Performance Analysis of a Duct With Multiple Internal Helmholtz Resonators

Wanderson Vinicius de O. Monteiro[1,2], Arthur F. Martins[1,2], Ubirajara A. S. Junnyor[1], Edilson D. Nóbrega[1] and José Maria C. Dos Santos[2]

[1]Federal University of Maranhão (UFMA), University City Dom Delgado, 65085-580, São Luís, Maranhão, Brazil
[2] University of Campinas (UNICAMP), 13083-860, Campinas, São Paulo, Brazil

Abstract: Noise control in environments is essential to achieve acoustic comfort. One way to control the noise is the use of mufflers composed of acoustic metamaterials, which are artificial materials with unconventional properties. Acoustic metamaterials arranged periodically have forbidden bands or bandgaps, which are frequency ranges where sound waves do not propagate. Therefore, this work aims to analyze Helmholtz Resonator devices internally allocated in a periodic structure like a duct and evaluate the transmission loss of the sound and the forced responses. The effects of wave propagation in the structure were evaluated using the Finite Element Mmethod (FEM) and the Wave Finite Element (WFE) method , obtaining the dispersion diagram of a cell and the sound pressure level in the periodic structure. Results showed a bandgap in the resonance frequency of the Helmholtz device from the dispersion diagram, consequently, in the same frequency range, the sound wave attenuation occurred in the sound transmission loss and sound pressure level responses.

Keywords: Acoustic metamaterial, Helmhotz Resonator, Wave Finite Element, Transmission Loss, Sound Pressure Level.

INTRODUCTION

Acoustic satisfaction is a dimension of well-being in the environment, *i.e.* a state of contentment with physical environmental conditions. Navai and Veitch (2003) , when compiling several works on acoustics, verifies that people characterize acoustic satisfaction in terms of noises or sounds that irritate and distract. Based on the definition of acoustic satisfaction, it is reasonable to assume that the greater the satisfaction the quieter (less noise) is the environment. Therefore, among the noise attenuation techniques, Helmholtz resonators (HR) stand out.

This device consists of a cavity connected to the system of interest by means of one or more narrow and short ducts. In other words, the large relatively volume cavity is connected to external space by a narrow neck or through an opening (Alster, 1972). This operation is based on the ability to absorb sound waves of a certain frequency, the so-called resonant frequency, making it possible to apply this device in different scenarios such as ventilation and cooling systems, acting in the reduction of noise (Chen *et al.*, 1998) and (Cai and Mark, 2018). On the other hand, HR's require a precise adjustment in their operating conditions in order to obtain better noise attenuation. For this reason, the frequency range in which Helmholtz resonators are effective is relatively narrow (Bedoult *et al.*, 1997).

A Helmholtz resonator is a reactive silencer, thus, attenuation frequency range can be adjusted by changing its geometric characteristics (Munjal, 1987). Several forms of HR are studied to improve its performance and increase its attenuation band, among them we have the extended neck (Zhao *et al.*,2018;WU; GUAN, 2021), union between HR and microperforated panels (Mahesh and Mini, 2021; GAI *et al.*, 2016), multiple Helmholtz resonators (Duan *et al.*, 2021;WU *et al.*, 2019), use of acoustic metasurfaces (Zhu and Assouar, 2019) and acoustic material plate (Yamato, 2018).

In this work, the performance of an acoustic metamaterial formed by a duct with internal Helmholtz resonators was investigated. For this analysis, the Wave Propagation Finite Element (WFE) method was used and the validation was performed using the Finite Element Methods. The results are show as the dispersion diagram, obtained from the Floquet-Bloch theorem, Transmission Loss and Sound Pressure Level (SPL) of the structure.

METHODOLOGY

The Wave Propagation Finite Element (WFE) method was proposed by Mencik (2008). Thus, WFE is an hybrid method that uses a single cell of the periodic structure produced from Finite Elements to make an approach from the propagation of waves.

Wave Propagation Finite Element

As presented by Kinsler *et al.*, (1999), the sound wave propagation equation can be represented according to the Eq. (1).

$$\nabla^2 \cdot \mathbf{p} = \frac{1}{c^2}\frac{\partial^2 \mathbf{p}}{\partial \mathbf{t}^2}, \tag{1}$$

Where ∇ is a differential operator, **p** the 3D acoustic pressure vector, c the sound velocity and t as time. By application Galerkin method as demonstrated by Cook *et al.* (2001), the system become as Eq. (2) form.

$$\mathbf{M_a}\ddot{\mathbf{p}} - \mathbf{K_a}\mathbf{p} = \mathbf{f}, \tag{2}$$

So that $\mathbf{M_a}$ represent the acoustic mass matrix and $\mathbf{K_a}$ acoustic stiffness matrix, **p** is acoustic pressure node vector and **f** represent the acoustic excitation vector. The acoustic dynamic stiffness matrix **D** is defined by Eq. (3)

$$\mathbf{D} = \mathbf{K_a} - \omega^2 \mathbf{M_a}, \tag{3}$$

Where ω is the angular frequency. There is possible to divide the acoustic dynamic stiffness matrix Eq. (3) into inner degrees of freedom $()_i$, left $()_l$ and right $()_r$.

$$\begin{bmatrix} \mathbf{D_{ii}} & \mathbf{D_{il}} & \mathbf{D_{ir}} \\ \mathbf{D_{li}} & \mathbf{D_{ll}} & \mathbf{D_{lr}} \\ \mathbf{D_{ri}} & \mathbf{D_{rl}} & \mathbf{D_{rr}} \end{bmatrix} \begin{Bmatrix} \mathbf{p_i} \\ \mathbf{p_l} \\ \mathbf{p_r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0_i} \\ \mathbf{f_l} \\ \mathbf{f_r} \end{Bmatrix}, \tag{4}$$

The Equation (4) presents the acoustic dynamic stiffness matrix in the simplify form, also called condensed acoustic dynamic stiffness matrix:

$$\begin{bmatrix} \mathbf{D_{ll}} & \mathbf{D_{lr}} \\ \mathbf{D_{rl}} & \mathbf{D_{rr}} \end{bmatrix} \begin{Bmatrix} \mathbf{p_l} \\ \mathbf{p_r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f_l} \\ \mathbf{f_r} \end{Bmatrix}, \tag{5}$$

Where $\mathbf{D_{ll}} = \mathbf{D_{ll}} - \mathbf{D_{li}}\mathbf{D_{ii}^{-1}}\mathbf{D_{il}}$, $\mathbf{D_{rl}} = \mathbf{D_{rl}} - \mathbf{D_{ri}}\mathbf{D_{ii}^{-1}}\mathbf{D_{il}}$, $\mathbf{D_{lr}} = \mathbf{D_{lr}} - \mathbf{D_{li}}\mathbf{D_{ii}^{-1}}\mathbf{D_{ir}}$ e $\mathbf{D_{rr}} = \mathbf{D_{rr}} - \mathbf{D_{ri}}\mathbf{D_{ii}^{-1}}\mathbf{D_{ir}}$.

The Equation (5) can to be organized as transfer matrix, so:

$$\begin{Bmatrix} \mathbf{p}_r \\ -\mathbf{f}_r \end{Bmatrix} = \begin{bmatrix} -\mathbf{D}_{lr}^{-1}\mathbf{D}_{ll} & -\mathbf{D}_{lr}^{-1} \\ \mathbf{D}_{rl} - \mathbf{D}_{rr}\mathbf{D}_{lr}^{-1}\mathbf{D}_{ll} & -\mathbf{D}_{rr}\mathbf{D}_{lr}^{-1} \end{bmatrix} \begin{Bmatrix} \mathbf{p}_l \\ \mathbf{f}_l \end{Bmatrix}. \tag{6}$$

The Equation (7) has been obtained by simplification of Eq. (6).

$$\mathbf{q_r} = \mathbf{T}_{WFE}\mathbf{q_l}, \tag{7}$$

Therefore, $\mathbf{T}_{WFE}$ corresponds to system unitary cell transfer matrix, $\mathbf{q_r}$ and $\mathbf{q_l}$ are unitary cell state vector. Considering consecutive unitary cells m and $m+1$, the continuity of the periodic structure guide to $\mathbf{q_r}^m = \mathbf{q_l}^{m+1}$.

The Equation (8) is obtained by applying the Floquet - Bloch theorem to propagate waves in periodic structures.

$$\mathbf{q_l}^{(m+1)} = e^{\mu}\mathbf{q_l}^{(m)}, \tag{8}$$

where $\mu = -jkL$ corresponds to propagation constant with j as imaginary number, k is the wave number and L cell length.

Replacing Eq. (7) in Eq. (8), it is possible to obtain the problem of eigenvalue (e^{μ}) and eigenvector (q_l), according to Eq. (9).

$$\mathbf{T}_{WFE}\mathbf{q_l}^{(m)} = e^{\mu}\mathbf{q_l}^{(m)}. \tag{9}$$

To avoid transfer matrix mis conditioning problems, Zhong e William (1995) proposes a stiffness matrix as a function of generalized coordinates, in the form:

$$\mathbf{q_l} = \underbrace{\begin{bmatrix} \mathbf{I_n} & 0 \\ -\mathbf{D_{ll}} & -\mathbf{D_{lr}} \end{bmatrix}}_{L} \underbrace{\begin{Bmatrix} \mathbf{p_l} \\ \mathbf{p_r} \end{Bmatrix}}_{w} \quad \text{and} \quad \mathbf{q_r} = \underbrace{\begin{bmatrix} 0 & \mathbf{I_n} \\ \mathbf{D_{rl}} & \mathbf{D_{rr}} \end{bmatrix}}_{N} \underbrace{\begin{Bmatrix} \mathbf{p_l} \\ \mathbf{p_r} \end{Bmatrix}}_{w}, \tag{10}$$

Where $\mathbf{I_n}$ represents an identity matrix of the order of degrees of freedom on the left (or right) side of the structure. replacing the Eq. (10) in Eq. (9), to obtain:

$$e^{\mu}\mathbf{L}\mathbf{w} = \mathbf{N}\mathbf{w} \tag{11}$$

where e^{μ} are the eigenvalues and $\mathbf{Lw}$ are the eigenvectors of Eq. (11). According to Silva *et al.* (2014) the state vectors $\mathbf{q}^m$ can be expressed by:

$$q^{(m)} = \sum_j \Phi_j Q_j{}^{(m+1)} = \sum_j \Phi_j e^{(-ik_jL)} Q_j{}^{(m)}, \tag{12}$$

where $\Phi_j = Lw_j$, $Q_j{}^{(m+1)}$ and $Q_j{}^{(m)}$ are the wave amplitude vectors of unit cells $m+1$ and m, respectively. From Eq. (12), we obtain the sound pressure level (SPL) at the end of the structure, which relates the sound pressure at the muffler outlet to the reference pressure.

The sound transmission loss is one of the main parameters to measure the performance of a muffler, and shows the relationship between transmitted and incident powers in the system (Munjal, 1987), Where a_t corresponds to the sound power transmission coefficient.

$$TL = 20\log\left(\frac{1}{a_t}\right) \tag{13}$$

The Fig. 1 shows a unit cell of the studied acoustic metamaterial, which consists of a duct with six internal Helmholtz Resonators. The Fig. 2 shows in more detail the geometric features of the HR in relation to the cell.

(a) **Front view.** (b) **Isometric view.**

Figure 1 – Structure of the duct with multiple internal Helmholtz Resonators.

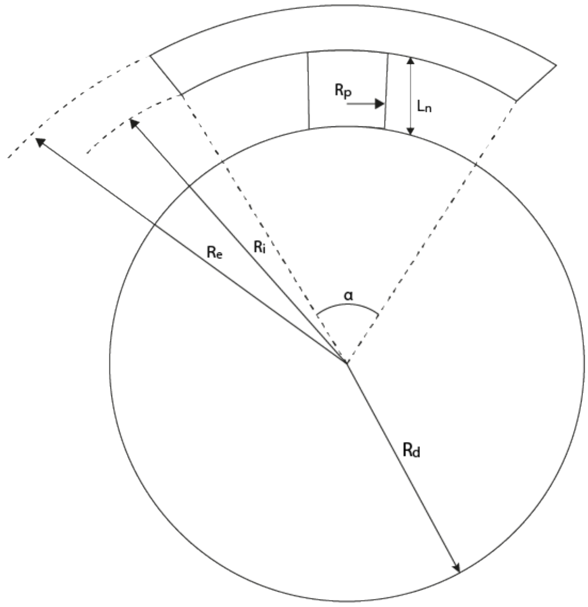
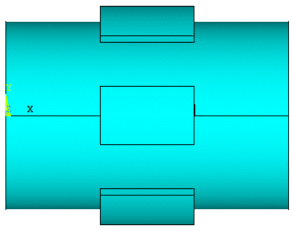
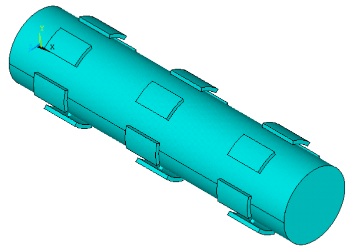


Figure 2 – cell characteristics.

Then, the control volume of the structure was modeled in the software *Ansys* using the element *fluid30* , as shown in Fig. 3 in order to appy WFE and FEM.



(a) **Structure Cell Model.**



(b) **Periodic Structure.**

Figure 3 – Duct control volume with inner Helmholtz Resonators.

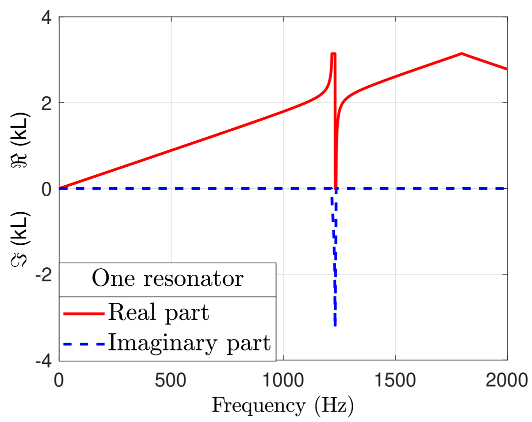
NUMERICAL RESULTS

Fluid and structural parameters are listed in table 1. Using a single cell of the structure, modeled by the WFE, to obtain the sound pressure level and dispersion diagram, and a three-cell structure modeled by FEM to obtain the TL and SPL.

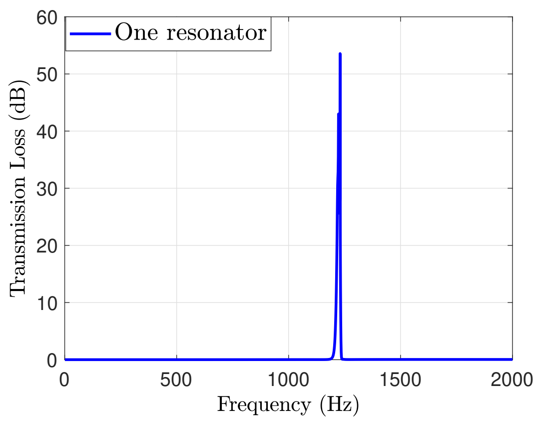
Table 1 – Geometry and properties of the duct-HR system.

Geometry and material	Variable	Value
Density (kg/m^3)	ρ	1.20
Velocity (m/s)	c	343.3
duct radius (m)	r_d	0.003
neck radius (m)	r_p	0.005
cavity internal radius (m)	r_i	0.045
cavity external radius (m)	r_e	0.050
Cavity opening angle (degrees)	α	45
duct length (m)	L	0.096
neck length (m)	L_n	0.005
cavity length (m)	h_e	0.032
number of cells	-	3

Figure 4 shows the diagram diagram obtained by the Floquet-Bloch theorem and the sound transmission loss for a model with three cells and one internal resonator per cell. The dispersion diagram (Fig. 4a) has a real part, which varies between 0 and π, in which it represents the propagating waves, and an imaginary part representing the evanescent waves. There is a band gap characteristic of structures with resonators at 1250 Hz, which corresponds to the resonant frequency of the Helmholtz Resonator. This phenomenon increases the transmission loss of the sound wave in the structure by 55 dB at the muffler output, as can be seen in the Fig. 4b.



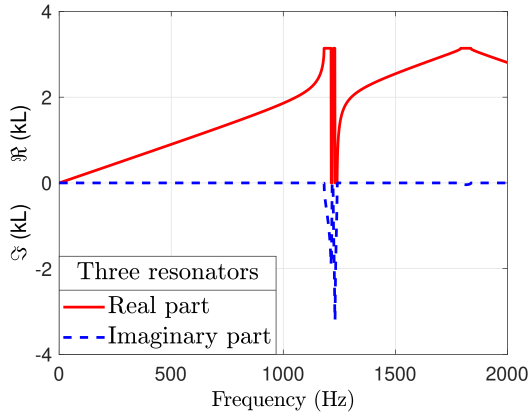
(a) **Dispersion diagram for cell with one Resonator.**



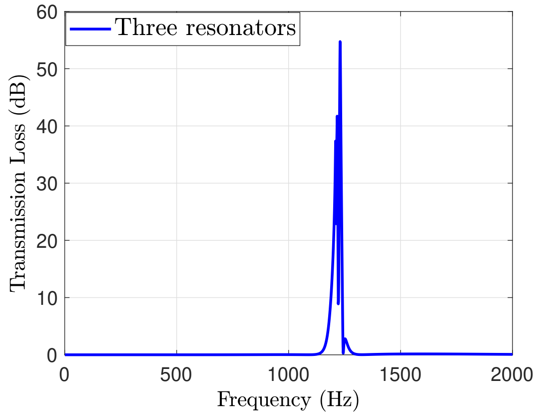
(b) **Transmission Loss for cell with one internal Resonator.**

Figure 4 – Dispersion diagram and Transmission Loss for the duct with one internal Resonators.

The Fig. 5 now displays the results for a structure with three cells and three resonators per cell. Note that with the increase in the number of resonators, there was an increase in the frequency range where band gaps occur, also resulting in a greater range of sound transmission loss, between 1130 Hz to 1300 Hz, however, with the increase in the number of resonators from one to three, there was no increase in the sound transmission loss of the muffler, which remains at 55 dB, but there was an improvement in performance as the width of the frequency band where attenuation occurs has increased.



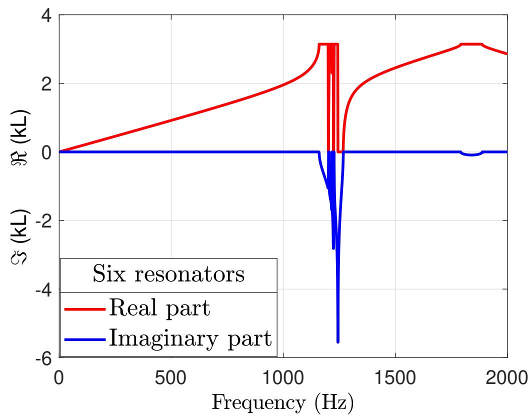
(a) **Diagram dispersion for cell with three internal Resonators.**



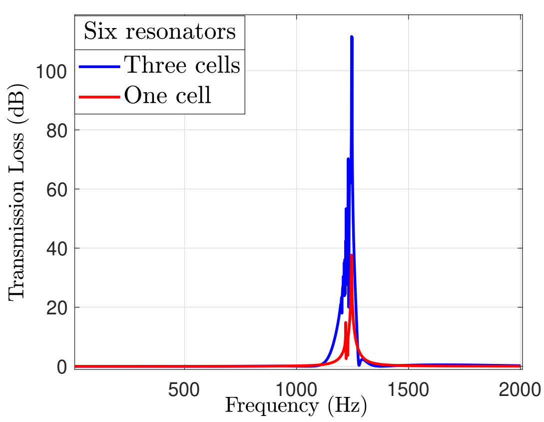
(b) **Transmission Loss for cell with three internal Resonators.**

Figure 5 – Dispersion diagram and Transmission Loss for the duct with three internal Resonators.

The Fig. 6 shows the dispersion diagram and the Transmission Loss for one and three cells for a configuration with six resonators per cell. The dispersion diagram (Fig. 6a) exhibits a bandgap at the resonance frequency in 1147 - 1268 Hz range. Despite having a frequency range close to that of 3 resonators, there is a significant increase in the value of the sound transmission loss.



(a) **Diagram dispersion for cell with six internal Resonators.**

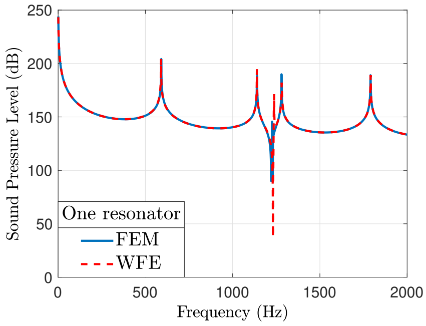


(b) **Transmission Loss for cell with six internal Resonators.**

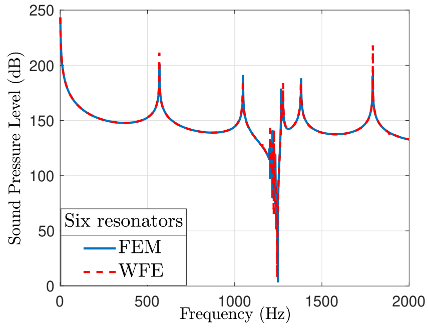
Figure 6 – Dispersion diagram and Transmission Loss for the duct with multiple internal Resonators.

In the Transmission Loss (Fig. 6b) , the attenuations for one cell was 37 dB and occurred in the range of badgaps; the three cell configuration showed 111 dB in same frequency.

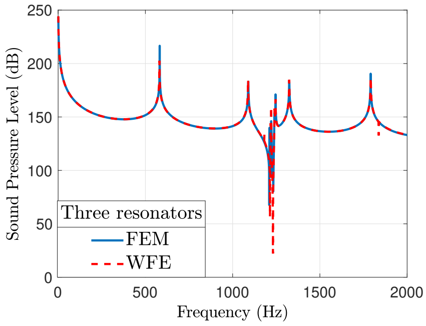
The Figure 7 shows the results SPL comparation for the four configurations between the WFE and FEM methods obtained a good convergence. The attenuations shown in the graphics TL and SPL occur in the same frequency range as the bandgaps, where there is no sound wave propagation. The reduction of the sound pressure level values (attenuation) occurs in the same proportions of the band gaps according to the imaginary part of the dispersion diagram. Figure 7d shows the SPL for a configuration without resonators, in which there is no attenuation.



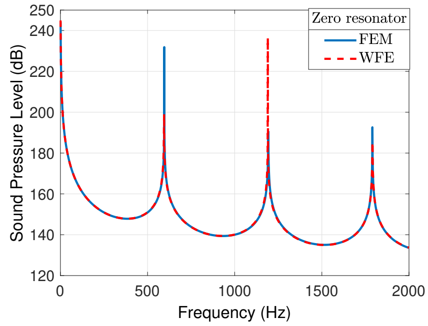
(a) **Sound Pressure Level for cell with one internal Resonators.**



(c) **Sound Pressure Level for cell with six internal Resonators.**



(b) **Sound Pressure Level for cell with three internal Resonators.**



(d) **Sound Pressure Level for cell with without Resonators.**

Figure 7 – Sound pressure level for one, three and six internal Resonators.

CONCLUSIONS

The present work investigated a duct system configuration with internal Helmholtz Resonators, from the Finite Element and Finite Element Wave Methods to estimate the dispersion diagram by the Floquet-Bloch theorem, transmission loss and sound pressure level. Through the dispersion diagram it was possible to identify the band gaps formed by the effect of the resonance of the HR, which represent the frequency range where the attenuation occur. It was observed in a structure with three cells with a resonator in each one, an attenuation of 55 dB occurs, increasing the number of resonators to three, there was no change in the sound transmission loss of the resonator, only in the frequency range of the attenuation. For a structure with six resonators, there is no significant change in the attenuation range, but the sound transmission loss value has increased sharply to 111 dB. Sound pressure levels decay in resonant frequency, according to the number of resonators in the structure.

REFERENCES

Alster, M. ”Improved calculation of resonant frequencies of helmholtz Resonators”. Journal of Sound and Vibration, v. 24, n. 1, p. 63–85, 1972.

Bedout, J. et al. ”Adaptive-passive noise control with self-tuning helmholtz Resonators”. Journal of Sound and Vibration, v. 202, n. 1, p. 109–123, 1997.

Cai, C.; Mak, C. M. ”Noise attenuation capacity of a helmholtz Resonator”. Advances in Engineering Software, v. 116, p. 60–66, 2018. ISSN 0965-9978.

Chen, K. et al. ”The improvement on the transmission loss of a duct by adding helmholtz Resonators”. Applied Acoustics, v. 54, n. 1, p. 71–82, 1998. ISSN 0003-682X.

Cook, R. D. et al. ”Concepts and Applications of Finite Element Analysis”, 4th Edition. 4. ed. [S.l.]: Wiley, 2001. ISBN 0471356050,9780471356059.

Duan, H. et al. ”Acoustic multi-layer helmholtz resonance metamaterials with multiple adjustable absorption peaks”. Applied Physics Letters, v. 118, n. 24, p. 241904, 2021.

GAI, X.-L. et al. Sound absorption of microperforated panel mounted with helmholtz Resonators. Applied Acoustics, v. 114, p. 260–265, 2016. ISSN 0003-682X.

Kinsler, L. E. et al. ”Fundamentals of acoustics”. 4. ed. [S.l.]: Wiley, 1999. ISBN 9780471847892,0471847895.

Mahesh, K.; Mini, R. ”Theoretical investigation on the acoustic performance of helmholtz Resonator integrated microperforated panel absorber”. Applied Acoustics, v. 178, p. 108012, 07 2021.

Mencik, J.M.” Approche numérique pour la propagation multi-modale guidée”. Université François Rabelais de Tours, 2008.

Navai, M.; Veitch, J. A. ”Acoustic Satisfaction in Open-Plan Offices: Review and Recommendations”. [S.l.], 2003.

Silva, P.; Mencik, J.-M.; Arruda, J. ”On the forced harmonic response of coupled systems via a wfe-based super-element approach”. In: . [S.l.: s.n.], 2014.

WU, D. et al. Hybrid noise control using multiple helmholtz Resonator arrays. Applied Acoustics, v. 143, p. 31–37, 2019. ISSN 0003-682X.

WU, W.; GUAN, Y. Numerical investigation on low-frequency noise damping performances of helmholtz Resonators with an extended neck in presence of a grazing flow. Journal of Low Frequency Noise, Vibration and Active Control, v. 40, n. 4, p. 2037–2053, 2021. D

Yamamoto, T. ”Acoustic metamaterial plate embedded with helmholtz Resonators for extraordinary sound transmission loss”. Journal of Applied Physics, v. 123, n. 21, p. 215110, 2018.

Zhao, H. et al. ”Effect of extended necks on transmission loss performances of helmholtz Resonators in presence of a grazing flow”. Aerospace Science and Technology, v. 77, p. 228–234, 2018. ISSN 1270-9638.

Zhong, W.; Williams, F. ”On the direct solution of wave propagation for repetitive structures”. Journal of Sound and Vibration, v. 181, n. 3, p. 485–501, 1995. ISSN 0022-460X

Zhu, Y.; Assouar, B. ”Multifunctional acoustic metasurface based on an array of helmholtz Resonators”. Phys. Rev. B, American Physical Society, v. 99, p. 174109, May 2019.

RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.