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Shaft flexibility and gyroscopic effects on the dynamic response of a geared rotor

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Abstract: This paper investigates the effect of shaft flexibility and gyroscopic effects on the dynamic response of the system by comparing the bearing vibration of a gear pair. In order to evaluate shafts flexibility effect, rigid and flexible shafts conditions were simulated. For the purpose of investigating gyroscopic effects, an additional disc, with the gear size, was included in the shaft. Results indicate that the flexible shaft presents a longer transient response. Additional disc also causes this effect. At permanent frequency responses, natural frequencies sidebands are present in the flexible systems. If gyroscopic effect is increased, another natural frequency is also present in the response.

Keywords: spur gear, gear pair, flexible shaft, time-varying mesh stiffness

INTRODUCTION

Özgüven and Houser (1988), divided gear mathematical models in five groups. One of these groups was considered to be the gear rotor dynamics model, in which the shaft transverse vibrations are considered in two perpendicular directions, so that the shaft can whirl.

Usually, geared rotordynamics models consist on a finite elements method model of the shaft and the gears are considered as rigid discs connected by a a spring-damper system, that represent gear mesh (Zhang et al., 2019). Mesh stiffness is a time-varying phenomenon and it is often modeled by a square wave function (Kahraman e Singh, 1991; Kahraman, 1994), or as a sine function (Theodossiades and Natsiavas, 2001; Shin and Palazzolo, 2020), since they are easy to implement and represent mesh stiffness variation without increase computational cost significantly. A more accurate representation of mesh stiffness is obtained by the potential energy method, which is also considered in many geared rotordynamics works, specially considering gear failure (Hu et al., 2016).

Geared rotors frequency response usually present acceleration amplitude peaks at the mesh frequency multiples. In the presence of a defect (crack, spalling, eccentricity), this response exhibits frequency sidebands (Chen et al., 2022; Huangfu et al., 2020; Zhao et al., 2020). However, the presence of sidebands was also found on a healthy geared rotor with a flexible shaft and an additional disc (Visnadi and Castro, 2022).

Studies considering geared rotordynamics often simulate systems with rigid shafts. This paper compares the bearing vertical acceleration signal of a gear pair assembled in rigid shafts and in flexible shafts. The only altered parameter was the shafts diameter and both time and frequency responses were analysed.

MATHEMATICAL MODEL

In order to obtain a detailed response of the shaft motion, the rotor system model is obtained by the finite elements method (FEM), considering the shaft's elements as Timoshenko beams, the gears as rigid discs coupled by the mesh force ($\mathbf{F_m}$) and the bearings are represented as constant stiffness and damping coefficients. This model was proposed by Rao, Shiau and Chang (1998).

The mesh force is represented by time-varying damping ($c_m(t)$) and stiffness ($k_m(t)$):

$$\mathbf{F_m} = c_m(t)\dot{\mathbf{q}}_{\mathbf{pg}} + k_m(t)\mathbf{q}_{\mathbf{pg}}, \tag{1}$$

where $\mathbf{q}_{\mathbf{pg}}$ is the displacement vector of the gear pair.

Mesh damping is often dependent on the mesh stiffness,($k_m(t)$), on a damping factor, (ξ_m), on pinion and gear polar moments of inetia (J_p, J_g) and on pinion and gear base radii (r_{bp} e r_{bg}), (Yi, Huang, Xiong, Sang, 2019):

$$c_m(t) = 2\xi_m\sqrt{k_m(t)J_pJ_g/(J_g r_{b_p}^2 + J_p r_{b_g}^2)}, \tag{2}$$

Mesh stiffness ($k_m(t)$) is calculated by considering tooth potential energy. It is time-varying because the tooth is a variable cross section beam and the number of pairs in contact at the same time, n also varies, since gears contact ratio must be greater than one in order to guarantee the gears contact and provide a smoother motion transmission (Ma, Song,

Table 1 – Simulation parameters

Parameter	Pinion	Gear
Young modulus [GPa]	212	
Density [kg/m³]	7800	
Poisson ratio	0.3	
Module [mm]	6	
Face width [mm]	16	
Pressure angle (α_0) [°]	20	
Mesh damping ratio (ξ_m)	0.03	
Bearings stiffness coefficients [N/m]	2.10^8	
Bearings damping coefficients [Ns/m]	150	
Number of teeth	25	25

Pang, Wen, 2014):

$$k_m(t) = \sum_{i=1}^{n} k_{pair_i} \tag{3}$$

The tooth pair stiffness is given by a series association of the teeth stiffness (Ma, Song, Pang, Wen, 2014):

$$k_{pair}(t) = (1/k_{tooth_1} + 1/k_{tooth_2})^{-1} \tag{4}$$

Tooth stiffness is the series association of bending (k_b), axial compressive (k_a), shear (k_s) and fillet foundation (k_f) stiffness:

$$\frac{1}{k_{tooth}} = \frac{1}{k_b} + \frac{1}{k_a} + \frac{1}{k_s} + \frac{1}{k_f} \tag{5}$$

Details about how to calculate each one of these stiffness components may be found on Ma, Song, Pang and Wen (2014).

Considering the FEM rotor model, the global equation of the entire system is (Rao, Shiau and Chang, 1998):

$$\mathbf{M}\ddot{\mathbf{q}} + [\dot{\theta}_{xp}\mathbf{G} + \mathbf{C}(\mathbf{t})]\dot{\mathbf{q}} + \mathbf{K}(\mathbf{t})\mathbf{q} = \mathbf{F}\,, \tag{6}$$

where $\mathbf{M}$, $\mathbf{G}$, $\mathbf{C}(\mathbf{t})$, $\mathbf{K}(\mathbf{t})$ are the mass, gyroscopic, damping and stiffness matrices, respectively. $\mathbf{F}$ is the external forces vector. The overdot indicates time derivative and $\mathbf{q}$ the displacement vector, for each node. Damping and stiffness matrices are time varying because of the gear pair terms, properly allocated in the corresponding nodes. Crossed terms of these matrices couple the shafts movement (see Rao, Shiau and Chang, 1998 for further details).

RESULTS

A system with two identical shafts supported by two identical bearings each one and with the gears mounted on it centers was considered. Both shafts have Young's Modulus of 200 GPA , density of 7800 kg/m^3 and total length of 0.55 m. It was considered that shaft damping is proportional to its stiffness by a coefficient of 3.10^-5. Gears and bearing parameters are presented in Tab. 1.

In order to verify the shaft flexibility effect on the dynamic response of the geared rotor, two different shaft conditions were tested: rigid (with a 0.056 m diameter) and flexible (with a 0.016 m diameter). The considered system is the same presented by Chen et. al (2022), and the nodes and gears positions are the same. Figure 1 displays the natural frequencies up to 500 Hz of both rigid (a) and flexible (b) shafts systems, in a Campbell diagram. The rigid rotor first natural frequency is 356.9 Hz, and the flexible shafts first natural frequency is much lower: 47.9 Hz. Systems time responses were obtained by the Newmark-beta integration method, with its coefficients set according to the middle point rule, which is unconditionally stable. The time step considered was 1.10^{-5} s.

A ten seconds time response of the same systems is presented in Fig. 2. The rigid rotor (a) presents a permanent response after 0.1 s, meanwhile the flexible geared system (b) presents a transient response in at the first 4 s. At permanent response, maximum amplitude vibrations are similar, for both systems. A 0.05 s of the permanent response of both systems is presented in Fig. 3. Both responses are periodic, but the flexible shaft response presents more oscillation in a same period.

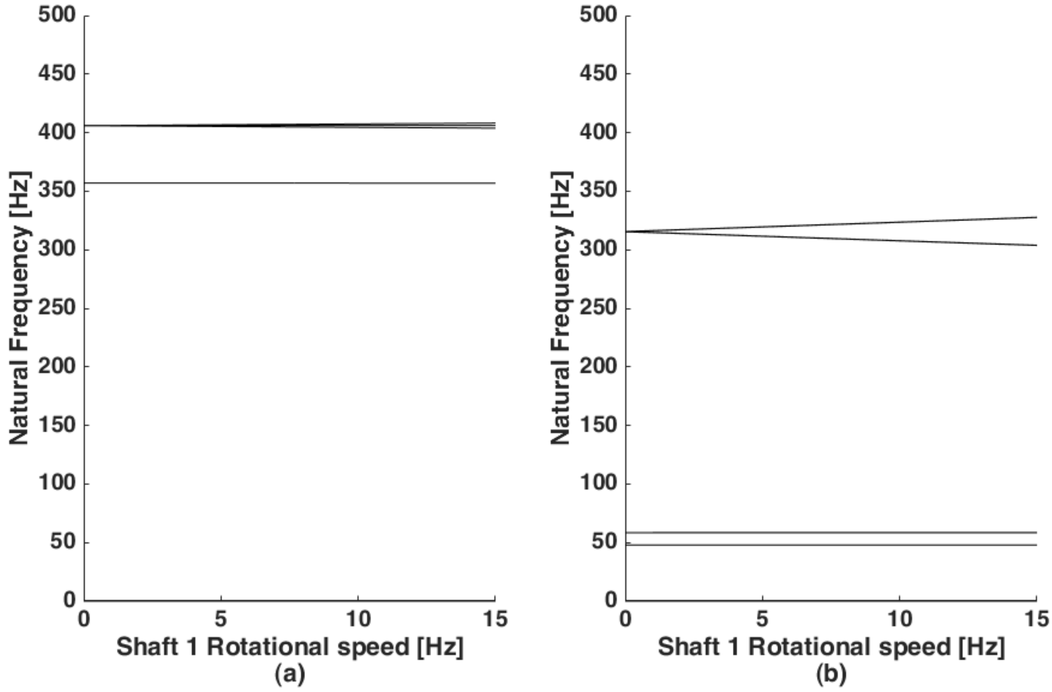

Figure 1 – Campbell diagram up to 500 Hz (a) in the rigid shaft system (b) in the flexible shaft system

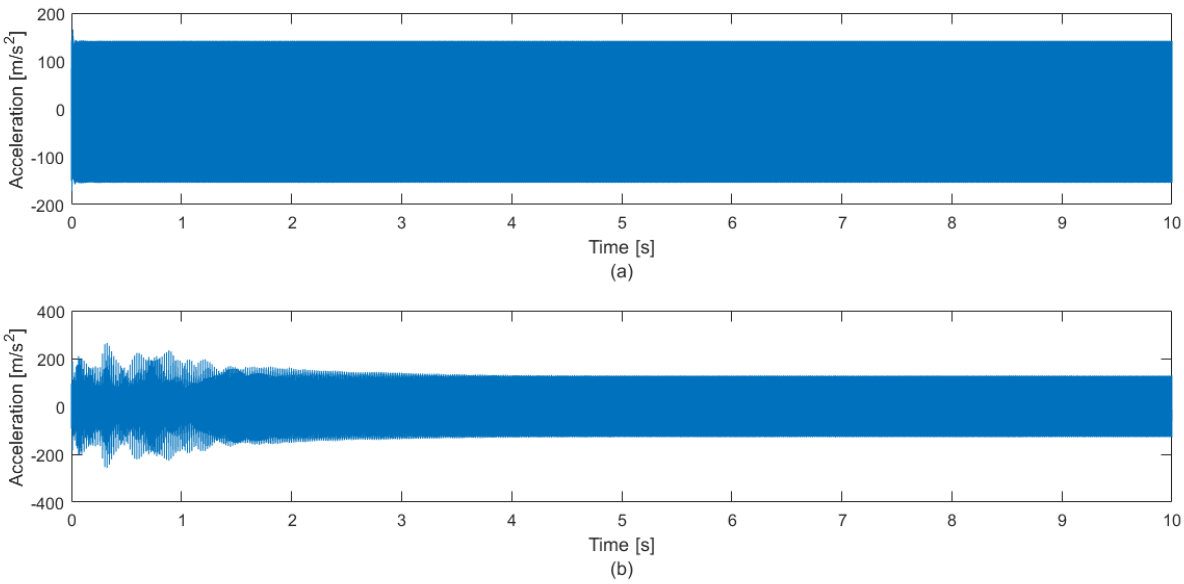

Figure 2 – Bearing 1 vertical acceleration 10 seconds time response (a) in the rigid shaft system (b) in the flexible shaft system

Rigid and flexible systems frequency responses are displayed in Fig. 4 (a) and (b), respectively. Both responses present amplitude peaks at multiple frequencies of the mesh frequency f_m, as indicated. In the rigid system response (a), there is a clear presence of shaft rotational speed sidebands. In the flexible shafts response (b), these sidebands are not so clear, but there is an evident amplitude peak at the system first natural frequency.

In order to understand gyroscopic effects on the dynamic response of the system, the same rigid and flexible shafts were considered, but in the first shaft the gears where placed in node 5 and an additional disc, with the same size as the the gear, was added in node 10. Note that the nodes references are the same as presented by Chen et. al (2022). A 10 s time response for both rigid (a) and flexible (b) shafts systems are presented in Fig. 5. Rigid shafts system (a) present a

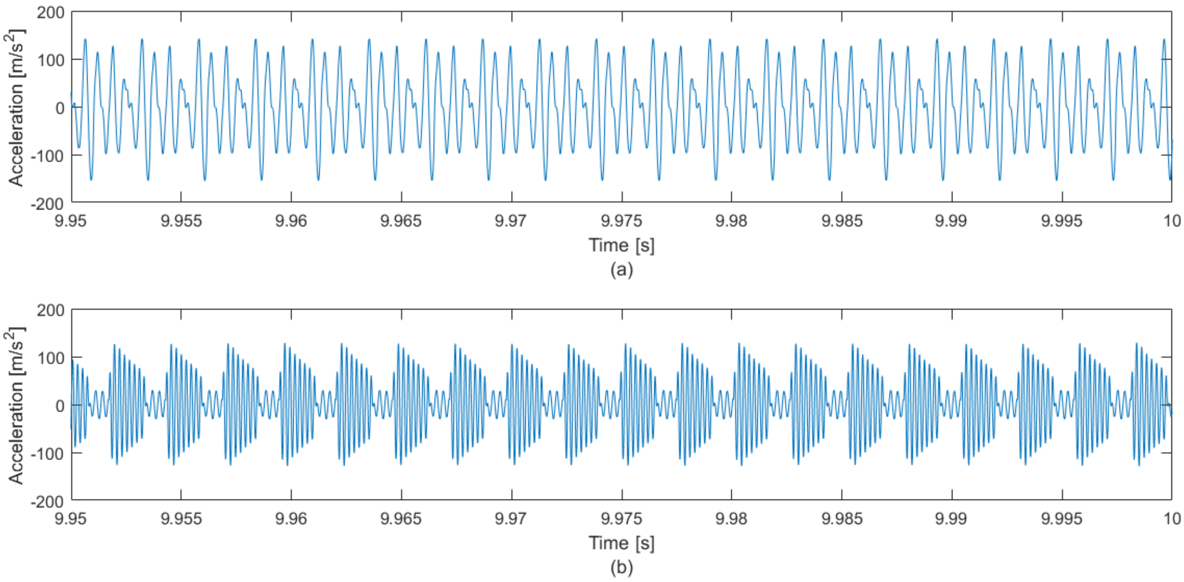

Figure 3 – Bearing 1 vertical acceleration 0.05 seconds permanent time response (a) in the rigid shaft system (b) in the flexible shaft system

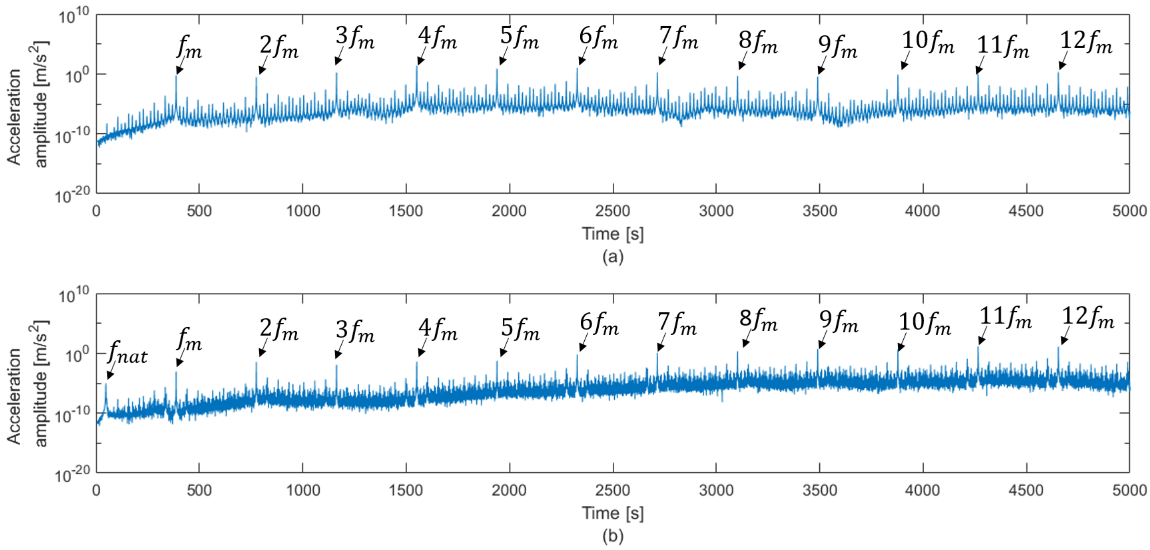

Figure 4 – Bearing 1 vertical acceleration permanent frequency response (a) in the rigid shaft system (b) in the flexible shaft system

permanent response after 1 *s*, meanwhile the flexible system presents a permanent response after 5 *s*. A 0.05 *s* permanent time response is displayed in Fig. 6. Responses are periodic, and the rigid system with additional disc presents more oscillations in a same time period when compared to the rigid system with no additional disc response (Fig. 3 (a)).

Figure 7 presents frequency response of rigid (a) and flexible (b) systems with additional disc. Similarly to the systems with no additional disc, there are amplitude peaks at multiples of the mesh frequency. For the rigid system (a), shaft rotational speed sidebands are evident. For the flexible system (b) two peaks at natural frequencies are present.

CONCLUSION

This paper investigated the influence of shaft flexibility and the gyroscopic effect on a geared rotor acceleration response. Time and frequency response of a rigid and a flexible shafts systems were compared. The only difference between

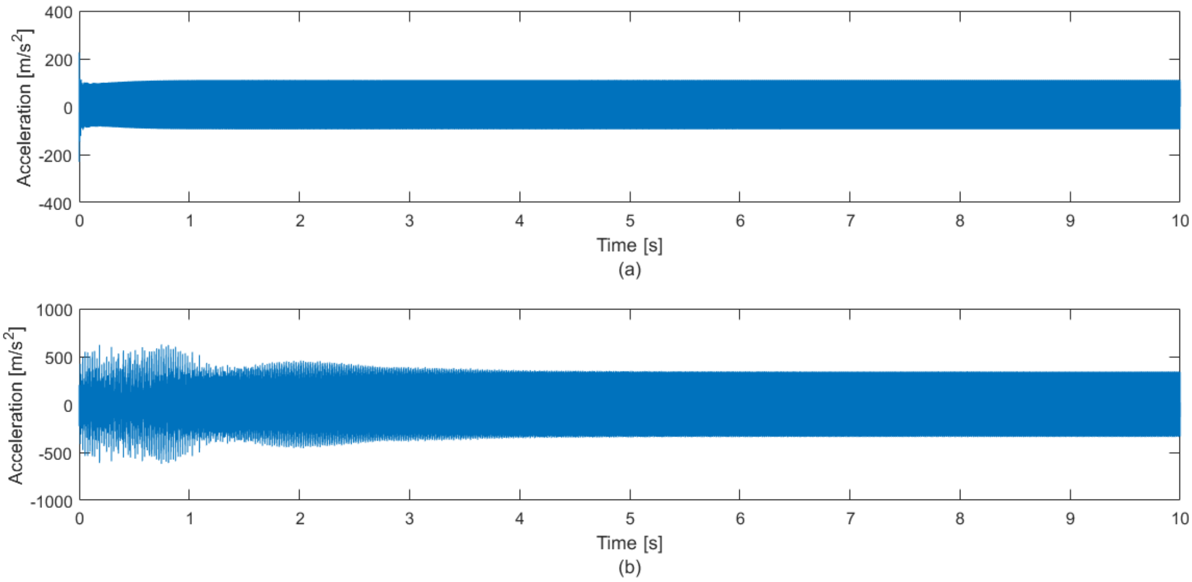

Figure 5 – Bearing 1 vertical acceleration 10 seconds time response (a) in the rigid shaft system (b) in the flexible shaft system, with additional disc

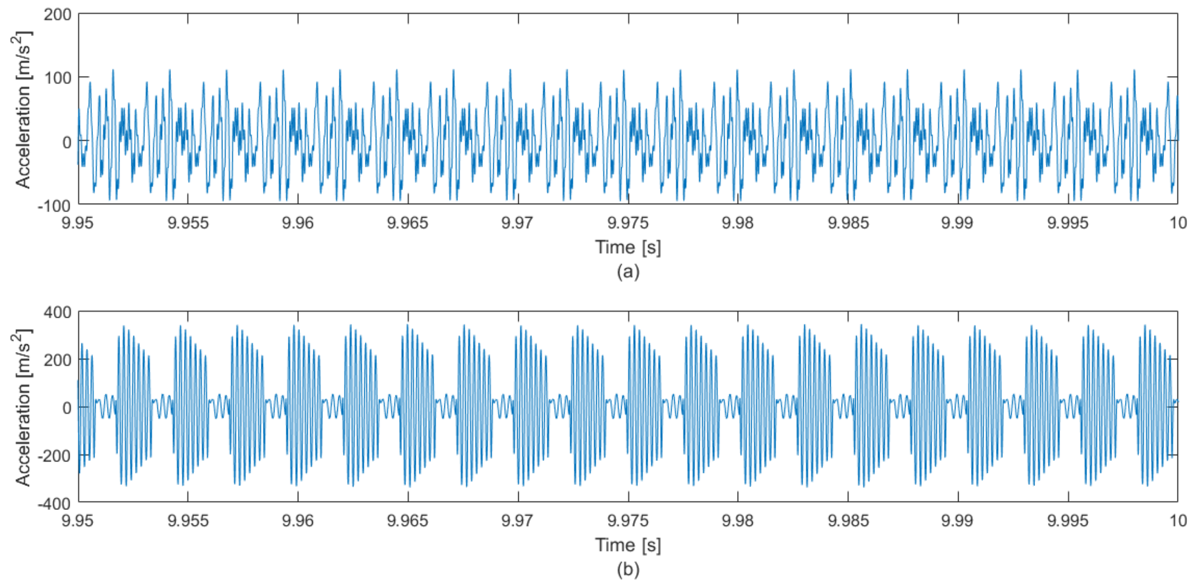

Figure 6 – Bearing 1 vertical acceleration 0.05 seconds permanent time response (a) in the rigid shaft system (b) in the flexible shaft system, with additional disc

the considered rotors were the shafts diameters.

Results indicate that the geared system dynamic response if affected by the shaft flexibility. A flexible shaft presents a bending motion, which corresponds to the first natural vibration mode, at a relative low frequency, i.e., the first natural frequency of the flexible shafts rotor is lower than the first natural frequency of the rigid shafts rotor.

As a consequence, the acceleration response of the flexible rotor presents a longer transient response. When an additional disc in included in the shaft, gyroscopic effects are higher, and the transient response period gets even longer. Flexible shaft response also presented and acceleration peak at a natural frequency, and in the presence of an additional disc, another natural frequency peak was present.

All of the considered systems presented shaft rotational speed sidebands. If the shafts are flexible, natural frequency

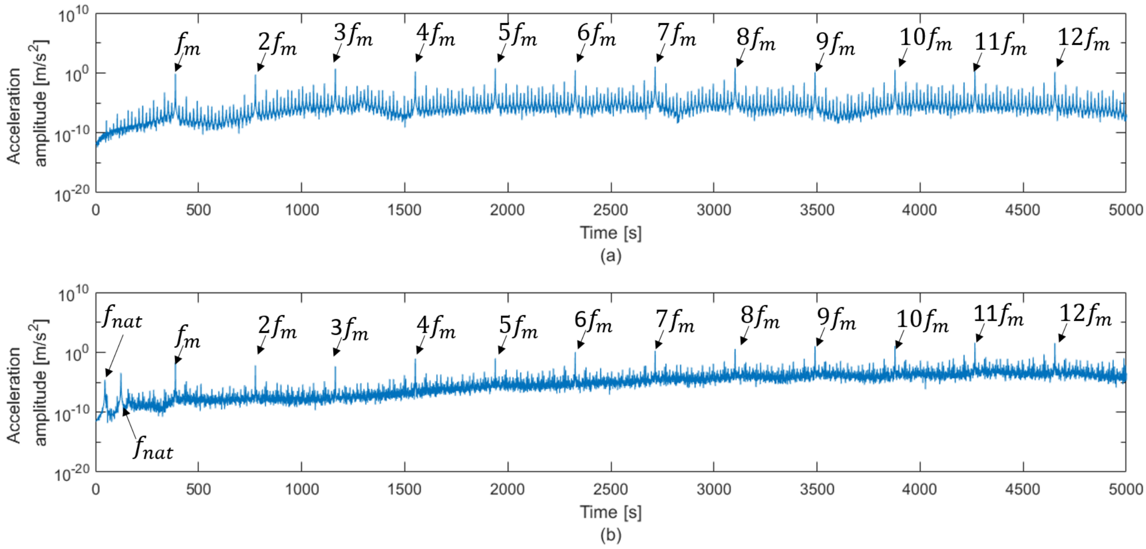

Figure 7 – Bearing 1 vertical acceleration permanent frequency response (a) in the rigid shaft system (b) in the flexible shaft system, with additional disc

sidebands are also present. If shafts are flexible and gyroscopic effects are potentiated, another natural frequency also contributes to sidebands. Since these sidebands usually indicates gear failure, the analysis of a flexible geared rotor response may lead to a more difficult diagnosis process.

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