Influence of shaft flexibilities and unbalance moment on a spur gear pair

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Abstract: Geared systems present a non-linear behaviour due to time varying mesh stiffness, which is a parametric excitation, backlash and the static transmission error. Most studies about these systems often do not consider the shaft flexibilities. Previous study showed that including these flexibilities on the model might cause great difference on the results for spur gears initially assembled horizontally. The present study intends to analyze the influence of shaft flexibilities on a spur gear pair, considering unbalance and anisotropy. Results indicate that, for the adopted system configuration, horizontal motion is highly affected by unbalance.

Keywords: gear pair dynamics, spur gear, unbalance moment, time-varying parameters

INTRODUCTION

Geared systems have been studied systematically since the 1920s, but more complex models, including the teeth stiffness and nonlinearities were introduced back on the 1970s (Özgüven and Houser, 1988). These systems present a non-linear behaviour due to time varying mesh stiffness, which is a parametric excitation, backlash and the static transmission error. Besides, the development of computers allowed the models to include more effects on dynamic models, obtaining more accurate simulated responses.

Özgüven and Houser (1988b) developed a non-linear model of a gear pair with a single degree of freedom, in order to analyse the mesh dynamics. Later, Özgüven (1991) developed a six-degree-of-freedom model for a spur gear pair with time-varying mesh stiffness, considering shaft and bearing flexibilities. He concluded that, if bearing and shaft are not stiff enough, their influence could not be neglected on the system response, so the single-degree-of-freedom model was not accurate in such problems. A more complete model considering shaft flexibilities for a helical gear train was published by Kahraman (1994). This model was used on the study of Kubur, Kahraman, Zini, and Kienzle (2004), that developed a model of finite elements for N flexible shafts, which was validated with experimental results.

These works considered a time-varying mesh stiffness, but they were approximated, not considering the tooth profile. More recent works (Pedrero, Pleguezuelos, Artés and Antona, 2010; Ma, Song, Pang, and Wen, 2014) developed this calculation according to the involute curve of the tooth and obtained the analytical expressions for the mesh stiffness considering the tooth as a non-uniform cantilever beam and applying the potential energy method. When considering bearing flexibilities on the geared system, the translational motions of the gears centers causes changes on their distance, so that the operating pressure angle and the contact ratio changes.

Kim, Yoo and Chung (2010) first approached this problem for a spur gear pair, providing a more complete model and obtaining more accurate results about the system response. Yi, Huang, Xiong and Sang (2019) included the variable backlash in this previous model. They showed that significant changes occur on the response of the system when compared to the model with rigid supports, including the possibility of a chaotic motion. The presented results neglected the effects of gravity and unbalanced masses.

This paper considers the same model proposed by Yi, Huang, Xiong and Sang (2019). Although they included unbalance moments and gravity effects on their models, they neglected it on the results. The present study considers both of them and intends to investigate the influences of unbalance moments, shaft flexibilities, shaft anisotropy and backlash on a spur gear pair. Preliminary results indicate that gear unbalance may have great influence on gear pair dynamics.

THE SIX DEGREE OF FREEDOM MODEL OF SPUR GEAR PAIRS

A spur gear pair is often modeled as a six degree of freedom system, if torsional deformations of the gear bodies and motions that occur on the axial plane are not considered. Figure 1 presents a scheme of a spur gear pair with support flexibilities, so that translational motions of their centers are possible.

Figure 1 also represents the gear mesh along the line of action as an association of a spring with variable stiffness and a damper (representing teeth deformations), a discontinuity (representing the backlash) and a time-dependent function (representing possible assembling or manufacturing errors).

The equations of motion of these six degrees of freedom system, considering unbalanced masses on the discs are (detailed explanation of the equations may be found on the work of Yi, Huang, Xiong and Sang, 2019):
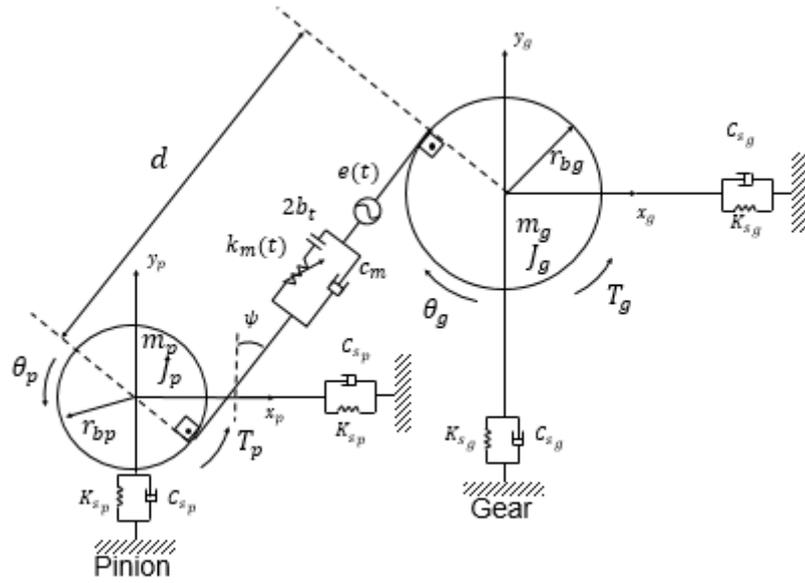
Figure 1 – Gear pair scheme

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F_{mesh}}(t) + \mathbf{F_{unb}}(t) + \mathbf{F_{grav}}(t) + \mathbf{T} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, support damping and support stiffness matrices. $\mathbf{u}$ the displacements vector and the dot above it indicates a time derivative. $\mathbf{F_{mesh}}$ is a vector of the gear mesh force, $\mathbf{F_{unb}}$ is a vector of the unbalance forces, $\mathbf{F_{grav}}$ is a vector of gravity forces (only has non-zero terms on the vertical direction) and $\mathbf{T}$ is the torque vector (only has non-zero terms on the rotational directions).

The unbalance forces on y, x and θ directions are given by Eq. (3), (4) and (5) , where me is the unbalance moment (product between of pinion mass and mass eccentricity), J is the polar moment of inertia and m is the mass:

$$F_{unb_{yk}} = -me_k(\ddot{\theta}_k cos\theta_k - \dot{\theta}_k^2 sin\theta_k) \quad k = p,g \tag{2}$$

$$F_{unb_{xk}} = me_k(\ddot{\theta}_k sin\theta_k + \dot{\theta}_k^2 cos\theta_k) \quad k = p,g \tag{3}$$

$$F_{unb_{\theta k}} = -J_k me_k^2/m_k \ddot{\theta}_k \quad k = p,g \tag{4}$$

The mesh force is on the direction of the line of action of gearing and is given by Eq. (6). The vector correspondent to this force on Eq. (1) has the components of this force properly decomposed in which direction, and with the proper positive or negative sign for which body.

$$F_{mesh} = c_m(t)\dot{h}(p(t)) + k_m(t)h(p(t)) \tag{5}$$

where $c_m(t)$ is the mesh damping coefficient, $k_m(t)$ is the time-varying mesh stiffness, $p(t)$ is the tooth deformation and $h(p(t))$ is the backlash function. These time-dependent parameters are detailed on the next section.

TIME-VARYIING PARAMETERS

The mesh damping coefficient is related to a damping ratio ξ_m and is dependent on the time-varying mesh stiffness (Yi, Huang, Xiong and Sang, 2019):

$$c_m(t) = 2\xi_m \sqrt{k_m(t)J_pJ_g/(J_gr_{bp}^2 + J_pr_{bg}^2)} \tag{6}$$

Mesh stiffness $k_m(t)$ is calculated by considering contact ratio and the pair stiffness as a series association of each tooth stiffness. Tooth stiffness is calculated by the potential energy method and it considers the tooth as a variable cross

section beam. Details about the mesh stiffness calculation method used in this work may be found in Ma, Song, Pang, and Wen (2014).

The tooth deformation along the line of action is also named as dynamic transmission error (DTE). Equation 14 describes how this deformation is calculated. In order to consider manufacturing and assembly errors, the DTE has an error term that is time-dependent $e(t)$, usually known as static transmission error (STE) and considered to be random, as described by Kim, Yoo and Chung (2010).

$$p(t) = y_p \cos(\psi) + x_p \sin(\psi) - y_g \cos(\psi) - x_g \sin(\psi) + r_{bp}\theta_p + r_{bg}\theta_g - e(t) \tag{7}$$

In order to prevent failure due to elastic deformations and thermal expansion, a microscopic clearance between pinion and gear teeth, known as backlash (b_t), is necessary on gear operations (Xiong et al., 2019). If the backlash is considered, the tooth deformation is no longer correspondent to $p(t)$. It is also possible to occur a contact loss, so it is necessary to define the backlash function:

$$h(p(t)) = \begin{cases} p(t) - b_t, & \text{if } p(t) > b_t \\ 0, & \text{if } |p(t)| \leq b_t \\ p(t) + b_t, & \text{if } p(t) < b_t \end{cases} \tag{8}$$

The centre distance of the gears is affected by their displacements (Kim, Yoo and Chung, 2010; Yi, Huang, Xiong and Sang, 2019):

$$d = \sqrt{(x_g - x_p + d_0)^2 + (y_g - y_p)^2} \tag{9}$$

where d_0 is the initial centre distance, calculated by the sum of base radius of pinion and gear. Since this distance is variable, the pressure angle (α), the position angle (ψ) and the backlash (b_t) also are, as shown in Equations 17, 18 and 19, respectively:

$$\alpha = \cos^{-1}\left(\frac{r_{bp} + r_{bg}}{d}\right) \tag{10}$$

$$\psi = \alpha - \frac{y_g - y_p}{x_g - x_p + d_0} \tag{11}$$

$$b_t = b_0 + (r_{bp} + r_{bg})(\text{inv}(\alpha) - \text{inv}(\alpha_0)) \tag{12}$$

If these considerations are assumed, the expression of the mesh force is not as simple as described on Equation (12), since both position angle (ψ) and backlash (b) are not constant anymore and their time derivatives must be considered. Full deduction of the expressions is provided by Yi, Huang, Xiong and Sang (2019)

RESULTS

A gear pair with flexible supports was simulated, and the considered constant parameters are presented in 1. A factorial design of experiments (DoE) was conducted, considering two levels of unbalance mass for each gear body and two levels of support stiffness. Table 2 shows the simulation number and the considered parameter value if each one.

System equation of motion was solved by the Newmark-beta method (Newmark, 1959), with the parameters set by the middle point rule and the time step of 1×10^{-9} s. Response was evaluated in the frequency domain, obtained by applying a Discrete Fourier Transform (DFT) to the time response. Influence of parameters was observed in terms of amplitude and frequency of the highest peak of the DFT, which are presented in Tab. 3.

Table 3 indicates that the frequencies of the highlighted peaks have a certain pattern. The frequency of the higher peak of the pinion in y direction is the same for all simulations (13220 rad/s). The pinion displacements in x direction when unbalance in the pinion were neglected also present the same frequency value (simulations 1 ,3, 5 and 7).This frequency is the mesh frequency, which is given by the body rotational speed multiplied by its number of teeth.

For the gear, the frequency of the higher peak is not the mesh frequency on any simulation. On the y direction the frequency varies according to the support flexibility. This frequency corresponds to natural frequency of the body-shaft system.

When comparing results of simulations that add an unbalance moment on the pinion (i.e. comparing simulations 2 to 1 and 6 to 5), the frequency of the x peak changes from mesh frequency to pinion rotational speed frequency, and there is

Table 1 – Simulation parameters

Parameter	Pinion	Gear
Young modulus [GPa]	206	
Density [kg/m³]	7850	
Poisson ratio	0.3	
Module [mm]	5	
Face width [mm]	40	
Initial pressure angle (α_0) [°]	20	
Initial relative position angle (ψ_0) [°]	20	
Mesh damping ratio (ξ_m)	0.07	
Gravity [m/s²]	9.81	
Shaft damping coefficient ($C_{s_{xp}}, C_{s_{yp}}, C_{s_{xg}}, C_{s_{yg}}$) [Ns/m]	512.64	
Initial contact ratio	1.67	
Initial centre distance (d_0)[mm]	190	
Maximum mesh stiffness ($k_{m_{max}}$)[N/m]	5.093×10^8	
Minimum mesh Stiffness ($k_{m_{min}}$) [N/m]	3.024×10^8	
Number of teeth (N_p/N_g)	21	55
Rotational Speed ($\dot{\theta}_p/\dot{\theta}_g$) [rad/s]	628.32	239.90
Torque [N.m]	300	785.71
Unbalance phase (if applied) [°]	0	0
Mass (m_p/m_g) [kg]	2.720	18.650

Table 2 – Unbalance moment and support stiffness considered in each simulation

Simulation number	Pinion unbalance moment [kg.m]	Gear unbalance moment [kg.m]	Support Stiffness [N/m]
1	0	0	1×10^8
2	1×10^{-4}	0	1×10^8
3	0	1×10^{-4}	1×10^8
4	$1 \times 10^{\times} 10^{-4}$	1×10^{-4}	1×10^8
5	0	0	5×10^7
6	1×10^{-4}	0	5×10^7
7	0	1×10^{-4}	5×10^7
8	1×10^{-4}	1×10^{-4}	5×10^7

a significant increase on amplitude. On the other hand, there is no significant change on the y direction, because the effect of mesh gear force is preponderant in this direction, so the unbalance does not cause an effect on the major peak.

Similarly, the addition of unbalance moment on the gear (i.e. comparing simulations 3 to 1 and 7 to 5), makes the frequency of the x peak to change from the natural frequency of the system to the frequency of the gear rotational speed.

When both pinion and gear have an unbalance moment, the same effects are observed, and there is no interaction between them. So, the simulations with unbalance moment on pinion and gear (i.e. simulations 4 and 8) are equivalent to a sum of the effects of the simulation with unbalance only on the pinion and only on the gear.

Figure 2 shows the displacements DFTs of pinion horizontal direction (a), pinion vertical direction (b), gear horizontal direction (c) and gear vertical direction (d) obtained as a result for simulation number 4, as an example. The markers indicate the body (pinion or gear) rotational speed (circle), the mesh frequency (asterisk) and the natural frequency of the body-shaft system (cross).

Analyzing the influence of the increasing shaft flexibility, i.e., comparing both simulations cases, the first from 1-4 to 5-8, it is noteworthy that the amplitudes of pinion vibration on the y direction are reduced. Moreover, the amplitudes of the pinion on the x direction and the amplitudes on both directions of the gear are increased. Finally, the change on the gear peak frequency when it corresponds to the natural frequency occurs because natural frequency depends on the support flexibility.

CONCLUSION

This paper intended to investigate the influences of unbalance and shaft flexibility on a spur gear system with transmission ratio greater than one, initially assembled horizontally, using a complete model that considers variable center distance

Table 3 – Amplitude and frequency of the higher peak of DFTs

| Simulation | Amplitude [μm] | | | | Frequency [rad/s] | | | |
| | Pinion | | Gear | | Pinion | | Gear | |
	y direction	x direction	y direction	x direction	y direction	x direction	y direction	x direction
1	1.456	0.001	0.578	0.007	13220	13220	2262	2262
2	1.455	0.260	0.606	0.007	13220	653.5	2262	2262
3	1.466	0.001	0.578	0.054	13220	13220	2262	251.3
4	1.466	0.259	0.578	0.054	13220	653.5	2262	251.3
5	1.374	0.003	0.615	0.011	13220	13220	1609	1659
6	1.373	0.525	0.615	0.011	13220	653.5	1609	1659
7	1.374	0.003	0.615	0.110	13220	13220	1609	251.3
8	1.373	0.525	0.615	0.110	13220	653.5	1609	251.3
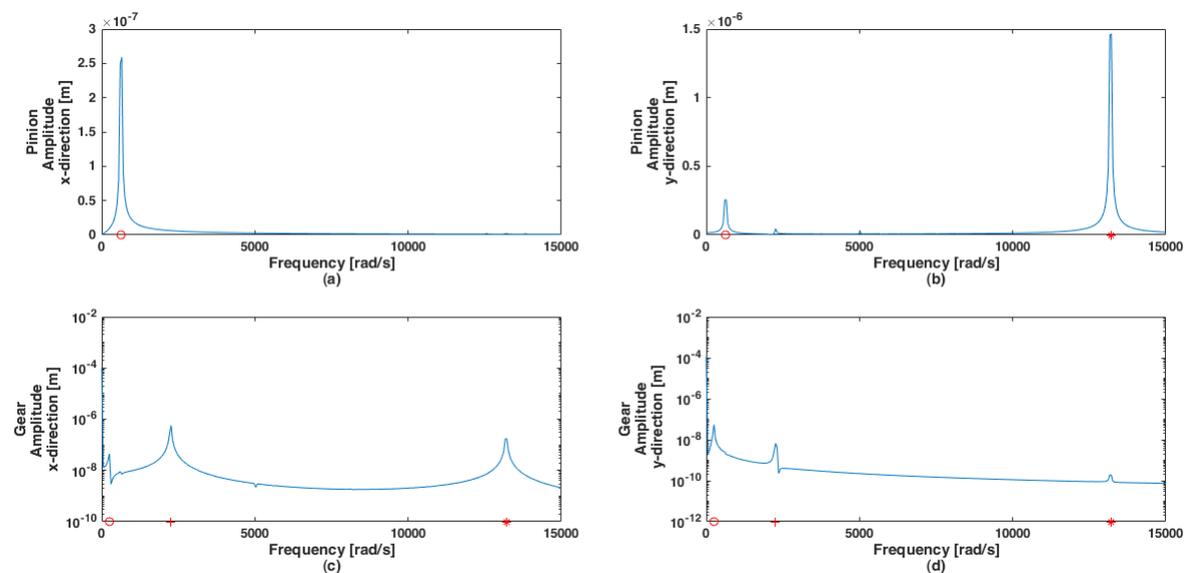
Figure 2 – Pinion and gear displacements DFTs of simulation number 4. Markers: (*) natural frequency; (o) body rotational speed; (+) natural frequency

and time-varying mesh stiffness, both calculated on each time step. The Newmark method was applied to integrate the time response and the effects were analyzed by considering the DFTs of both pinion and gear responses.

According to results presented on the previous section, some preliminary conclusions may be done:

a) The dominant excitation frequency on the pinion, when there is no unbalance, is the mesh frequency.

b) The dominant excitation frequency on the gear, when there is no unbalance, is the natural frequency of the shaft-gear system, which is maybe caused by the non-linear effects of the system.

c) When unbalance moment is added, the predominant excitation frequency on the x direction changes to the body rotational speed.

d) Since the mesh forces are much higher on the y direction, unbalance has no significant effect on this direction.

e) The decrease of shaft stiffness causes a reduction of amplitude of pinion vibration on the y direction, but the amplitudes of the pinion on the x direction and the amplitudes on both directions of the gear increase.

f) The change on the shaft flexibility changes the balanced gear excitation frequency, since it is related to its natural frequency.

Based on these conclusions, it is suggested for future works:

a) The influence of unbalance should be investigated on systems with different configurations, so the mesh forces on x and y directions are different from the present study.

b) To investigate why the decrease of shaft stiffness causes a decrease of vibration amplitude of pinion on y direction, associated to the mesh frequency

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