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Dynamic Response of Wind Turbine Tower Induced by Wheel Unbalance

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Abstract: The main actions that occur in wind towers come from the incidence of wind. However, there are adverse situations that can lead to an increase in the loads on the towers; an example is the unbalance of the blades. This paper aims to study the behavior of a wind turbine tower considering the unbalance of the blades and its influence on the dynamic response of the tower. Some results show that small variation of mass distribution in one of the blades increases considerably the dynamic response of the tower, mainly if the interaction between rotor-tower is considered.

Keywords: Dynamic analysis, Wind Turbine Tower, Unbalanced mass.

INTRODUCTION

When the three blades of a wind turbine have the same mass distribution, there is no additional load on the tower due to their rotation in operation, because the applied loads are balanced. In case of any manufacturing defect in the geometry, or change in the relative position in one of the mass distributions of the balancing set; this rotating imbalance generates additional excitation forces on the tower. A historical review of wind tower collapse was performed by (Ma et al., 2019). The authors identified that most of the incidents occurred due to the extreme wind loads and storms, however problems related to the unbalance of the blades were also pointed out as an important factor to be studied in order to avoid the collapse of wind towers. It has also an important influence in power output.

Over the years, several works have dealt with the problem of unbalance in wind turbines and its influence on the dynamic response of the support tower. Some simulations were performed on wind towers using commercial software, evaluating three scenarios: normal operation, with mass unbalance and with aerodynamic asymmetry and noticed that the unbalance increases the dynamic response of the tower compared to its normal operation (Gardels et al., 2010). Similarly, an analytical analysis of a wind tower model considering unbalance was performed and, in parallel, some experimental measurements carried out to corroborate these studies, evidencing the increase in displacement imposed by the unbalance of the blades on the tower (Jiang et al., 2009).

All these works mentioned above carried out their simulations considering the forces due to unbalanced blades uncoupled to the tower response. However, a more rigorous approach must consider the coupled system, since the blades are coupled to the tower at the point of highest tower displacements, and these displacements can interfere in the frequency of rotation, that is, the system performs as a non-ideal system. Here, some numerical studies are carried out considering this tower-blades coupling considering unbalance and its influence on the dynamic response is explored. for the response of the tower under the dynamic action of unbalanced blades.

MATHEMATICAL MODEL

In this section an analytical model is developed for the tower with the attached blades. Figure 1 shows the model of the tower with the blades attached at the top and the equivalent simplified model with two degrees of freedom. Here L is the height of the tower, r is the length of the blades; A, the cross-sectional area of the tower; ρ, the density of the material of the tower; E, the modulus of elasticity, I, the moment of inertia of the tower and M is the equivalent total mass of the tower and rotor at the top of the tower. The blades rotate with a speed Ω in a counterclockwise direction. The degrees of freedom considered for the model are: the transverse displacement of the tower, w, and the angular displacement of the blades with respect to a vertical reference line, θ.

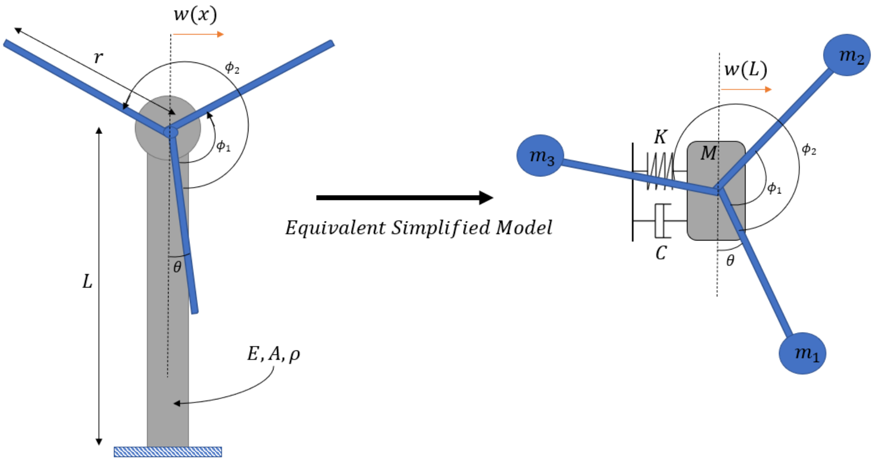


Figure 1 – Wind turbine tower and simplified model.

Tower Energy Functional

The kinematic and elastic potential energy of the tower are defined as

$$T_t = \frac{1}{2} M \dot{w}^2 \tag{1}$$

$$\Pi_t = \frac{1}{2} K w^2 \tag{2}$$

In above equations, M and K are the equivalent mass and stiffness of the tower, respectively. The notation $\dot{w}$ denotes a derivative of displacement with respect to time. The Lagrangian function of the tower can then be written as

$$\Im_t = T_t - \Pi_t = \frac{1}{2} M \dot{w}^2 - \frac{1}{2} K w^2 \tag{3}$$

Blades Energy Functional

The kinematic and potential energy of the blades are defined as

$$T_b = \sum_i \frac{1}{2} m_i v_i^2 \tag{4}$$

$$\Pi_b = \sum_i m_i g h_i \tag{5}$$

where the index i denotes the number of blades; $m_i = 33\rho_b A_{bi} r / 140$ is the equivalent mass of the each blade, $v_i^2 = \dot{w}^2 + 2\Omega r \dot{w} \cos(\theta + \phi_i) + \Omega^2 r^2$, is the velocity, with $\Omega = \dot{\theta}$; and the potential height is defined as $h_i = r[1 - \cos(\theta + \phi_i)]$. As we are defining the contribution of blades, ϕ_i is the offset angle between each blade, in this case $\phi_1 = 0^\circ$, $\phi_2 = 120^\circ$ and $\phi_3 = 240^\circ$. The Lagrangian function of the blades then can be written as

$$\Im_b = T_b - \Pi_b = \sum_i \frac{1}{2} m_i \left(\dot{w}^2 + 2\Omega r \dot{w} \cos(\theta + \phi_i) + \Omega^2 r^2 \right) - \sum_i m_i g r \left[1 - \cos\left(\theta + \phi_i \right) \right] \tag{6}$$

Structural System Functional and Equation of Motion

The total energy of the system is sum of each functional associated with the tower and blades:

$$\Im = \Im_t + \Im_b \tag{7}$$

By applying Hamilton's principles and Eq. (7), the coupled nonlinear equations of motion are given by

$$\frac{d}{dt}\left(\frac{\partial \Im}{\partial \dot{w}}\right)-\frac{\partial \Im}{\partial w}=F_d \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial \Im}{\partial \Omega}\right)-\frac{\partial \Im}{\partial \theta}=0 \tag{9}$$

where $F_d = -C\dot{w}$ is the viscous damping force (nonconservative, dissipative force). The equations of motion, Eq. (8) and (9), are explicitly given in state space as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \left(M+\sum_i m_i\right) & 0 & \sum_i m_i r\cos\left(\theta+\phi_i\right) \\ 0 & 0 & 1 & 0 \\ 0 & \sum_i m_i r\cos\left(\theta+\phi_i\right) & 0 & \sum_i m_i r^2 \end{bmatrix} \cdot \begin{Bmatrix} \dot{w} \\ \ddot{w} \\ \Omega \\ \ddot{\theta} \end{Bmatrix} = \begin{Bmatrix} \dot{w} \\ -Kw - C\dot{w} + \sum_i m_i r \Omega^2 \sin\left(\theta+\phi_i\right) \\ \dot{\theta} \\ -\sum_i m_i g r \sin\left(\theta+\phi_i\right) \end{Bmatrix} \tag{10}$$

Observing the above equations, it's possible to notice that the system tower-blades is coupled, that is, the displacement of the tower influences the rotation of the blades and these ones reintroduce new forces upon the towers. They are numerically solved with the Runge Kutta algorithm implemented in Matlab (Dormand & Prince, 1986).

NUMERICAL STUDY AND SOME RESULTS

For numerical purposes, in this section, some analyses will be carried out in order to understand the coupled behavior of the tower-blade system. Table 1 shows the data adopted for the tower in the analysis and Table 2 the data for the wind turbine. For this study a reduction of 5% in the mass of one of the blades is considered; the equivalent stiffness is calculated based on the simplified formulation presented in Ko *et al*. (Ko, 2020) for a tower with varying cross-section. For the analysis, a rotational frequency of blades equal to $\Omega = 1.05\omega$ where ω is the lowest natural frequency of the tower is adopted. Solving the Eq. (10) the time history response of the tower and the blades rotational frequency are obtained, as illustrated in Figure 2.

As the tower moves horizontally, it imposes a retroactive inertial action on the blades, modifying its rotation frequency, so that this interaction decreases continually the rotation of the blades with time which in turn modifies the tower exerted by the blades on the tower (Figure 2b). Figure 2a compare the time response considering the tower-rotor interaction (in black) with the classical formulation of an ideal unbalance mass where the blade rotation doesn't change with time (in green). The differences between the two responses highlights the importance of the dynamic coupling and unbalance in the analysis of wind turbine towers.

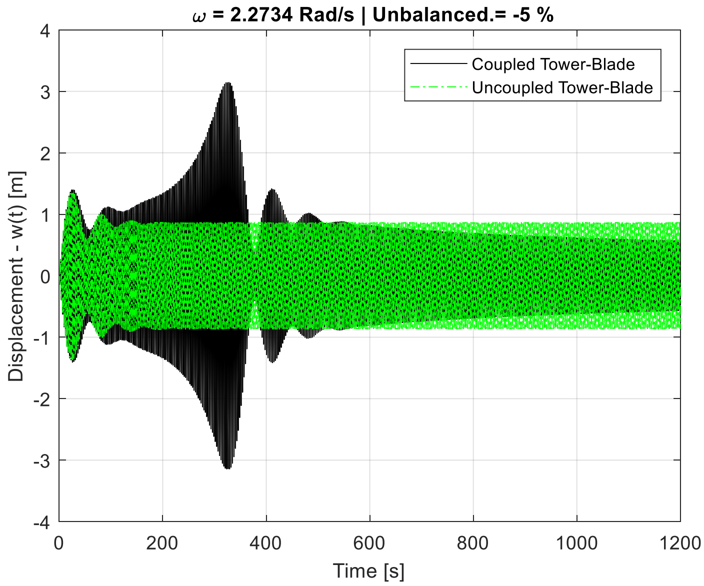
Table 1 – Tower Structural Parameters (Ko, 2020)

Parameter	Value
Height (m)	87.6
Base external diameter (m)	6.0
Top external diameter (m)	3.87
Wall thickness (m)	0.027
Young Modulus (GPa)	210
Mass density (kg/m³)	8500*

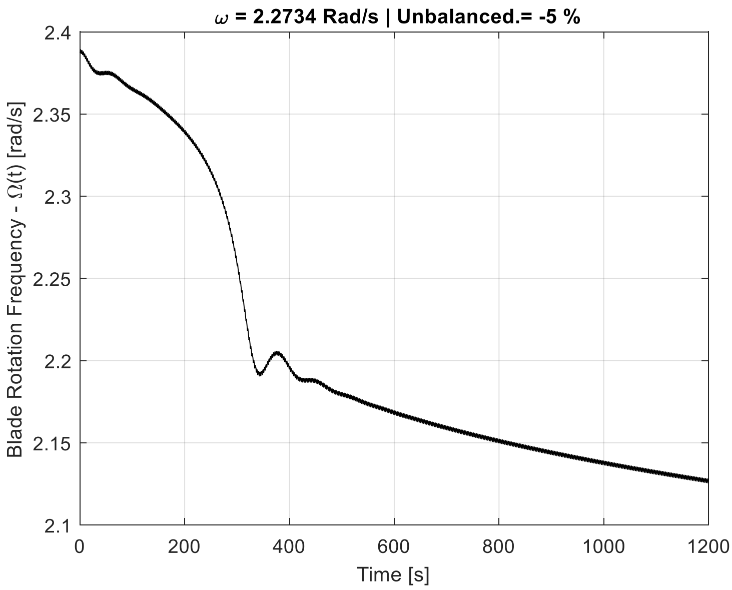
* it was increased mass to account eventually contributions of joints and accumulated dust

Table 2 – Wind Turbine Parameters (Ko, 2020)

Parameter	Value
Nacelle mass (kg)	239150.0
Blade length (m)	63.0
Blade cross section near the nacelle (m²)	0.6
Blade cross section at the tip (m²)	0.05
Mass density (kg/m³)	1800



(a)



(b)

Figure 2 - Dynamic response of tower due to unbalanced mass blade. (a) displacement at the top of the tower with coupled and uncoupled tower-blade interaction; (b) variation of blade rotational frequency due to coupled interaction.

As shown in Figure 2, the transient response that arises due to a coupled tower-blade system is important and cannot be disregarded in the dynamic analysis. Figure 3 shows the variation of the maximum lateral displacement of the tower as a function of the normalized frequency. There is a plateau around the natural frequency due to resonance followed by a discontinuity, leading to jumps as the frequency increases or decreases.

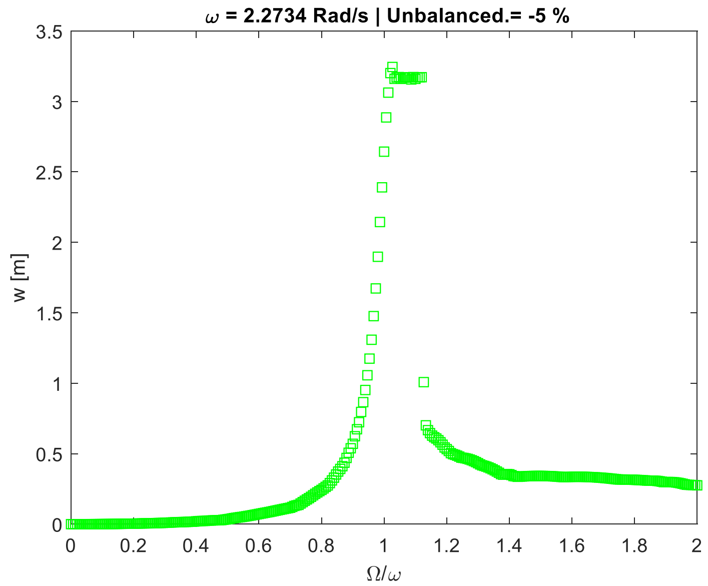


Figure 3 – Dynamic response of maximum displacement versus normalized frequency (coupled tower-blade)

CONCLUSIONS

A simplified model of a wind turbine using a 3-bladed rotor is developed analytically to study the influence of the wind wheel unbalance on the vibrations of the tower considering the interaction between the tower and the 3-bladed rotor. The results show non-ideal vibrations with a strong interaction influencing not only the tower but also the rotor response and, consequently, their safety and energy output.

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