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A topology optimization study of structural foundations considering dynamic loading

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Abstract: This work presents a topology optimization study of structural foundations subject to dynamic loading from in-soil sources. In this approach, a large square region within the soil is defined as a design domain, where the foundation should exist and provide support for a tower above the soil. The physical problem is solved via the Finite Element Method (FEM). The perfectly matched layer (PML) approach is employed to absorb dynamic waves and simulate the infinite domain. Decision variables indicate the material properties for each finite element in the design domain mesh, being either concrete (variable 1) or soil (variable 0). The goal is to find the structural foundation topology (0,1) that minimizes the squared difference of the displacements between the top and bottom points of the tower subject to a maximum mass of concrete used. Dynamic loading is applied in different locations, under and at the soil surface. The optimization problem is solved by employing sequential integer linear programming via the Topology Optimization of Binary Structures (TOBS) method. Sensitivities are calculated by semi-automatic differentiation and the optimizer is based on the branch-and-bound algorithm. Numerical results show the potential of topology optimization to provide insights about the soil-structure interaction and the design of structural foundations subject to dynamic loads.

Keywords: topology optimization, structural design, dynamic loading, PML, soil-structure interaction

INTRODUCTION

Structural solutions with better performance and quality are continuously sought in the area of structural engineering. The design of complex structures is a challenge and requires adequate tools for its elaboration. Optimization methods have been constantly employed in several engineering areas aiming at the development of ideal designs, taking into account economic aspects and the necessary performance requirements. Despite the large potential, optimization methods are rarely applied in geotechnical field such as in the design of structural foundations.

Structural topology optimization in foundation designs can provide optimized layouts and produce designs with better performance to support the loads imposed on the system, in addition to lower material use. Topology optimization (TO) is a powerful numerical method that provides an effective structural layout and can be applied to different types of structures. Obtaining optimized layouts in foundations design is still an underexplored area into topology optimization framework. Seitz (2015) obtained optimized structures for some basic geotechnical problems using the SIMP (Solid Isotropic Material with Penalization) method. Subsequently, the method was extended to applications in 3D problems (Seitz and Grabe, 2016). The authors proposed the optimization of a 3D shallow foundation in a granular soil. Kammoun et al. (2019) employed an optimization approach based on direct limit analysis to optimize shallow and deep foundation designs. In addition, the authors considered the use of two materials in the optimization problem. Since there are the presence of specific materials (soil and foundation structural material) in the same domain, the inclusion of two materials in the problem of optimizing geotechnical structures proves to be advantageous. The classic TO approach considers only one type of material in the optimization process, generally characterized by solid-void. However, the inclusion of more than one material during the optimization process requires the adoption of robust numerical methods, which are capable of capturing the boundaries of each material in a distinct way. Recently, Sadeghi et al. (2021) published the first study including including dynamic loads in foundations design.

In this context, this work aims to find the optimized soil-structure designs subject to dynamic loads. For that, the TOBS (Topology Optimization of Binary Structures) method (Sivapuram and Picelli, 2018) is used. The method employs binary design variables and consider sequential integer linear programming to solve the optimization problem. Fully discrete optimized structural foundations with clear physical boundaries are provided. The objective is to minimize the difference squared of the top and bottom displacements of a tower over a soil. Volume is constrained to a certain percentage of the design domain.

FREQUENCY DOMAIN ANALYSIS

The frequency domain analysis addresses the study of the structural response to a harmonic steady state load for certain frequency. Thus, considering no body forces and any acceleration, the mechanics structural domain is governed by

$$\rho\omega^2(\mathbf{u}) + \sigma_s = -\mathbf{F}e^{i\phi} \tag{1}$$

Where ω corresponds to the excitation angular frequency, $\mathbf{u}$ is the structural displacement field and ϕ is the is the phase angle. All forces and responses consist of complex values. In order to absorb the wave and avoid reflections, Perfectly Matched Layer's (PML's) are applied to the soil boundaries in this case.

OPTIMIZATION FORMULATION

THE TOBS METHOD

The TOBS method (Sivapuram and Picelli, 2018) applies binary {0,1} design variables that can be useful to address different materials into the optimization framework. This methodology uses a linear approximation of the objective and constraint functions to generate the suboptimization problems associated with integer linear programming. Therefore, the linearized optimization problem to be solved is given by:

$$\begin{aligned} \underset{\Delta\mathbf{x}^k}{\text{Minimize}} \quad & \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{x}^k} \Delta\mathbf{x}^k \\ \text{Subject to} \quad & \left.\frac{\partial g_i}{\partial \mathbf{x}}\right|_{\mathbf{x}^k} \Delta\mathbf{x}^k \le \bar{g}_i - g_i^k \quad i \in [1, N_g] \\ & \Delta x_j \in \{-x_j, 1-x_j\} \quad j \in [1, N_d] \end{aligned} \tag{2}$$

where $f(\mathbf{x})$ is the objective function, constrained by $g_i(\mathbf{x}) \le \bar{g}_i$. $i \in [1, N_g]$ and N_g and N_d are respectively the number of inequality constraints and elements in design variables vector. The term g_i^k is the value of constraint g_i at iteration k of optimization. The ILP solver will be used to find the optimal change $\Delta\mathbf{x}$ for the integer design variables $\mathbf{x}$. After each iteration, the design variables are updated as $\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$.

TOBS METHOD CONSIDERING TWO MATERIALS

The topology optimization problem considering two materials is modeled using an interpolation between materials. For this, the extended SIMP interpolation is chosen. Considering two materials (soil and solid) in the optimization, the interpolation is described by

$$E(\mathbf{x}_i) = E_{soil} + \mathbf{x}_i^p(E_{structure} - E_{soil}), \tag{3}$$

where $\mathbf{x_i}$ is the design variable, p is a penalty factor, E_{soil} corresponds to the Young's modulus of the surrounding subsoil and E_{solid} is the Young's modulus of the solid, e.g, steel. Thus, using extended SIMP interpolation and discrete design variables the structural topology optimization problem for two materials adds a new condition to the design variables in Eq. 2, being

$$\Delta x_{jk} \in \{-x_{jk}, 1-x_{jk}\} \quad j \in [1, N_d]\ k \in [1,\ N_m]. \tag{4}$$

Where N_m is the number of materials available for optimization.

INTEGRATION WITH EXTERNAL fem SOLVER

The present study employs the TOBS method to update the design variables in the problem and integrate it with a FEA package, herein COMSOL Multiphysics. A diagram illustrating the steps of the algorithm is presented in the Fig. 1. A grid of points of interest is used in order to establish a communication between both modules. The grid of points is located in the design domain in order to compute the required sensitivities field. The sensitivities are provided by the FEA package and the optimization module (TOBS) provides the correct distribution of each material based on the grid of points described by binary design variables {0,1}. A fixed quadrilateral mesh is considered in the design domain and in the PML's domain. The rest of the domain is freely meshed considering triangular elements. Herein, we consider a quadratic Lagrange approximation for the structural mechanics analysis. Binary variables prescribe each phase of the material, being (1) for the structure domain (foundation) and (0) for the soil domain. This proposed methodology is implemented in COMSOL LiveLink with MATLAB.

SENSITIVITY ANALYSIS

Sensitivities are calculated through a stationary study solver in FEA module, herein the COMSOL multiphysics commercial software. The objective functions are defined in the software and model sensitivities are solved via automatic differentiation. Comsol's automatic differentiation employs the adjoint method to calculate sensitivities. The generic adjoint equation is expressed as:

$$\left(\frac{\partial \mathbf{R}}{\partial \mathbf{u}}\right)^{\mathrm{T}} \lambda = -\left(\frac{\partial f}{\partial \mathbf{u}}\right)^{\mathrm{T}}, \tag{5}$$

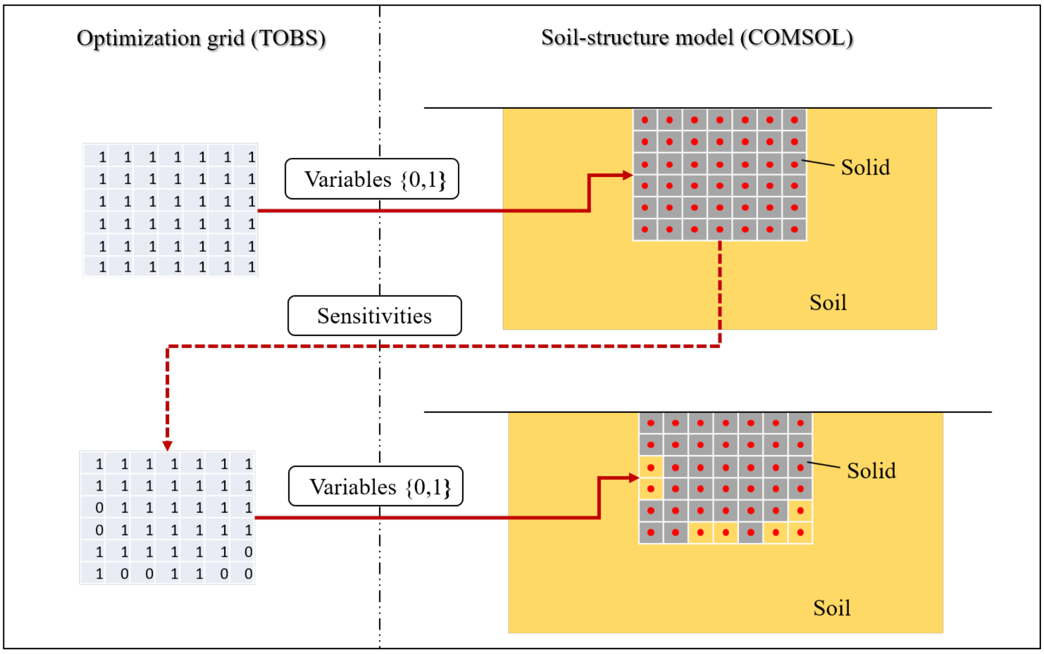

Figure 1 – Illustration of the TOBS method for foundation structural design considering soil-structure interaction.

where λ is the vector of adjoint variables, f is the vector of objective function and R is the vector of constraints. Sensitivities can then be computed as:

$$\left(\frac{\mathrm{d}L}{\mathrm{d}\mathbf{x}}\right) = \left(\frac{\partial f}{\partial \mathbf{x}}\right)^{\mathrm{T}} + \lambda^{\mathrm{T}}\frac{\partial \mathbf{R}}{\partial \mathbf{x}}. \tag{6}$$

This is a general formulation to compute the sensitivities of any function f. The respective sensitivities can be exported using the function `fsens(dtopo1.theta_c)/dvol`. A set of grid points defined by binary variables $\{0,1\}$ is generated by the optimizer and passed on to the FEA module, the sensitivities are calculated and then passed back to the optimizer. A spatial filter is applied to the sensitivities to smooth out the problem and avoid numerical problems, like the chessboard.

The structural volume sensitivities with to respect to the design variable ρ_j are expressed as

$$\frac{\partial V}{\partial \rho_j} = V_j, \tag{7}$$

where V_j is the volume fraction referring to the design variable j.

OBJECTIVE FUNCTION

We consider the minimization of the displacement between two points. Thus, the objective function is defined as

$$f(\mathbf{x}) = (u_2 - u_1)^2 \tag{8}$$

Where u_1 and u_2 corresponds to the displacement in two different points. Both objective functions are subject to the volume fraction constraint into the optimization problem.

Numerical example

In this section, we show the numerical result obtained by using the TOBS method. The goal is to find an optimized design of a structural foundation of a tower with dimensions of 5×0.4 m, that is located above the soil domain of dimension 45×25 m (see Fig. 2). The foundation design is obtained from a design domain of 12×9 m. An optimization grid of 240×180 is employed. The convergence is defined by averaging the changes in the objective function over 6 consecutive iterations for an tolerance of $\tau = 0.001$. We consider an excitation load at the center of the soil domain. The objective is to find the minimum difference between the displacements of points u_2 and u_1 subject to a volume constraint of 20%. The harmonic excitation load is defined as $\mathbf{F} = 1 \cdot 10^7$ N and is applied to an angular frequency of $\omega = 10$[Hz]. Regarding material properties, the considered Young's modulus are $E_{solid} = 2 \cdot 10^{11}$ and $E_{soil} = 1 \cdot 10^{1}1$ Pa, and densities $\rho_{solid} = 7800$ kg/m^3 and $\rho_{soil} = 100$ kg/m^3 for the structure and soil domains, respectively. The Poisson's ratio is $\nu = 0.3$ in both domains. The optimization parameters used are $\varepsilon = 0.02$, $\beta = 0.05$ and penalty factor $p = 5$. Fig. 3 presents the final optimized topology obtained. As noted, the solid material is distributed densely below the tower structure.

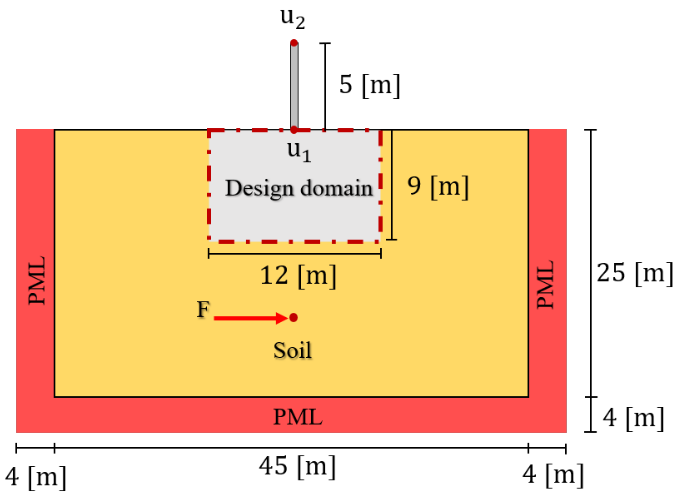

Figure 2 – Illustration of the soil-structure problem considering an excitation harmonic load at the center of the soil domain.

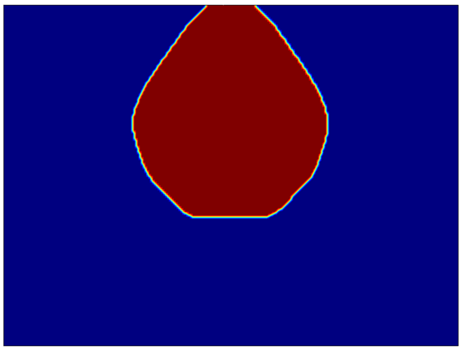

Figure 3 – Final topology obtained considering harmonic excitation at the top of the tower structure. The red domain corresponds to the solid material domain (structure) and the blue domain to the soil domain.

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