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# Fatigue Analysis in a Drillstring under Torsional Vibrations

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*Abstract: In oil exploration and production, drilling is one of the most important processes. Undesirable vibrations, intrinsic to the process, can lead to fatigue failure, resulting in loss of efficiency due to downtime and, consequently, loss of productivity and high costs. In this work we present an analysis of the fatigue life of a drillstring subject to torsional vibrations and the influence of operating conditions on the occurrence of Stick-Slip. The mathematical model that describes this movement is presented, the stresses are calculated to determine the fatigue life. The results are presented indicating that the weight on the bit has a great influence on the number of cycles in relation to the angular velocity imposed during the operation.*

***Keywords: Drill string dynamics, Torsional vibration, Bit-rock interaction, Fatigue, Stick-Slip***

## INTRODUCTION

In the exploration and production sector in the oil industry, drilling is one of the most important and complex activities, in addition to presenting a high cost associated with this process. Undesirable vibrations occur during drilling in the interaction between the rock formation and the bit, these vibrations being of three types: axial, torsional and lateral. In the present work we deal with torsional vibration and the *Stick-Slip* phenomenon, characterized by a severe state of this type of vibration and leading to low drilling efficiency, increased operating costs and time spent in the process.

Since one of the greatest costs is associated with downtime due to fatigue failures [1, 2, 3] this work deals with fatigue analysis based on operating conditions, in particular, the speed imposed by the rotary table and the weight on the bit ($WOB$). We start presenting the torsional model proposed in [4] (see too [5]) and then calculate the stresses due to the difference in speed resulting from the rock/bit interaction process to determine the number of cycles until fatigue failure and thus, consequently, the time of operation.

## MATHEMATICAL MODEL

### Torcional Model

The model adopted to describe the torsion dynamics proposed in [6] describes the drill string as a simple torsion pendulum composed of a spring of stiffness $k_t$, inertia $J$ [7] and damping $c_t$ (as show in Figure 1). The constant rotation $\Omega$ is imposed by the *top drive*, where $\theta$ is the angle of the bit.

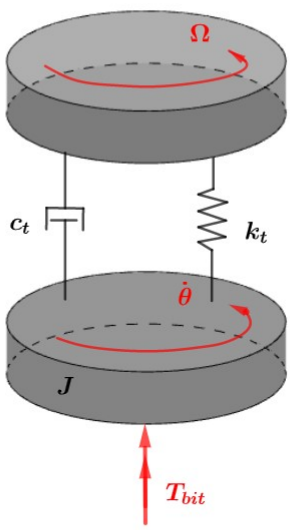


**Figure 1 – Torsional Pendulum Diagram**

The governing equations of the systen shown in Figure 1 are expressed by

$$J\ddot{\theta}_{bit}(t) + c_t\dot{\theta}_{bit}(t) + k_t\theta_{bit}(t) = c_t\Omega + k_t\Omega t + T_{bit}, \tag{1}$$

where $T_{bit}$ is the torque of the the drill bit [4] given by

$$T_{bit} = -b_0\left(tanh(b_1\dot{\theta}_{bit}) + \frac{b_2\dot{\theta}_{bit}}{1+b_3\theta_{bit}^2}\right), \tag{2}$$

being $b_0$, $b_1$, $b_2$ and $b_3$ positive non-linear constants that depend on the weight on bit and the rock properties and characteristics of the bit.

## Fatigue Analysis

For the fatigue analysis, the steady state is considered in which the stress amplitude is constant for the problem. Goodman's criterion is adopted, in which the limit of fatigue resistance for infinite life $S_e$ is replaced by $S_f$ for finite life, once $S_f > S_e$ [8]. In this way, rearranging we have

$$S_f = \frac{\sigma_a}{1-(\sigma_m/S_{ut})}, \tag{3}$$

where $\sigma_a$ is the stress amplitude, $\sigma_m$ is the average stress and $S_{ut}$ the tensile strength limit, such that

$$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} \quad \text{and} \quad \sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2}. \tag{4}$$

The variation of stress, in the shear case (making $\sigma = \tau$) gives $\sigma_{max}$ and $\sigma_{min}$ for the calculations of $\sigma_m$ and $\sigma_a$. The shear stress $\tau$ is a function of the difference in the angular position between the ends, the shear modulus $G$, the length $L$ of the column and the drill pipe outer radius $r_{dp}^{ext}$, given by

$$\tau = G\frac{(\Omega t - \theta_{bit})}{L}r_{dp}^{ext}. \tag{5}$$

To determine the number of cycles to failure, the Wohler model according to [9] is used. This model is given by

$$S_f = aN^b \tag{6}$$

such that

$$N = \left(\frac{S_f}{a}\right)^{1/b}, \tag{7}$$

where $a$ and $b$ are constants [8] given by

$$a = \frac{(fS_{ut})^2}{S_e} \quad \text{and} \quad b = -\frac{1}{3}log\left(\frac{fS_{ut}}{S_e}\right) \tag{8}$$

with $f$ being the fatigue strength fraction of $S_{ut}$.

The endurance limit for infinite life $S_e$ of the machine element is given by

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S_e', \tag{9}$$

being $S_e'$ the material fatigue strength limit and $k_a ... k_f$ the modifying factors whose reference values and calculations can be found in [8] and defined as follows.

$k_a$: surface condition modification factor
$k_b$: size modification factor
$k_c$: load modification factor
$k_d$: temperature modification factor
$k_e$: reliability factor
$k_f$: modification factor for varied effects

Finally, the simplified flowchart (Figure 2) illustrates the procedure for calculating fatigue life.

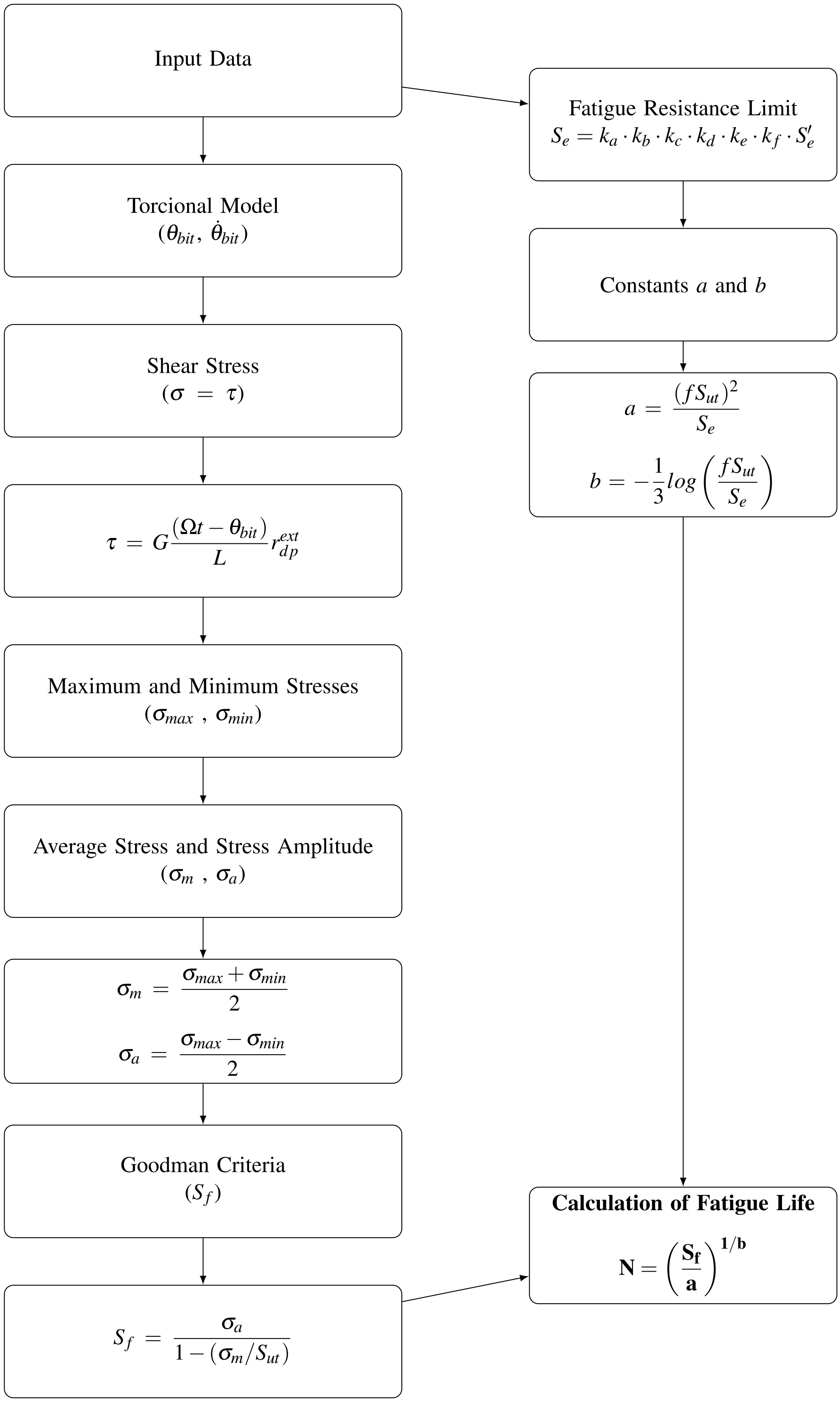


**Figure 2 – Flowchart for Calculating Fatigue Life**

## NUMERICAL RESULTS

The values associated with the mechanical properties, dimensions and constants used in the presented model were obtained from [4] and [5] and are presented in Table 1 and the factors used for the fatigue calculations are presented in Table 2. Note that for the stress concentration factor we used the unit value since such parameters will be treated again in future works involving stochastic analysis. Other parameters based on experimental values can be found, for example in [9, 10].

**Table 1 – Data used in the simulation: Physical and Geometric Parameters**

| | | |
|---|---|---|
| $L$ | 4733.6m | *Pipe Lenght* |
| $r_i$ | 0.0595m | *Inside Radius of the Drill Pipe* |
| $r_e$ | 0.07m | *Outside Radius of the Drill Pipe* |
| $E$ | $220.00GPa$ | *Modulus of Elasticity* |
| $S_{ut}$ | $690.0GPa$ | *Tensile Strength* |
| $S_y$ | $580.0GPa$ | *Yield Strength* |
| $\xi$ | 0.25 | *Damping Rate* |
| $\rho$ | $7800.00kg/m^3$ | *Density* |
| $\nu$ | 0.29 | *Poisson's Ratio* |
| $b_0$ | 5671.0 | *Model Parameter* |
| $b_1$ | 0.4775 | *Model Parameter* |
| $b_2$ | 8.8754 | *Model Parameter* |
| $b_3$ | 4.5595 | *Model Parameter* |

**Table 2 – Data used in the simulation: Parameters for Calculating Fatigue Life**

| | | |
|---|---|---|
| | **Surface Condition Modification Factor** | |
| Cold Lamination | $a = 4.51$ and $b = -0.265$, $k_a = aS_{ut}^b$ | $k_a = 0.797$ |
| | **Size Modification Factor** | |
| $d = 140.0mm$ | $51.0 <= d <= 254.0mm$, $k_b = 1.51d^{-0.157}$ | $k_b = 0.695$ |
| | **Load Modification Factor** | |
| | Loading Type: Torsional Stress | $k_c = 0.59$ |
| | **Temperature Modification Factor** | |
| | $T < 300^\circ$ C | $k_d = 1.0$ |
| | **Reliability Factor** | |
| | Reliability: 0.95 | $k_e = 0.868$ |
| | **Modification Factor for Varied Effect** | |
| | Concentration Stress and Others | $k_f = 1.0$ |
| | **Material Fatigue Strength Limit** | |
| | $S_{ut} = 690.0MPa \leq 1400.0MPa$ , $S'_e = 0.5S_{ut}$ | $S'_e = 345.0MPa$ |
| | **Fatigue Strength Limit** | |
| | $97.9MPa$ | |

Figure 3 presents the angular velocity and the shear stresses at $16.0rad/s$ for different values of $WOB$, characterizing a normal operating scenario where the velocity converges to $\Omega$, which implies the convergence of the shear stresses with the reduction of its amplitude and the condition of infinite life. Figure 4 presents scenarios at $10.0rad/s$ where the *Stick-Slip* phenomenon takes place due to low rotational speeds associated with increased weight on the bit.

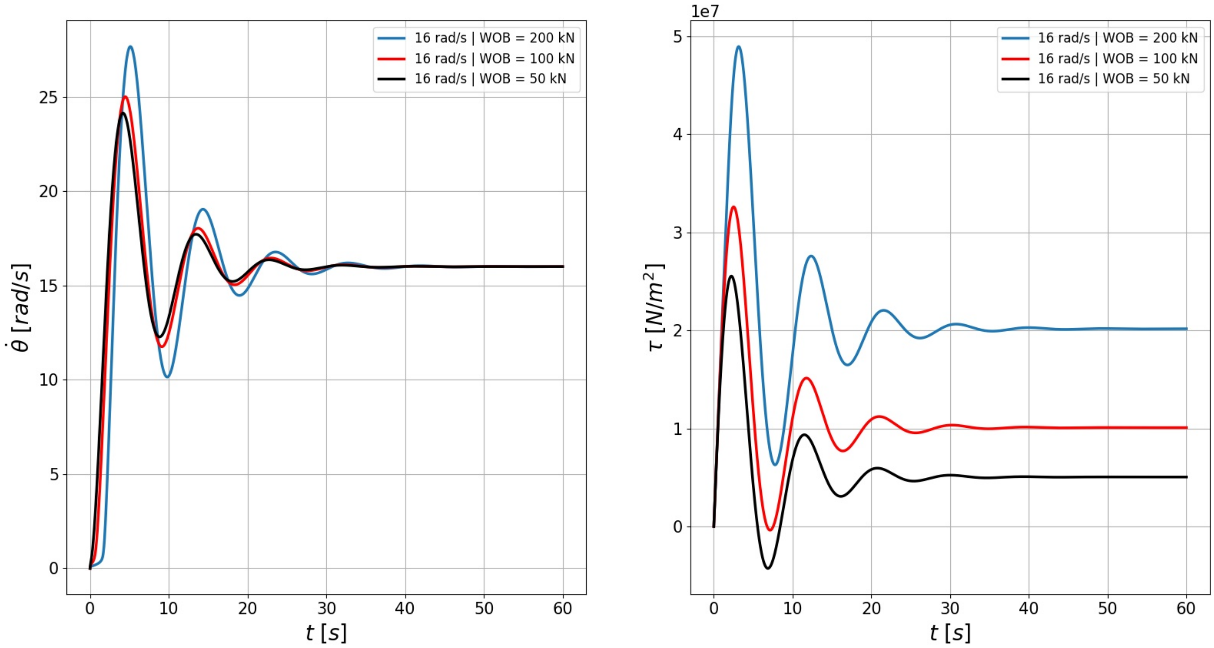


**Figure 3 – Influence of weight on bit without occurrence of *Stick-Slip*** ($\Omega = 16 rad/s$)

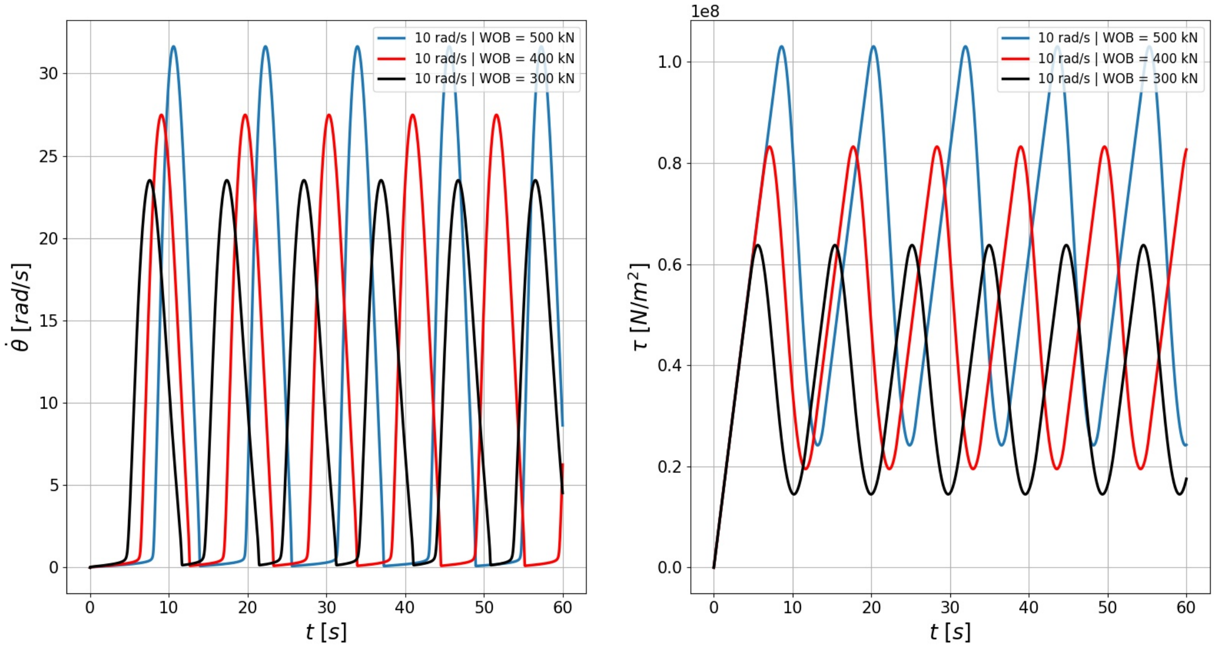


**Figure 4 – Influence of weight on bit with occurrence of *Stick-Slip*** ($\Omega = 10 rad/s$)

Finally, Figure 5 presents a parametric study of fatigue life as a function of imposed velocity and weight on bit *WOB*. Within the finite life domain, ones observes a decrease in fatigue life with either an increase in velocity $\Omega$ or a increase in weight on bit *WOB*. A better visualization of this behavior can be see in Figure 6.

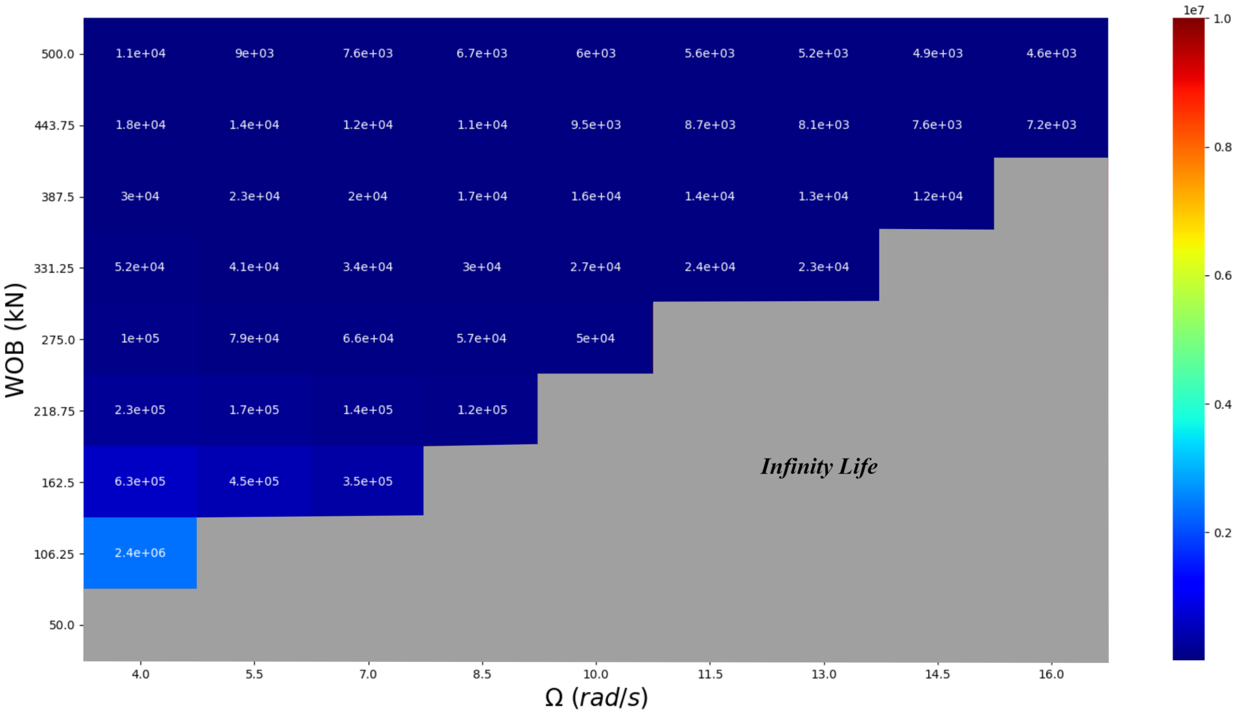


**Figure 5 – Fatigue Life depending on Rotation and *WOB* (in hours)**

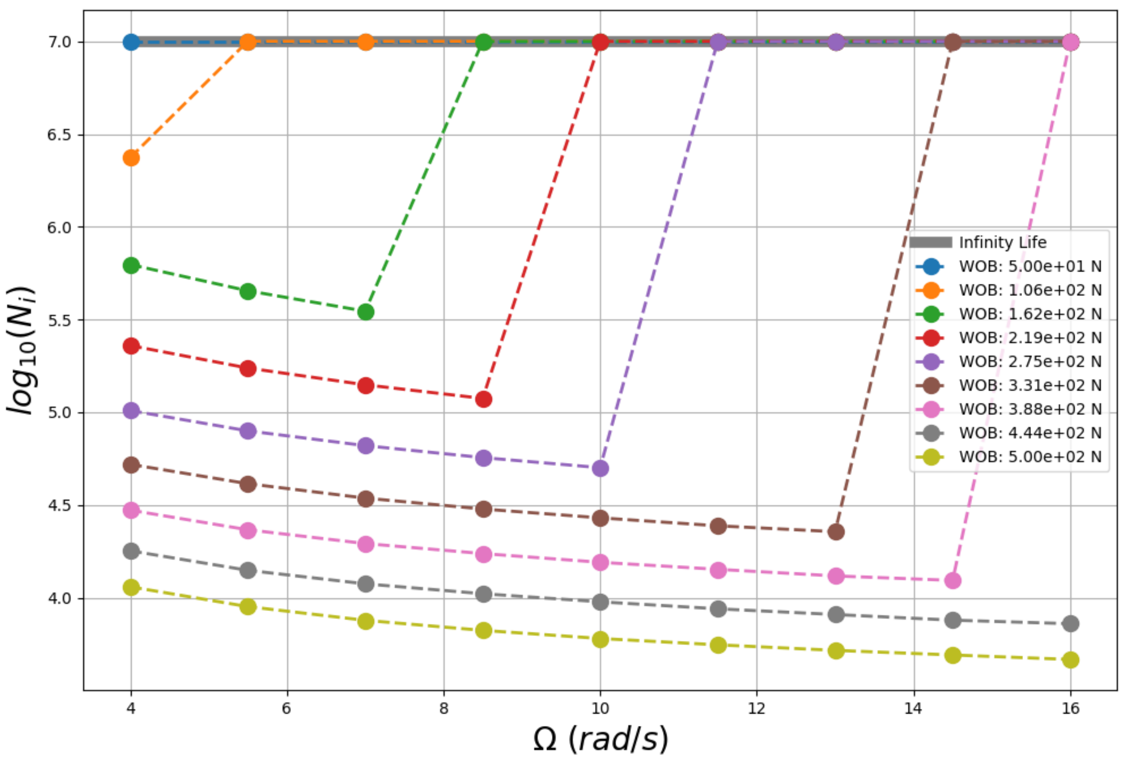


**Figure 6 – Fatigue Life ($N_i$) in logarithmic scale depending on Rotation and *WOB* (in hours)**

## CONCLUSIONS

In this work, the dynamic behavior of a drill string subject to torsional vibrations is analyzed. Through the proposed model and fatigue analysis, the aim is to evaluate how the angular velocity imposed by the top drive and the weight applied to the drill bit influence the occurrence of the *Stick-Slip* phenomenon and, in turn, the life in fatigue. The results indicate that high weights on the drill associated with low angular velocities exert a significant influence on the reduction of fatigue life.

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