas turbine conditions by comparing these models of gas turbine unbalance under different severity classes.

## ROTOR DYNAMIC MODEL

A rotordynamic model of an industrial gas turbine was used to generate the data sets, which provided the basis for the following steps to obtain the data-driven model. These rotordynamic model aims to describe the lateral vibration of an industrial gas turbine, supported by two journal bearings. Moreover, it considers a linear relationship between load and deformation in the bearings; additionally, there is a cross-coupling dynamic stiffness and damping effect. The rotordynamic model used in this work describes a system composed of disks, a shaft, and bearings. Through the numerical method of Rayleigh-Ritz, the discretization of the continuous model in finite elements is done. The rotor presents the geometric characteristics listed in table 1 and the following material properties: Modulus of elasticity of 211 (MPa), Shear Modulus of Elasticity of 81.2 (GPa), and Density of 7810 ( $kg/m^3$ ).

Table 1: Geometric properties of the model

Properties	Value	Description
$\Omega$	3600 rpm	Rotor speed
L	8.364 m	Rotor length
$D_{min}$	0.420 m	Rotor minimum diameter
$D_{max}$	1.733 m	Rotor maximum diameter
$D_{disc_{max}}$	1.680 m	Maximum diameter of rotor/blade ensemble
$D_{disc_{min}}$	1.341 m	Minimum diameter of rotor/blade ensemble

The finite element model adopted for the rotodynamic description of the system with out-of-balance forces and moments has the terms of the gyroscopic effect described explicitly, through the matrix  $\mathbf{G}$ , the equilibrium position of rotor center be  $\dot{\mathbf{q}}_e$  and can be listed in equation 1, where  $\mathbf{M}$  are the mass system matrix,  $\mathbf{q}$  the system degrees of freedom (DOF),  $\mathbf{C}$  the damping system matrix and  $\mathbf{K}$  the system stiffness matrix.

$$\mathbf{M}\ddot{\mathbf{q}} + \Omega\mathbf{G}\dot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \Omega^2\mathbf{M}\dot{\mathbf{q}}_e - \Omega\mathbf{G}\dot{\mathbf{q}}_e \quad (1)$$

The above equation 1 can be described with unbalance masses as external forces, represented by an offset and by a tilt  $\beta$  as shown in Eqn. 2.

$$\mathbf{M}\ddot{\mathbf{q}} + \Omega\mathbf{G}\dot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \Re(\Omega^2 \begin{Bmatrix} m\epsilon e^{j\delta} \\ -im\epsilon e^{j\delta} \\ iI_{dk}\beta e^{j\gamma} \\ I_{dk}\beta e^{j\gamma} \end{Bmatrix} e^{i\Omega t}) = \Re(\Omega^2 \mathbf{b}_{0e} e^{i\Omega t}) \quad (2)$$

The under-analysis rotor can be seen in figure 1, where the asymmetric aspect mainly represents the compression and expansion of the mixture gas flow. As this work aims to represent a detailed turbine, the model description used as a guide the API 684 standard, and from these definitions, the convergence of the model was analyzed through the first forward natural frequency, which established a 62-node model as a reasonable compromise between increasing computational cost and convergence.

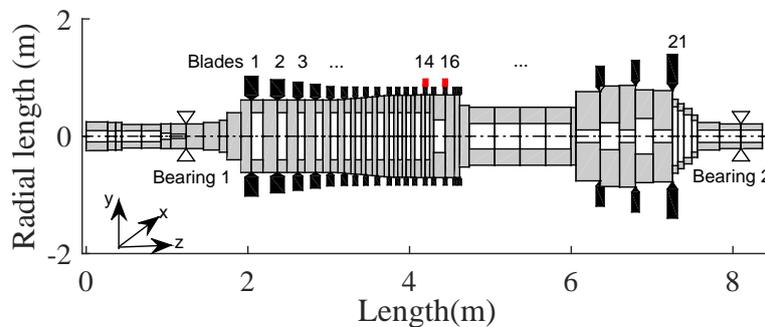


Figure 1: Gas turbine model - 62 nodes.

Once the rotor was established, the bearing parameters were obtained through a inverse problem, with the Approximate Bayesian Computation (ABC) strategy, where bode results were set as the parameter value for convergence and bode field

data from a industrial gas turbine where used as input. A Bayesian approach consist of generating massive amount of data through a Monte Carlo technique, where it can or can not evaluate a likelihood function – a density function of probability described through a parameter  $\theta$ , with observation set of  $x$ . As already approached in industrial applications by other authors for parameter estimation of bit-rock interaction model of a drill-string.

## CLASSIFICATION DATA-DRIVEN MODEL

Most of the current studies involving fault classification use data-oriented methodologies, given the amount of available information, where companies have increasingly more extensive and more organized databases, which allowed the advancement of techniques for their treatment. The so-called machine learning (ML) aims to find a rule that explains the behavior of data from a limited sample - The learning problem. The classification model of gas turbine unbalanced severity condition was built using data generated from the model presented in Eq.(1), where the severity is associated with the unbalance mass allocated at the tip of the blades positioned in the center of the rotor – as seen in Fig.(1).

### Data generation

Obtaining ML models requires a large amount of data, as these will compose the data set to train and validate the classification model. The different class label definition was set as a function of the unbalance mass positioned at the tip of the gas turbine blades, as can be seen in figure 1, where the red rectangles represent the unbalance mass. Where class 1 corresponds to light unbalance caused by a mass of  $m_{d1} = 300(g)$ , and class 2 to a heavy unbalance caused by a mass of  $m_{d2} = 500(g)$ . The data generation also considered an uncertainty on the bearing parameters around an already identified bearing and quantified by a Monte Carlo simulation.

The algorithm of data generation is presented in Algorithm (1), and the generated data set can be seen in Fig.(2), where it can be observed that class 1 and 2 are more overlapped in bearing 1 than in bearing 2 – behavior mainly influenced by the rotor asymmetry (see Fig.(1)) – rotordynamic simulations employed the rotordynamic open source software - ROSS (Timbó, et al. 2020).

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#### Algorithm 1 Data generation

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- 1: Given: class 1 as light unbalance mass of 300(g) and class 2 as high unbalance mass of 500(g)
  - 2: Given:  $nMC$  as the Monte Carlo observations of Eq.1, considering a *Uniform* distribution of  $\mathbf{K}_b$  and  $\mathbf{C}_b$  parameters – previously identified
  - 3: Set:  $\mathbf{P}_{ref}$  as an array of the reference parameters,  $\mathbf{K}_b$  and  $\mathbf{C}_b$
  - 4: Set:  $\mathbf{P}_{min}$  as a lower bound of  $\mathbf{P}_{ref}$ , where  $\mathbf{P}_{min} = lb \times \mathbf{P}_{ref}$
  - 5: Set:  $\mathbf{P}_{max}$  as an upper bound of  $\mathbf{P}_{ref}$ , where  $\mathbf{P}_{max} = ub \times \mathbf{P}_{ref}$
  - 6: **Step 1:** for  $i = 1$  to  $nMC$  do
  - 7: Generate  $\mathbf{P}$  as a *Uniform*( $\mathbf{P}_{min}, \mathbf{P}_{max}$ )
  - 8: Get  $\mathbf{q}_b(\mathbf{P})$  in X and Y directions, for both bearings –  $\mathbf{q}_b$  are the bearing DOF of eq. 1
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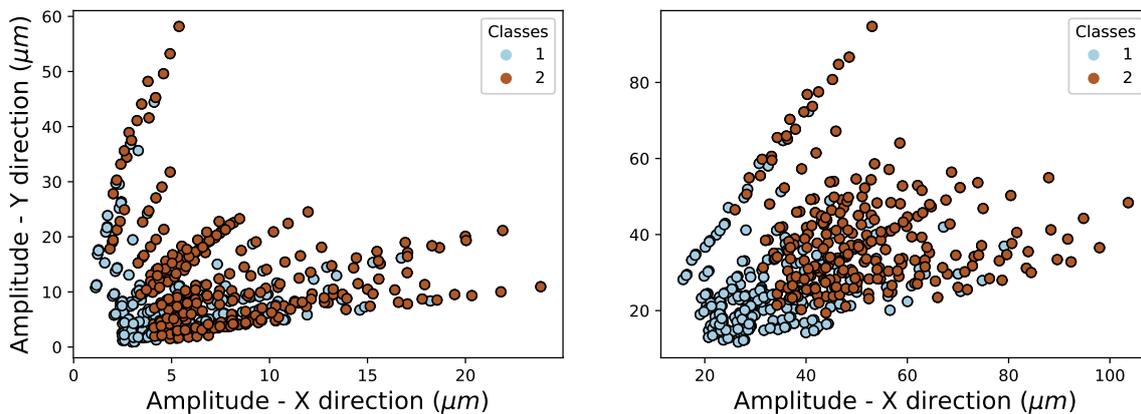


Figure 2: Generated data – First characteristic frequency (60(Hz)) at both X and Y directions, for both bearings 1 (a) and 2 (b).

### POD and SOD

The main purpose of POD is to extract, from a set of variables, a low dimensional representation (of uncorrelated variables), while retaining as much as possible the variation present in the data set. The smooth orthogonal decomposition

(SOD) has a slightly different characteristic since it considers the process of interest and its derivative.

Following the development of Bellizzi and Sampaio (2012), let us define the smooth decomposition of the random field  $\{\mathbf{X}(t,x), (t,x) \in R \times \mathcal{D}_x\}$ , where  $t$  and  $x$  are the independent variables related to time and location,  $R$  is the field of real numbers, and  $\mathcal{D}_x$  is the  $x$ -domain.

Let  $C_{\mathbf{X}}(0,x,x')$  and  $C_{\dot{\mathbf{X}}}(0,x,x')$  be the covariance functions of  $\mathbf{X}(t,x)$  and  $\dot{\mathbf{X}}(t,x)$ . The random field  $\mathbf{X}(t,x)$  might be written as an infinite sum

$$\mathbf{X}(t,x) = \sum_{i=1}^{\infty} \zeta_i(t) \phi_i(x). \quad (3)$$

Some particularities of the decompositions are related to the application in the present work, which uses discrete points for measurements and from these selects the desired features –  $Y$  and  $X$  radial displacements. Each feature  $x^{(i)}$  is obtained in a time window  $w$  of each data generation set, in which  $i = \text{bearing location, displacement direction}$ . Given a centralized feature observed in each window  $\mathbf{X}$ , the estimation for the covariance matrix is given by equation 4 and the covariance matrix for the derivation of data in time is give by equation 5

$$\mathbf{C}_{\mathbf{X}} = \frac{1}{n-1} \mathbf{X}\mathbf{X}^T. \quad (4)$$

$$\mathbf{C}_{\dot{\mathbf{X}}} = \frac{1}{n-1} \dot{\mathbf{X}}\dot{\mathbf{X}}^T. \quad (5)$$

Applying both objectives (maximize variance and smoothness), we can write the following optimization problem as presented by equation 6

$$\phi^* = \operatorname{argmax}_{\phi} \left\{ \frac{\|\mathbf{X}^T \phi\|^2}{\|\dot{\mathbf{X}}^T \phi\|^2} \right\}. \quad (6)$$

The eigenvalue problem can be written as:

$$\mathbf{C}_{\mathbf{X}} \phi_i = v_i \mathbf{C}_{\dot{\mathbf{X}}} \phi_i \quad i = 1, \dots, m, \quad (7)$$

where, as in the continuous case,  $v_i$  are the smooth values and  $\phi_i$  are the smooth modes. The proper orthogonal values  $\lambda_i$ , and modes  $\psi_i$  are obtained similarly if we consider the maximization of  $\|\mathbf{X}^T \phi\|^2 / \|\phi\|^2$ . This yields the following discretized eigenvalue problem,

$$\mathbf{C}_{\mathbf{X}} \psi_i = \lambda_i \psi_i \quad i = 1, \dots, m. \quad (8)$$

To exemplify the resulted features obtained through POD and SOD, the time response of the rotordynamic model were analysed. That can be seen in Fig.(3) presents the time response measured in both bearings, and both directions,  $X$  and  $Y$ , and also demonstrate how POD and SOD modes behaves along their components in Fig.(4a) and Fig.(4b).

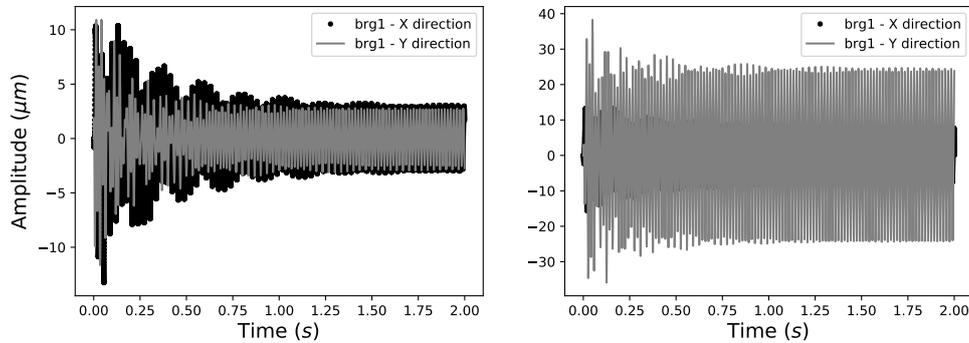


Figure 3: Time response due to light unbalance mass in both bearings.

As the main propose is to classify during steady state regime, the transient state portion was removed from the analysis. The modal components (MC) of each proper orthogonal mode (POM) and for each smooth orthogonal mode (SOM) change as the energy presented in the mode decreases – the way that modal components relate with one another reflects the dynamic response measured in both bearings, and for that reason can improve classification results.

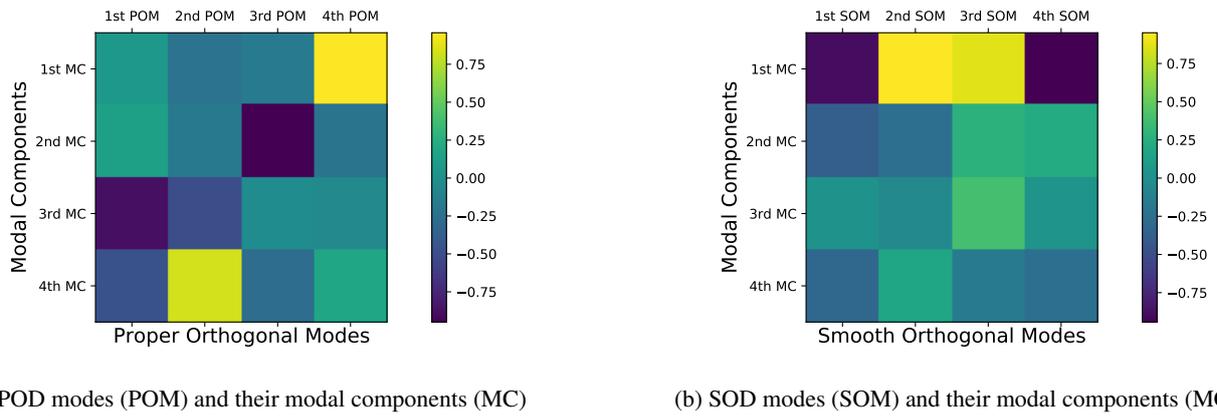


Figure 4: POD and SOD modes for a light unbalance case.

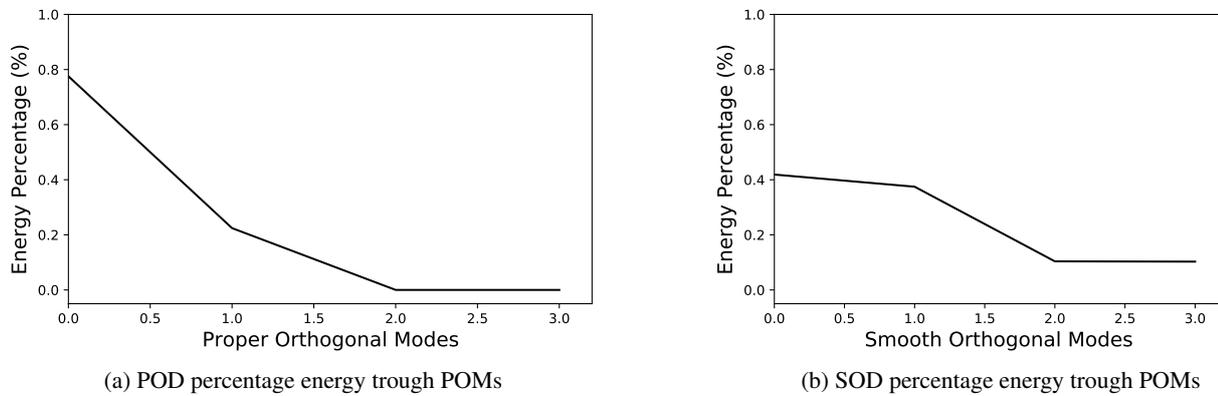


Figure 5: Energy percentage of POD and SOD for a light unbalance case.

## Feature selection

The essence of ML classification models is the selection of features that represent the phenomena desired to classify, and the validity of a feature subset must be related to a measure of the classification result. The features of the unbalance classification model extract information from radial displacement and velocity of both bearings in  $Y$  and  $X$  directions.

The data were defined in the time domain and excluded the transient period since the objective is to determine if an already in base load gas turbine is out-of-balance and how intense is this unbalance condition, and disregards transient effects during machine start-up. The features can be divided into frequency, statistical moments, and orthogonal decomposition.

The first group is represented by the Fast Fourier Transform (FFT) and power spectral density (PSD), both obtained in the first characteristic frequency ( $60 \text{ (Hz)}$ ) of rotor velocity ( $\Omega$ ), as these are often used to characterize rotordynamic machine's fault – unbalance, misalignment, looseness, and others (Bently and Hatch, 2002). The second group is composed of the first moment, second central moment, the moment coefficient of skewness, and kurtosis, all evaluated in the time domain. The third group aims to extract information on data directions of higher variance. For this objective, a proper orthogonal decomposition (POD) and a smooth orthogonal decomposition (SOD) were used as features, composed of sorted eigenvalues of POD and SOD eigenvalue problems.

## RESULTS

The first classification model considered a subset of features formed by the first group of features, represented by FFT and PSD of radial displacement and velocity of both bearings, at  $Y$  and  $X$  directions – a total of sixteen features, FFT and PSD of four displacement frequency response, at the first characteristic frequency ( $60 \text{ (Hz)}$ ), and four velocity frequency response, also at ( $60 \text{ (Hz)}$ ). The second model considered the first and second group of features; the third model considered the first, second, and third group of features.

Table 2 presents the accuracy results of a random forest decision tree algorithm, for the three feature group approaches,

where it can be seen the diminish of deviation in 3% comparing models 2 and 1, and an increase of 3% in the mean value of accuracy when model 3 and 1 are compared.

Table 2: Accuracy results for a random decision tree algorithm, evaluated in three groups of features – as described above.

Data driven models	Feature group	Accuracy
Model 1	1	$0.77 \pm 0.15$
Model 2	1 and 2	$0.77 \pm 0.12$
Model 3	1, 2 and 3	$0.8 \pm 0.15$

## CONCLUSIONS

It can be verified that the inclusion of the eigenvalues contained in the  $i$ -th mode of the POD decomposition and the smoothed SOD decomposition improved the performance of the models, even in the condition in which there is a pronounced separation of the classes in the displacement amplitudes of the bearing 2, as shown in Fig.(2) – which can, by itself, facilitate the classification task, and make improvement more challenging. Cases with other configurations of the rotodynamic model and unbalanced conditions can be investigated, as well as other ways to quantify POD and SOD decomposition features.

## REFERENCES

- Bently, D. E. and Hatch, C. T., 2002, "Fundamentals of Rotation Machinery Diagnostics", Bently Pressurized Bearing Company, Minden, NV, USA.
- Bellizzi, S. and Sampaio, R., 2012, "Reduced models based on smooth decomposition for random mechanical systems", International Review of Mechanical Engineering, Vol. 6, pp. 74-85.
- Cook, R. D., Malkus, D. S. and Plesha, M. E., 1989, "Concepts and applications of finite element analysis", John Wiley and Sons, Madison, WI, USA.
- Kuehl, J. J., DiMarco, S. F., Spencer, L. J. and Guinasso Jr, N. L., 2014, "Application of the smooth orthogonal decomposition to oceanographic data set", Geophys. Res. Lett, Vol.41.
- Lalanne, M. and Ferraris, G., 1989, "Rotordynamics prediction in Engineering", John Wiley and Sons, New York, NY, USA.
- Nembhard, A. D., Sinha, J. K. and Yunusa-Kaltungo, A., 2014, "Development of a generic rotating machinery fault diagnosis approach insensitive to machine speed and support type", Journal of Sound and Vibration, Vol.337, pp. 321-341.
- Parish, E. J. and Duraisamy, K., 2016, "A paradigm for data-driven predictive modeling using field inversion and machine learning", Journal of Computational Physics, Vol.305, pp.758-774.
- Pereira, L.V. and Ritto, T.G., 2021, "Proper orthogonal decomposition and smooth orthogonal decomposition approaches for pattern recognition: Application to a gas turbine rub-impact fault", Journal of Sound and Vibration, Vol.511.
- Randall, R. B., 2011, "Vibration-based condition monitoring: industrial, aerospace and automotive applications", John Wiley and Sons, Chichester, West Sussex, UK.
- Ritto, T.G. and Rochinha, F.A., 2020, "Digital twin, physics-based model, and machine learning applied to damage detection in structures", from the arXiv database.
- R. Timbó, R. Martins, G. Bachmann, F. Rangel, J. Mota, J. Valério and T.G. Ritto, 2020, "Ross - rotordynamic open source software", J. Open Sour. Softw., Vol.5, pp. 237-268.
- Wagg, D. J., Worden, K., Barthorpe, R. J. and Gardner, P., 2020, "Digital Twins: State-of-the-Art and Future Directions for Modeling and Simulation in Engineering Dynamics Applications", ASCE-ASME J Risk and Uncert in Engrg Sys Part B Mech Engrg, Vol.6.
- Zhong, M., Xue T. and Ding, S. X., 2018, "A survey on model-based fault diagnosis for linear discrete time-varying systems", Neurocomputing, Vol.306, pp. 51-60.

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