



# A Synergistic Vibration Absorber using Magnetorheological and Shape Memory Alloy

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**Abstract:** *Smart materials represent an alternative technology for solving engineering problems that demand either an autonomous performance or an adaptive behavior. Regarding this, the target of this work is to model and numerically simulate a passive-adaptive vibration absorber system, containing two degrees of freedom – one related to a shape memory alloy (SMA) primary system to be controlled; and a secondary actuator system composed by a magnetorheological (MR) damper that can be tuned by an electric current. In order to describe smart material behaviors, a polynomial constitutive model is used for SMA and Bouc-Wen model is adopted for the MR damper. Numerical simulations are performed employing the fourth order Runge-Kutta method. A multiparametric analysis is carried out by means of parameter spaces varying the excitation quantities, SMA temperature and the MR electric current. Results attest the absorber capacity of modifying the primary system dynamical pattern, which provide the necessary information to enable both passive and active control, in a further study.*

**Keywords:** *Passive-adaptive absorber, magnetorheological damper, shape memory alloy, nonlinear dynamics.*

## INTRODUCTION

Smart material behavior presents a multiphysics coupling between two or more physical fields, which enables their use as sensors and actuators in the smart structures. This remarkable property propitiates an adaptive behavior that can be employed for either passive or adaptive control. Vibration mitigation in oscillatory systems is an important application of these materials, exploiting the changes of some coupling parameter, modifying properties such as: geometry, stiffness, viscosity, thermal conductivity, among others. Besides that, while incorporating these materials into dynamical systems, their nonlinear effects provide a dynamical richness that encourages chaos control through small perturbations of some parameter, taking advantage of unstable orbits embedded within chaotic attractor.

There are plenty of works in the literature dealing with dynamical applications involving smart materials. Among the works concerning shape memory alloy (SMA) devices, it is possible to highlight: Bernardini & Rega (2010), Enemark et al. (2014), Carpineto et al. (2014) and Gur et al. (2022); besides some review articles should be highlighted: Savi (2015) and Ibrahim (2015). Regarding magnetorheological MR fluids, the following works should be pointed out: Prabakar et al. (2009) and Zhao et al. (2021).

The synergistic use of smart materials has a growing interest in order to gather both sensing and actuation capacities. Silva et al. (2015) numerically investigated the nonlinear response of an SMA oscillator combined with a piezoelectric element for energy harvesting purposes. The idea is to tune the SMA temperature to maximize the vibration amplitudes and, thus, enlarge the harvested energy by the piezo.

The present work deals with a passive-adaptive vibration absorber exploiting the synergistic use of SMA and MR damper. The idea is to investigate a primary oscillator connected to a secondary MR system that may be controlled by electrical current. The goal is that the secondary system can inhibit complex behaviors originally presented by the SMA system alone. Two situations are of concern: the simple addition of the secondary system, without current application; and the use of electrical current applied to the MR damper.

Concerning the methodology, a polynomial constitutive polynomial model is employed to describe the SMA thermomechanical behavior (Falk, 1980), while the Bouc-Wen constitutive model is used to describe the hysteretic behavior of the MR damper (Bouc, 1971; Wen, 1976). The fourth order Runge-Kutta method is employed to perform numerical simulations of the dynamical equations of motion. Results are based upon parameter spaces simultaneously varying different quantities, mapping the dynamical behavior pattern (periodic, chaotic, hyperchaotic). Lyapunov exponents are used to classify these behaviors, using the cloned dynamics algorithm. Moreover, a brute force algorithm is developed to classify some selected periodicities, by real time monitoring of the state variables.

## MATHEMATICAL FORMULATION

This section is dedicated to model and formulate the synergistic dynamical system including both SMA element and MR damper. Figure 1 shows the schematic archetype for the passive-adaptive vibration absorber, where  $u_1$  is the primary mass  $m_1$  absolute displacement;  $u_2$  is the absolute displacement of the secondary mass  $m_2$ ;  $c_1$  and  $c_2$  are the primary and the secondary damping coefficients, respectively;  $k_2$  is the secondary stiffness coefficient;  $F(t)$  is the external harmonic force applied to the primary system, besides SMA and Bouc-Wen elements.

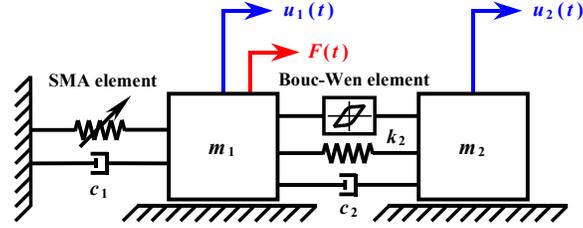


Figure 1 – Synergistic SMA-MR dynamical system employed for passive-adaptive smart vibration absorber.

Consider the SMA restoring element modeled as a helical spring, such that the force  $F_{SMA}$  stem from the integral given by Eq. (1), where  $\tau$  is the shear stress acting upon the spring wire cross-sectional area;  $\gamma$  is the shear strain, which is assumed to linearly vary with the radius  $r$  from zero to a maximum value  $\gamma_{max}$ ;  $d$ ,  $D$  and  $N$  are geometrical spring parameters related to spring wire diameter, nominal external diameter and number of active coils, respectively. After proper integration, the relation force-displacement for  $F_{SMA}$  in its final form is given by Eq. (2).

$$F_{SMA} = \frac{4\pi}{D} \int_0^{d/2} (\tau r^2) dr \quad (1)$$

$$\gamma(r) = \frac{r}{d/2} \gamma_{max}; \quad \gamma_{max} = \frac{d}{\pi D^2 N} u_1$$

The SMA constitutive thermomechanical behavior is described by a fifth-degree polynomial model (Falk, 1980), where  $a$  and  $b$  are material parameters;  $T$  is the absolute temperature,  $T_M$  is the temperature below which martensite is stable, while  $T_A$  is the temperature above which austenite is stable. On this basis, the stress-strain-temperature equation and the force-displacement-temperature equations are given by Eq. (2), as follows:

$$\tau(\gamma, T) = a(T - T_M)\gamma - b\gamma^3 + \frac{b^2}{4a(T_A - T_M)}\gamma^5$$

$$F_{SMA} = \frac{a(T - T_M)d^4}{8D^3N} u_1 - \frac{bd^6}{12\pi^2 D^7 N^3} u_1^3 + \frac{b^2 d^8}{64\pi^4 a(T_A - T_M)D^{11}N^5} u_1^5 \quad (2)$$

Concerning the MR damper, its entire force  $F_{MR}$  is composed by a linear restoring force, combined with a linear viscous dissipative force and a magneto-mechanical force described by the Bouc-Wen element, being  $z$  an internal variable responsible for the MR hysteresis described by a first-order differential equation, given by Eq. (3).

$$F_{MR} = k_2(u_2 - u_1) + c_2(\dot{u}_2 - \dot{u}_1) + \alpha z \quad (3)$$

$$\dot{z} = -\chi z \left| \dot{u}_2 - \dot{u}_1 \right| |z|^{n-1} - \beta (\dot{u}_2 - \dot{u}_1) |z|^n + A (\dot{u}_2 - \dot{u}_1)$$

where  $\alpha$ ,  $\chi$ ,  $\beta$ ,  $A$  and  $n$  are MR material parameters.

Liu et al. (2011) conducted a numerical-experimental study for an MR damper employing the Bouc-Wen model. They proposed empirical expressions for non-constant material parameters  $\alpha$ ,  $\beta$  and  $A$ , depending on electrical current  $I$ , according to Eq. (4).

$$\alpha = \alpha_a + \alpha_b I; \quad \beta = \beta_a \exp(\beta_b I); \quad A = A_a - A_b I + A_c I^2 \quad (4)$$

Once all necessary forces are defined, it is possible to write equations of motion based on the Newton's second law:

$$\begin{cases} -F_{\text{SMA}} - F_{\text{D}} + F_{\text{MR}} + F(t) = m_1 \ddot{u}_1 \\ -F_{\text{MR}} = m_2 \ddot{u}_2 \end{cases} \quad (5)$$

After some algebraic manipulation, the following set of ordinary differential equations of motion arises:

$$\begin{cases} 192 \pi^4 D^{11} N^5 m_1 \ddot{u}_1 + 192 \pi^4 D^{11} N^5 (c_1 + c_2) \dot{u}_1 - 192 \pi^4 D^{11} N^5 c_2 \dot{u}_2 + \\ + 24 \pi^4 D^8 N^4 [a (T - T_M) d^4 + 8 D^3 N k_2] u_1 - 16 \pi^2 b d^6 D^4 N^2 u_1^3 + 3 e d^8 u_1^5 - \\ - 192 \pi^4 D^{11} N^5 k_2 u_2 - 192 \pi^4 D^{11} N^5 \alpha z = 192 \pi^4 D^{11} N^5 f \cos(\Omega t) \\ m_2 \ddot{u}_2 - c_2 \dot{u}_1 + c_2 \dot{u}_2 - k_2 u_1 + k_2 u_2 + \alpha z = 0 \\ \dot{z} = -\chi z |\dot{u}_2 - \dot{u}_1| |z|^{n-1} - \beta (\dot{u}_2 - \dot{u}_1) |z|^n + A (\dot{u}_2 - \dot{u}_1) \end{cases} \quad (6)$$

## RESULTS AND DISCUSSIONS

This section presents parameter spaces (Fig. 2) mapping the dynamical pattern evolution simultaneously varying both excitation amplitude  $f$  and frequency  $\Omega$ . Besides, the electrical current  $I$  applied to the MR damper is varied, and constant values of primary damping coefficient  $c_1 = 0.0625$  Ns/m and intermediate SMA temperature  $T = 318$  K are assumed. Table 1 presents parameters related to the SMA helical spring, while Table 2 shows the dynamical parameters of the physical model.

**Table 1 – Parameters for the SMA helical spring.**

$D$ (m)	$d$ (m)	$N$	$a$ (Pa/K)	$b$ (Pa)	$T_M$ (K)	$T_A$ (K)
0,006	$0,750 \times 10^{-3}$	20	$0,350 \times 10^9$	$9 \times 10^{12}$	306	339

**Table 2 – Dynamical model parameters.**

$m_1$ (kg)	$m_2$ (kg)	$k_2$ (N/m)	$c_1$ (Ns/m)	$c_2$ (Ns/m)
1	0.100	1	0,0625	0.250

Figure 2 establishes a comparison of the system dynamics using  $\Omega$ - $f$  space. At the top, the primary system (1-DOF) is shown alone, serving as a reference case, while the 2-DOF response (including the absorber) is shown in the bottom part. Concerning passive control, comparing the 1-DOF reference case with Fig. 2(a) without current ( $I = 0$  A), there is a clear complexity loss, except for a vertical stripe for very low frequencies (that remains chaotic but now scattered) and; a unmodified lower rightward region comprising other kinds of behaviors. The original colored lamellas of the 1-DOF map gives rise to a mostly period-1 matrix, despite of some sparse period-2 and period-3 vertical stripes for  $10 < \Omega < 30$  rad/s, indicating lower richness of possible dynamical behaviors. Regarding active control, while increasing the electrical current, the general standard found for  $I = 0$  A remains, regardless of a rightward growing vertical stripe of other kinds of behaviors for low frequencies and; a lower rightward region enlargement comprising other kinds of behaviors, as well. For higher current values ( $I = 8$  and  $10$  A), a discrete vertical hyperchaotic lamella arises for  $10 < \Omega < 20$  rad/s, which does not appear in the 1-DOF system alone.

## CONCLUDING REMARKS

This paper focuses on the numerical simulation of a synergic SMA-MR dynamical system employed for passive-adaptive absorber control. According to the parameter spaces, the MR absorber insertion allows both passive and active control implementation. In terms of passive control, a wide range of the parameter space is changed eliminating chaotic regions. Applying an electric current, enables tune up the desired resulting dynamical pattern, towards a possible active control. In general, the MR absorber causes a loss of complexity and richness on the primary system.

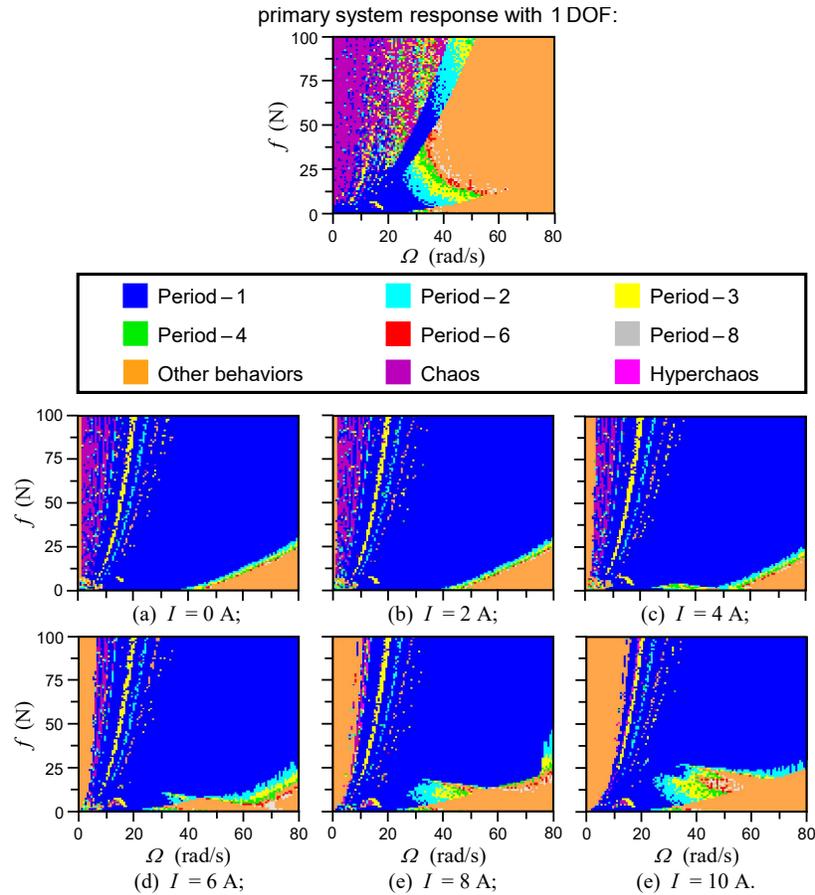


Figure 2 – Parameter  $\Omega$  -  $f$  space mapping dynamical pattern evolution varying electrical current.

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