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Malfunction Parameters Determination using Bayesian Neural Networks applied to a Multi-Fault Rotor

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Abstract: In addition to the qualitative diagnosis of rotating machinery, it is of great importance to determine the severity and characteristics of the malfunctions in order to provide sufficient information for its correction. In this sense, the present work aims to use Bayesian neural networks to develop meta-models for regression of these parameters. The methodology used simulated data from a multi-fault rotor. The displacement signals as a function of time in the four bearings were measured. After this, a full spectrum analysis was applied to extract the amplitude values of the six positive and negative harmonics. From these data, models were formulated for the regression of the following parameters: transverse shaft crack size, crack location, angle and parallel distance misalignment of the coupling. From the Bayesian model the uncertainties related to the predictions were extracted. Finally, the results were satisfactory given the expected physical behavior of the model.

Keywords: Bayesian Neural Networks, Parameter Regression, Angular and Parallel Misalignment, Crack Shaft, Multi-Fault Rotor

INTRODUCTION

In the context of the study of rotating machinery and its operation, it is of general interest to observe the dissipative means of energy present in the system. The manifestation of the general operating conditions of the machinery is demonstrated in the manner in which vibrations occur during routine use. In this sense, throughout the history of rotor dynamics, there has been the search for understanding these periodic phenomena (Krämer, 1993) and more recently with the increase in processing capacity, data collection and storage techniques of Artificial Intelligence have been employed to formulate models for the diagnosis of rotating machinery (Liu et al., 2018).

Among several techniques for fault detection already employed as in the works presented by the authors' review (Jardine et al., 2006) on condition-based maintenance, Bayesian methods take advantage by assigning an uncertainty to their prediction providing a metric for understanding the behavior of the models during and after their training. The increase in uncertainty in these models is directly related to the assertiveness of the technique (Dürr et al., 2020).

For this reason, the present work aims to conduct a study about the possibilities of formulating models for regression of fault parameters using Bayesian neural networks. Deep Learning techniques will be applied to the following faults: shaft crack, angular and parallel misalignment. After that, the performance of the regressions and the behavior of the uncertainty related to the predictions will be evaluated. In this work, the approach to Bayesian inference used was by applying the Monte Carlo Dropout technique that was first described in (Gal & Ghahramani, 2015). This method has been employed in situations where a higher diagnostic reliability is needed, such as in the medical field as in the works of (Lee & Kim, 2022) and (Ju et al., 2022). Other applications can be found in the reliability of nuclear power plants in (Bae et al., 2022) and for monitoring of rotor-craft icing from aeroacoustics time-series data in (Tong et al., 2022).

In the field of models for determining the parameters, there are several works related to each type of failure. For the determination of unbalance parameters, it is worth mentioning (Xie et al., 2016), which regressed unbalance data on a helicopter rotor, the paper by (Zang et al., 2018) in which the unbalance of a rotor based on sparsity control of the residual model was estimated and finally the paper by (Luu & Hai, 2020) which used a multiple regression model for the determination of the unbalance for a dynamic balancing system. The work of (Srinivas et al., 2021) proposes a model for the determination of angular and parallel misalignment parameters. For shaft cracking, the work of (Ellis et al., 2022) proposes a model to infer the crack size in turbine blades and thereby calculate the remaining life before failure.

OBJECTIVES

The objective of this paper is to present a relevant and condensed study about the condition monitoring of a rotor aimed at determining the failure parameters using Bayesian neural networks with the application of Monte Carlo Dropout as models for multi-variable regression. The models will be fed with amplitude data of the first six harmonics extracted from a full spectrum analysis. From these data, models for the regression of the following parameters were formulated: transverse crack size on the shaft, crack location, misalignment angle and distance from the parallel coupling misalignment. The uncertainties related to the predictions were extracted from the Bayesian model. Finally, the relationship of the degree of uncertainty with the accuracy of the models will be appreciated.

METHODOLOGY

Multi-Fault Rotor Model

The basic model for simulating and modeling rotating machines is composed of a rotor, bearings and supporting structures. Many other variations can be included in the model to better represent the physics of the problem. In these theoretical models there are parameter inputs that characterize the machine such as the mass, rotational speed, damping coefficient related to energy dissipation, and stiffness coefficient. This mathematical model is formulated for modeling application with the Finite Element Method (FEM). The modeling of the Multi-Fault Rotor will follow the paper (Garoli et al., 2019).

The shaft elements are modeled with Timoshenko beams of circular section which makes it possible to include the phenomenon of shear and rotational inertia. The formulation used for determines the mass matrix, rotation inertia matrix, gyroscopic matrix and the stiffness matrix follow (Lalanne & Ferraris, 1998). The global equation is shown bellow in 1

$$\mathbf{M}\cdot\ddot{\mathbf{q}}+(\mathbf{C}+\Omega\cdot\mathbf{G})\cdot\dot{\mathbf{q}}+\mathbf{K}\cdot\mathbf{q}=\mathbf{F} \tag{1}$$

Where the variables $\mathbf{M}$, $\mathbf{C}$, $\mathbf{G}$ and $\mathbf{K}$ are the global mass, damping, gyroscopic effect and stiffness matrices. The vector $\mathbf{F}$ is the forces that are applied to the model. The variable $\mathbf{q}$, Ω are the displacement and is the angular velocity of the rotor, respectively. The construction data used for the rotor simulation is shown in Table 1:

Table 1 – Table of Rotor Constructional Data

Constructive Parameters of the Simulated Rotor		
	Dimensions	**Unit**
Shaft Lengths	631,5	[mm]
Bearing Diameter	31	[mm]
Shaft Diameter	12	[mm]
Lubricating Oil Viscosity	4	[pa.s]
Inner Diameter of the Disks	12	[mm]
Inner Diameter of the Coupling Discs	4	[mm]
Outer Diameter of the Discs	90	[mm]
Number of Bolts in Coupling	4	unid.
Disc Thickness	45	[mm]
Shear Modulus (G)	79,6	[Mpa]
Modulus of Elasticity (E)	211	[Gpa]
Density	7860	[kg/m^3]

The diagram representing the physical aspects of the rotor is shown below in Figure 1:

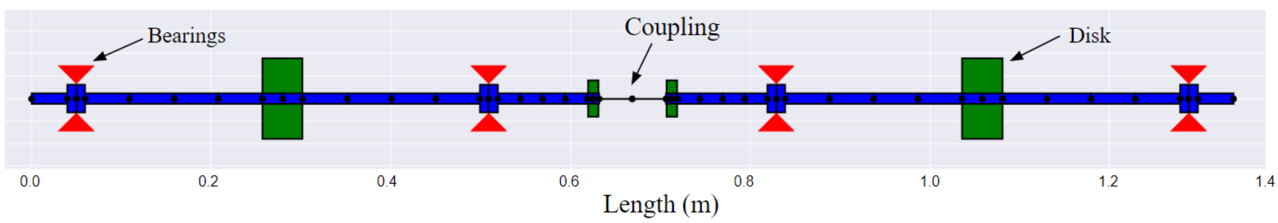

Figure 1 – Rotor's Diagram.

Journal Bearing Modeling

The modeling of hydrodynamic bearings is realized by changing the stiffness and damping matrices of the elements that model them. In a simplified way, the Timoshenko beam elements in which the bearings are located have the behavior of their degrees of freedom altered by the inclusion of stiffness and damping coefficients that characterize the bearing. This proposition was made by (Lund & Sternlicht, 1962) for the modeling of hydrodynamic bearings. In purpose of simplifying the mathematical modeling, the short bearing assumption proposed by (Ocvirk, 1952) is assumed, in which it is possible to disregard the pressure variation in the bearing circumference direction and thus obtain an analytical solution by Reynolds.To include the effect of the short hydrodynamic bearing in the model just add the coefficients to the respective degrees of freedom in the finite element. The mathematical approach to this proposition is found in (Krämer, 1993). Finally, the foundation is considered to be infinitely rigid and for this reason has no influence on this model.

Model for Faults Simulation

Unbalance is one of the most common faults in rotating machines, since some level of unbalance is always found. No matter how perfectly a machine is manufactured, some assembly deviation always occurs. (adapted from (Friswell, 2010)) The presence of unbalance in the rotor generates a radial force causing an increase in the oscillatory motion of the rotor increasing the possibility of machine degradation for this reason. (adapted from (Mohanty, 1989)) For its correction it is usually balanced the rotor. Besides causing vibrations at the synchronous frequency (1X) (Hatch, 2002). The unbalancing force matrix was add to the model as described according to (Lalanne & Ferraris, 1998). In the equation 2 is shown the formulation of the unbalance force.

$$\begin{bmatrix} 0 \\ \vdots \\ F_u \\ F_w \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ m_u d\Omega^2 & 0 \\ 0 & m_u d\Omega^2 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \sin(\Omega t + \beta) \\ \cos(\Omega t + \beta) \end{bmatrix} \tag{2}$$

In the equation m_u, d and β are respectively the unbalance mass, the mass eccentricity and as the unbalance phase angle.

Misalignment can be characterized as an angular or parallel deviation with respect to the coaxial shaft of a coupling connecting two rotors. In the presence of this shaft irregularity in the machine connection preload forces are introduced into the coupling which are then transmitted to the different components of the machine reducing its useful life. (adapted from (Piotrowski, 1952)) In the work of (Dewell & Mitchell, 1984), a spectral analysis was used for the determination of misalignment in flexible couplings. The most relevant measurements performed were the amplitudes of the second and fourth harmonics. The authors pointed out that amplitude measurements related to the rotational frequency of the rotor are not relevant for determining this type of failure. The modeling was done according to the (Less, 2007). For the angular misalignment there is the equation 3 and for parallel misalignment there is the equation 4.

$$f_{ang} = \frac{1}{2}\alpha_b r_b^2 \cdot \begin{bmatrix} 0 \\ 0 \\ 3k_a + k'(1+\cos(2\Omega t)) \\ k'\sin(2\Omega t) \end{bmatrix} \tag{3}$$

$$f_{par} = N_b \cdot k_b \cdot \delta_b \cdot \begin{bmatrix} \sin(\Omega t) \\ 1-\cos(\Omega t) \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

For the case of angular misalignment in the formulation 4, the model proposes two rotors coupled to each other with an angle α_b. Due to this, the stiffness of one of the coupling bolts is modeled as $k_a + k'$ with the stiffness of the other bolts kept as k_a. In the model of parallel misalignment N_b, δ_b, k_b are the number of bolts in the coupling, the distance of misalignment and the stiffness of the coupling. The input data for the simulation was sampled from the parameter distributions described in Table 2 and 3.

Table 2 – Data Sampling for the Simulation of Angular Misalignment.

Angular Misalignment Simulation Parameters for Sampling						
Rotor Parameters	**Distribution**	**Parameters**	**Unit**	**Node**	**Phase (rad)**	**Quantity**
Angular Velocity	Normal	N(50,3)	$[rad/s]$	-	-	1
Misalignment Angle (α_b)	Uniform	$[0.001, 1]$	$[Graus]$	22	-	1
Unbalance	Uniform	$\mathscr{U}[1,10]\cdot 10^{-6}$	$[Kg \cdot m]$	$\mathscr{U}[1,45]$	$[-\pi,\pi]$	2

Table 3 – Data Sampling for Parallel Misalignment Simulation.

Simulation Parameters for Sampling Parallel Misalignment						
Rotor Parameters	**Distribution**	**Parameters**	**Unit**	**Node**	**Phase (rad)**	**Quantity**
Angular Velocity	Normal	$\mathscr{N}(50,3)$	$[rad/s]$	-	-	1
Misalignment Distance (δ_b)	Uniform	$\mathscr{U}[1,10]$	$[mm]$	22	-	1
Unbalance	Uniform	$\mathscr{U}[1,10]\cdot 10^{-6}$	$[Kg\cdot m]$	$\mathscr{U}[1,45]$	$\mathscr{U}[-\pi,\pi]$	2

Shafts are very susceptible to fatigue crack nucleation and propagation, so it is of interest to monitor this type of failure. In (GREEN, 2005) it was verified that the second harmonic (2X) is affected in the presence of shaft cracking. According to (AL-Shudeifat, 2013) it is possible to simulate the presence of a transverse crack in a rotor-bearing-disk system. The author reports that during rotation, the crack opens on the shaft causing radial asymmetry due to the variation of the moment of inertia in time. Therefore, the stiffness matrix is formulated so as to generate a linear periodic system and thus introduce the crack effect into the model.

The stiffness matrices of the cracked element are shown below in the matrices 5 and 6:

$$K_{c_r} = \frac{E}{l^3}\begin{bmatrix} 12I_{\bar{x}} & & & & & & & \\ 0 & -6lI_{\bar{y}} & & & & & & \\ 0 & -6lI_{\bar{y}} & 4l^2 & & & & sym. & \\ 6lI_{\bar{x}} & 0 & 0 & 4l^2 & & & & \\ -12lI_{\bar{x}} & 0 & 0 & -6lI_x & 12I_x & & & \\ 0 & -12I_{\bar{y}} & 6lI_{\bar{y}} & 0 & 0 & 12I_y & & \\ 0 & -12I_{\bar{y}} & 6lI_y & 0 & 0 & 6lI_{\bar{y}} & 4l^2 & \\ 6lI_{\bar{x}} & 0 & 0 & 2l^2I_{\bar{x}} & -6lI_x & 0 & 0 & 4l^2I_{\bar{x}} \end{bmatrix} \tag{5}$$

$$K_e = \frac{E}{l^3}\cdot\begin{bmatrix} 0 & & & & & & & \\ -12I_{\bar{X}\bar{Y}} & 0 & & & & & & \\ 6lI_{\bar{Y}\bar{X}} & 0 & 0 & & & sym. & & \\ 0 & -6lI_{\bar{Y}\bar{X}} & 4l^2I_{\bar{Y}\bar{X}} & 0 & & & & \\ 0 & 12I_{\bar{Y}\bar{X}} & -6lI_{\bar{Y}\bar{X}} & 0 & 0 & & & \\ 12I_{\bar{Y}\bar{X}} & 0 & 0 & 6l^2I_{\bar{Y}\bar{X}} & -12lI_{\bar{Y}\bar{X}} & 0 & & \\ 6lI_{\bar{Y}\bar{X}} & 0 & 0 & 2l^2I_{\bar{Y}\bar{X}} & -6lI_{\bar{X}\bar{Y}} & 0 & 0 & \\ 0 & -6lI_{\bar{Y}\bar{X}} & 2l^2I_{\bar{Y}\bar{X}} & 0 & 0 & 6lI_{\bar{Y}\bar{X}} & 4l^2I_{\bar{Y}\bar{X}} & 0 \end{bmatrix} \tag{6}$$

Where, $I_{\bar{x}} = -A_{ce}e^2$, $I_{\bar{y}} = I_y$ where A_{ce} is the cross-sectional area of the crack, e is the centroid at Y.The area moments of inertia with respect to X and Y are written as I_x and I_y, respectively. Finally, the element stiffness matrix should be replaced by the sum between the element's modified stiffness matrix by the crack stiffness matrix as exemplified below by the equation 7.

$$K = K_e + K_{c_r} \tag{7}$$

The input data for the simulation was sampled from the parameter distributions described in Table 4.

Table 4 – Data Sampling for Transverse Crack Simulation.

Data Sampling for Transverse Crack Simulation.						
Rotor Parameters	**Distribution**	**Parameters**	**Unit**	**Nodes**	**Phase (rad)**	**Quantity**
Angular Velocity	Normal	$\mathscr{N}$(50,3)	[rad/s]	-	-	1
Crack Size	Uniform	$\mathscr{U}[0.01,0.4]$	-	$\mathscr{U}[4,14]\cup\mathscr{U}[32,42]$	-	1
Unbalance	Uniform	$\mathscr{U}[1,10]\cdot 10^{-6}$	$[Kg\cdot m]$	[9,37]	$\mathscr{U}[-\pi,\pi]$	2

Data Processing - Full Spectrum Features

The measurements used for the diagnosis with the Full Spectrum preprocessing come from time series of the X and Y displacements of the four bearings in a time window of 0.5 seconds of the rotor system in stedy state regime. From this a data processing is done applying a Frequency analysis with a Fourier transform in x and y to obtain values of the amplitudes of the $\pm 1X$, $\pm 2X$, $\pm 3X$, $\pm 4X$, $\pm 5X$ and $\pm 6X$ harmonics, where the positve ones are related to forward and the negatives to backward.

Bayesian Neural Networks applying Monte Carlo Dropout

The dropout procedure was formalized by (Srivastava et al., 2014) as a regularization method for neural networks, and consists of modeling the probability of a neuron participating or not in the model by a Bernoulli distribution. By applying the Monte Carlo test to neural network multiple predictions are obtained on the neural network with dropout. According to (Gal & Ghahramani, 2015) optimization based on stochastic gradient descent combined with the application of dropout during training produces the occurrence of the deep Gaussian process described by (Damianou & Lawrence, 2013) responsible for approximating Bayesian inference in the model. To determine the uncertainty related to prediction, multiple predictions are sampled from a neural network with dropout, and the uncertainty can be determined by calculating the standard deviation, the entropy, and the negative log likelihood.

RESULTS AND DISCUSSION

Once the signal processing techniques and the conditions for preparing the training data in the model were well defined, the neural networks were trained and their architecture was optimized in a common setup for all models. Table 5 shows the common characteristics of the models. In addition, Table 6 shows the errors obtained for each model in the Mean Absolute Percentage Error (MAPE) unit.

Table 5 – Neural Network Characteristics

Neural Network Characteristics for Fault Parameter Regression	
Number of Inputs	48
Data Collection	X and Y Displacements in the Bearings 1, 2, 3 and 4
Data Treatment	Full Spectrum
Number of Neurons	50
Number of Hidden Layers	2
Number of Outputs	1
Bias	Sim
Activation Function	Hyperbolic Tangent
Activation Function (Output Layer)	Linear
Loss Function	Mean Absolute Percentage Error (MAPE)
Optimizer	ADAM
Number of Epochs	2000
Learning Rate	0,0001
Bacth Size	10
Dropout	50%
Initialization Weights Type	Random Uniform
Amount of data used for Training	7500
Amount of data used for Testing	2000
Normalization Type	(0,1) and log10

Table 6 – Error of Failure Parameter Regression for each Model Trained

Error of Failure Parameter Regression Models	
Model	**MAPE (Error)**
Regression of Misalignment Angle	20.8%
Regression of Misalignment Distance	12%
Regression of Crack Size	19.22%
Regression of Crack Location	15.47%

Below are shown partial results of the regressions performed with their uncertainty indicated by the purple shaded area.

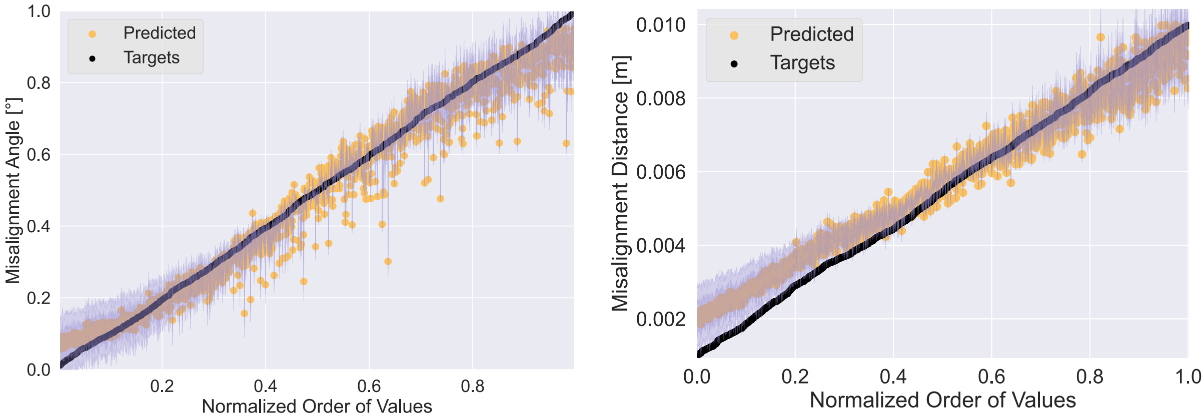

Figure 2 – a) Regression of Angular Misalignment b) Regression of Parallel Misalignment Distance.

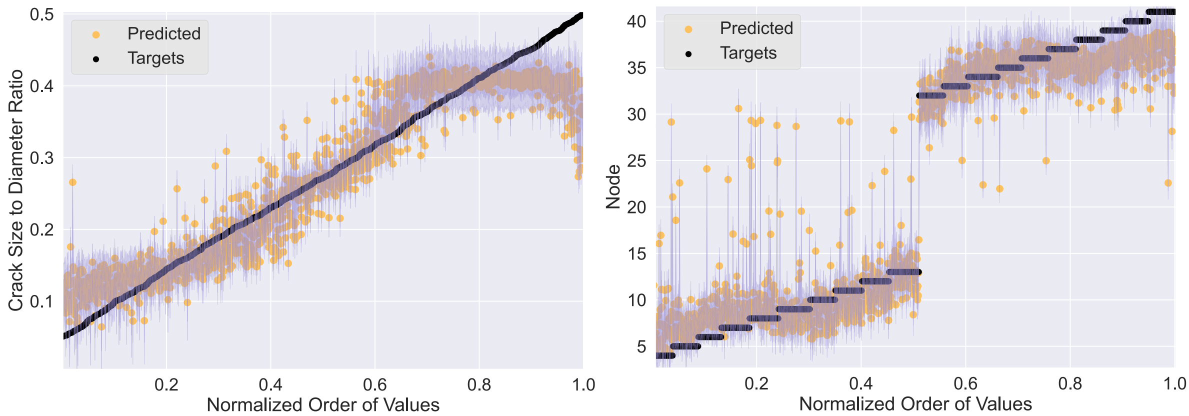

Figure 3 – a) Relative Crack Size Regression b) Crack Location Regression.

In all three fault severity regression graphs it was observed that the uncertainty tends to increase at the extremes of the fault severity. A probable reason is that in the region where the fault is not very severe its influence is small on the vibrational behavior of the machine and in the regions where the severity reaches its highest values it is due to the fact that non-linear phenomena start to interfere in the process. The regression performed to determine the location of the fault was successful in that it clearly determined the axis in which the fault was located.

It is interesting to point out that the model for predicting cracks size was able to estimate the size of cracks that changed position longitudinally and between the two shafts. Demonstrating great versatility and adaptability to the inherent characteristics of the mechanical system.

To study the behavior of the uncertainty coming from the models, graphs were generated that represent the increase in accumulated uncertainty by the accumulated error in the model predictions for the entire data set. All counts are performed from the ordering of the predictions by the uncertainty expressed in increasing form. It is expected that the uncertainty is directly proportional to the increase in error if this does not occur it means that the model is not able to represent its uncertainty and this occurs when there is lack of adherence of the model to the training data. Below in figures 4 and 5.

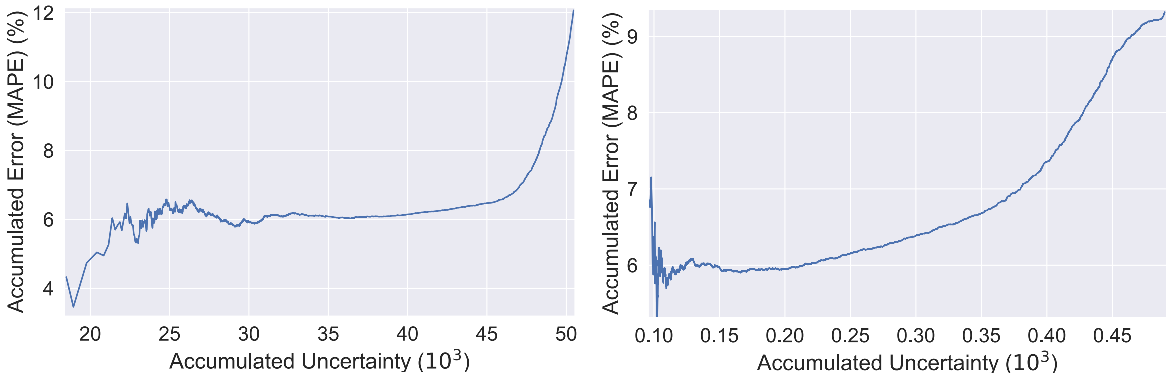

Figure 4 – a) Behavior of Uncertainty in the model Regression of Angular Misalignment b) Behavior of Uncertainty in the model Regression of Parallel Misalignment Distance.

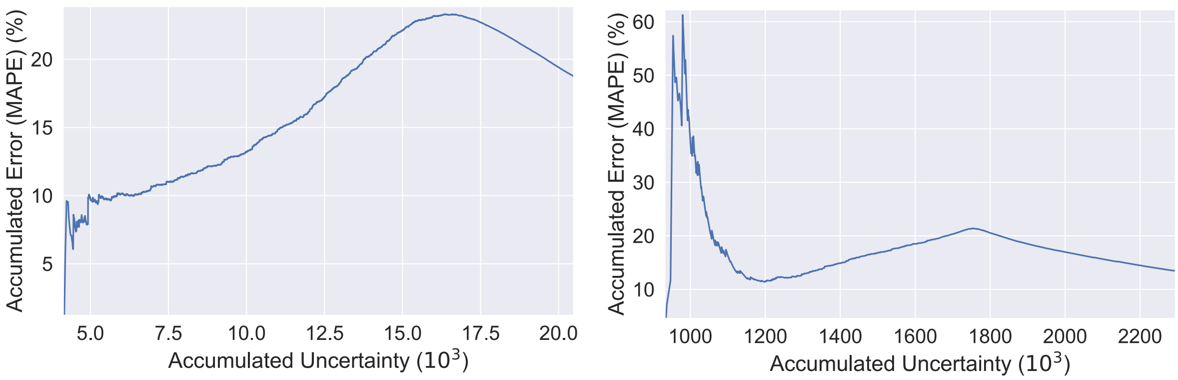

Figure 5 – a) Behavior of Uncertainty in the Model Relative Crack Size Regression b) Behavior of Uncertainty in the Model Crack Location Regression.

With the exception of the model for determining crack location, all other models showed the expected behavior trend for uncertainty and error.

CONCLUSION

In general, the behavior of the models was satisfactory in view of the fact that the greatest uncertainties were generated at the extremes of the regressed severity intervals. The trend of the severity curve was followed by all three models. The model that regressed the crack location was successful in determining in which axis the crack is, but, for determining the location of the crack node has proven ineffective in its expression of uncertainty, and for this reason should be improved in future work.

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