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A stochastic model for intermittent two-phase flow in horizontal pipes

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Abstract: Intermittent flows are common flow patterns in gas-liquid horizontal flow and attract attention and great research effort due to its importance for industrial and engineering applications. The slug flow is typically modelled based on a unit cell varying from an elongated air bubble with a liquid film in segregated flow pattern and an aerated liquid plug, the slug region, with remarkable stochastic characteristics of its alternating regions. In this paper, a two-state Markov chain model is proposed to represent the stochastic dynamics of developed slug flow in horizontal pipes. Each state represents either the liquid slug or the elongated bubble regions and the transition probabilities dictate the change of the given discrete time measurement to stay at a given state or change. This simple but insightful description of the phenomenon allows an analytical treatment of the statistics of Markov chain stochastic process. Measurement stations with two double wire resistive sensors are used to obtain the void fraction time series and a corresponding two-state representation. It is shown that the Markov chain model can successfully represent second-order statistics of the measurement, such as the autocorrelation and power spectral density, given an appropriate choice of the chain order. Subsequently, statistics of some slug flow features are estimated using the proposed approach and their interpretation as random variables derived from the void fraction stochastic process is discussed.

Keywords: Two-phase flow, Slug Flow Pattern, Markov Chain, Experimental Characterisation, Stochastic Process

INTRODUCTION

Multiphase flows are of common occurrence in numerous natural and industrial processes. One of the most common is the gas-liquid flow, where the phases are distributed in different geometric arrangements, which are called flow patterns. These patterns depend on fluids properties (density, viscosity, surface tension) and flow conditions (flow rates, pipe diameter and slope, etc) (Shoham, 2006; Ishii and Hibiki, 2011). In gas-liquid horizontal flows the stratified, annular, bubbles and intermittent flows are the most common patterns, each one with its own characteristics. The correct definition of the flow pattern and its characteristics is of ultmost importance for industrial purposes, as the presence of an specific pattern may lead to severe problems. In the case of the oil industry, flow assurance problems are related to the occurrence of an specific flow pattern (Shippen and Bailey, 2012). Efforts have been made during the last decades to propose models for the transition between such flow patterns (Taitel and Dukler, 1976; Barnea, 1987), as well as modelling the flow pattern itself leading to pressure drop and void fraction calculation, for example (Shoham, 2006; Ishii and Hibiki, 2011).

One of the most common and at same time complex gas-liquid flow pattern is the slug flow, as depicted in the schematic Fig. 1. The slug flow is typically modelled based on a unit cell varying from an elongated air bubble with a liquid film in segregated flow pattern (considered stratified flow) of length L_f, to a liquid slug with/without dispersed gas bubbles, of length L_s. Both patterns compose a whole periodic structure with length L_U called a unit cell (Taitel and Barnea (1990); Fabre and Liné (1992); Netto et al. (1999)). These structures are connected by a turbulent recirculating zone with length L_m (Wallis (1969); Shoham (2006); Ishii and Hibiki (2011)). One remarkable interesting aspect of the slug flow is its stochastic characteristic of alternating regions (Sarica et al. (2011); Soedarmo et al. (2019)).

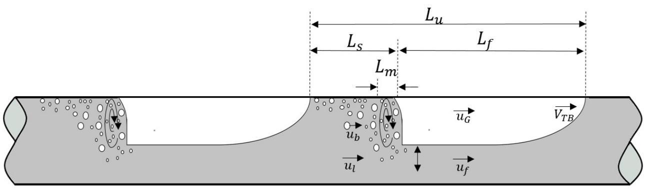


Figure 1: Schematic representation of the unit-cell model for the slug flow pattern.

In this paper, a two-state Markov chain model is proposed to represent the stochastic dynamics of developed slug flow in horizontal pipes. Each state represents either the liquid slug or the elongated bubble regions and the transition probabilities dictate the change of the given discrete time measurement to stay at a given state or change. This simple but intuitive description of the phenomenon allows an analytical treatment of the statistics of Markov chain stochastic process. Consequently, it can be used to investigate the physics of this rather complex flow dynamics and further comprehend several fundamental aspects of reducing the representation of this system by a simple stochastic process.

TWO-STATE MARKOV CHAIN MODEL

Assuming the void fraction measurements are made at a constant sampling rate Δ and the void fraction, i.e. $\alpha_n = \alpha(t = n\Delta)$, a very simple model for this process can be cast in the form of a two-state Markov chain Norris (1997); Soize (2017), which follows closely the representation given by Fabre et al. Fabre et al. (1989) for the time evolution of the flow structure. Each state represents either the elongated air bubble with liquid film or the liquid slug and the Markov chain (MC) is characterised by the probabilities of the sequence maintain, or changing, its state at the n-th sample given the previous sample, i.e. $P(X_n = x_n \mid X_{n-1} = x_{n-1})$ for a first order Markov chain and $x_n = 1$ for the air bubble and $x_n = 0$ for the liquid film. This classification is given from the time series α_n and it is discussed in the next section. This model is schematically represented in Figure 2.

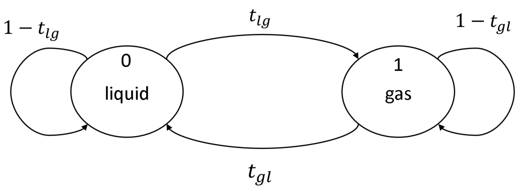


Figure 2: Two-state Markov chain model diagram for liquid slug ($X_n = 0$) and gas bubble ($X_n = 1$). State transition probability from liquid to gas t_α and from gas to liquid t_β.

By increasing the order Markov Chain model, the state of m previous samples other than the immediately previous are also taken into account at the probabilities of the sequence Katz (1981). This introduces a finite memory to the chain closely related to its order. Although it can, in principle, improve the Markov Chain model, it also significantly increases the number of parameters for estimation, with the increasing number of possible transition probabilities. It might require significantly longer measurements times which limits its practical uses and imposes a parsimonious approach for the order selection of the model. Ideally, a first order model should succeed in adequately representing a certain phenomenon even with longer time dependency Rafteryt (1985).

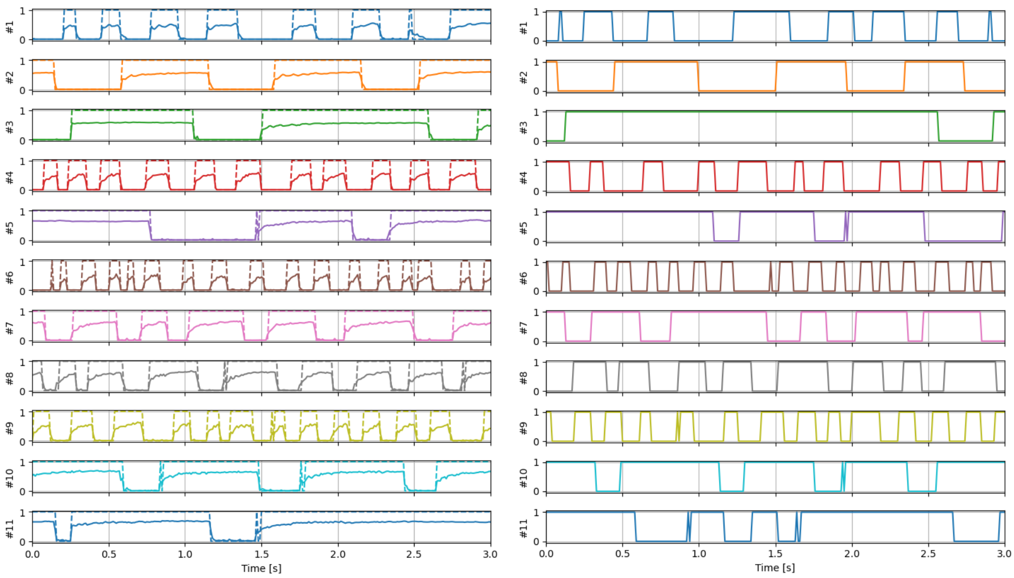
RESULTS

For estimation of the two-state Markov chain model, it is important to establish a threshold for classifying a measurement sample as either elongated bubble or a liquid slug. This threshold is dependent on the experimental point thus a completely data-driven approach is proposed. The Otsu's approach (Otsu, 1979) is used to find the best threshold for every experimental point. It is a non-parametric and unsupervised method of automatic threshold selection. It is typically used in computer vision and image processing but it can be directly applied to unsupervised decision problems in pattern recognition. It is based on minimising the variance between two classes and can be straightforwardly extended to multiple thresholds for several patterns.

The Akaike Information Criterion (AIC) Akaike (1974), originally proposed as a means of selecting competing models, can be used to determine the order m of the Markov Chain Tong (1975) that best suits the data by minimises the function Rafteryt (1985) $AIC(m) = -2L_L + 2m$, where $L_L = \sum_i n_{ti} \log t_i$ is the log-likelihood function of the transition probabilities and n_{ti} is the number of transitions occurring in a sequence and t_i is the corresponding transition probability. Similarly, the Bayesian Information Criterion (BIC) also establishes a metric for model selection G (1978) and has been proposed as a consistent estimator Katz (1981), unlike the AIC. The selected order m is such that it minimises Rafteryt (1985) $BIC(m) = 2L_L + m \log N_T$, where N_T is the sample size. It is shown both AIC and BIC criteria fails to give a clear consistent minima for all of the experimental points, which indicates that both information criteria might not be suited for this particular problem. The main objective of order identification is to include the long term effects of the chain and, consequently, to represent the behaviour of the passage of the unit cell. From the previous section, this is closely related to the zero-crossing of the autocorrelation function of the time series. Consequently, it can be argued that the order of the Markov Chain must be such that it can capture the lags at the first autocorrelation zero-crossing. Following this rational, the order of the chain is chosen that it is twice the number of lags until the first zero-crossing.

Figure 3 presents, on the left, the time series for 3 seconds of the void fraction measurements and the corresponding time series from the Otsu thresholding and, on the right, the synthetically generated time series from the higher order

Markov Chain model. It illustrates the suitability of the proposed order selection criterion. Figure 4 presents the histogram of f_u with 100 bins obtained from the measured time series and classified by the Otsu's threshold (Experimental - MC) and also from synthesised time series generated by a random sample of the Markov Chain (Sample - MC). Similar to the previous case, the histograms in both cases present a very good agreement. The mean value for each case is also shown in the figure.



(a) (b)

Figure 3: (a) Experimental void fraction for each experimental point (full line) and time series from the Otsu threshold (dashed line). (b) Synthetically generated void fraction for each experimental point with higher order Markov Chain model.

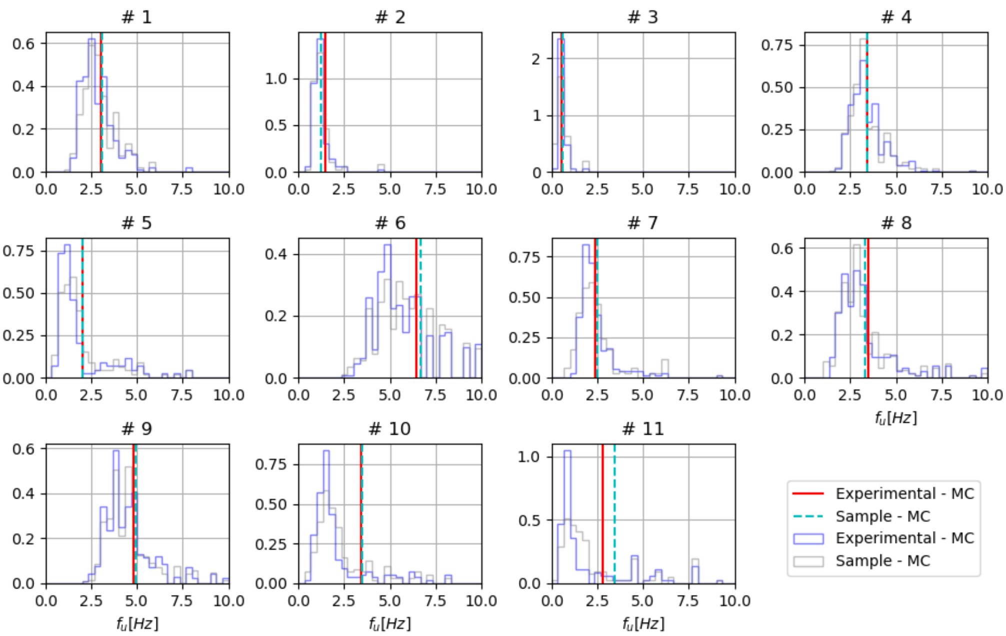


Figure 4: Histogram of the unit cell frequency estimated from the experimental data (grey) em from the Markov Chain sample (blue). Vertical lines show the median value of the experimental data (red full vertical line), the Markov Chain sample (cyan dashed vertical line) and the maximum PSD value.

CONCLUSION

In this paper, a two-state Markov chain model was proposed to represent the stochastic dynamics of developed slug flow in horizontal pipes aiming at a simple but intuitive description of the phenomenon. For the flow conditions used in this paper, the void fraction presented marginal PDF with multimodal features. Then, the Otsu's approach was proposed as an unsupervised and non-parametric classification for liquid slugs and elongated gas bubbles in the measured time series. This approach can be straightforwardly extended to include additional classes in the pattern classification. It is shown that this two-state representation is a reduced order representation that is suitable to describe second-order statistics of the two-phase flow. The proposed approach opens the way for further physical interpretation and insights on the complex dynamics of the slug flow.

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