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Adapting Deep Neural Networks for Rotating Machine Balancing Without Employment of Trial Weights

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Abstract: During normal operation of rotating machines, irregular mass distribution along the rotor yields unbalancing forces, causing high vibration responses significantly affecting the system's health and safe operation. Hence, balancing procedures are periodically applied to rotating machines to reduce vibration amplitudes, thus keeping the system operating within acceptable safety limits. Over the years, various methods have been developed to balance rotating machines, usually relying on the use of trial weights; the assumption of linearity between unbalance forces and measured vibration; or the use of high-fidelity models of the rotating system. In this contribution, we proposed a novel balancing procedure that does not employ trial weights or require the use of a representative model of the rotating system. Here, based on scarce data sets derived from distinct operational conditions, deep neural networks are trained to model the unknown relationship between unbalanced forces and correction masses. To illustrate the capabilities of the proposed methodology, a numerical case study is presented. In the presented numerical example, an unbalanced rotor finite element model combined with an established balancing procedure (the coefficient of influence method) is used to generate a reduced data set for the considered neural networks. A such example illustrates the use of available historical balancing data from the rotor system to train the proposed artificial neural networks. Additionally, a convergence analysis is also presented, evaluating the number of unbalanced responses required to train the artificial neural networks to achieve satisfactory vibration reduction. Obtained results show that with relatively small data sets, the proposed methodology can achieve low levels of vibration, without requiring the use of trial weights, representative models, or the assumption of linearity

Keywords: rotor balancing, neural networks, no trial weights, adapting deep neural networks, rotating machine.

INTRODUCTION

During the operation of rotating machines, irregular mass distribution along the rotor yields unbalancing forces, that can lead to significant vibration levels, affecting the system's health and safe operation. Such fault can increase with time due to component wear; the combined effects of other failure modes (e.g., misalignment, loss of lubrication, bearing defects, cracks, and fractures, etc); foreign material accumulation (e.g., dirt, rust, etc); and a miscellaneous of other factors. Therefore, balancing procedures are periodically applied to approximate the barycenter of the system to its geometric centerline, thus reducing vibration levels and, consequently, the forces supported by the bearings and associated structures (Eisemann, 1998). The balancing of rotating machines commonly requires the definition and evaluation of aspects such as balancing planes, positions in the rotor system where correction masses will be applied; measurement planes, where in the rotor, vibration signals will be measured; and trial weights, test values to further evaluate the relationship between unbalance excitation and measured vibration amplitude. The latter is usually the main bottleneck of existing balancing methodologies for industrial applications. Such trial tests are considered as being time-consuming, in addition to potentially imposing financial constraints due to the requirement of stopping the machine's regular operation, and in some instances, can be very complex to perform, depending on the locations of sensors and accessibility to some components (Kang et al., 2008).

Rotating machine balancing techniques are usually classified into signal-based and model-based techniques. Four or seven runs without phase, modal balancing, and the influence coefficients are some examples of signal-based methods (Steffen & Lacerda, 1996) e (Bently et al., 2002). Many of these techniques assume that the relationship between unbalance excitation and measured vibration amplitude is linear, and constrained to small displacements. However, if the rotor presents nonlinear behavior or large vibration amplitudes, the obtained correction weights and corresponding angular positions will not reduce vibration responses satisfactorily. To overcome such limitations, alternatives as presented in (Edwards et al., 2000), (El-Shafei et al., 2004), (Tiwari & Chakravarthy, 2006) e (Villafane Saldarriaga et al., 2011) have been investigated in recent years. Modal methods, as in (El-Shafei et al., 2004), require information about the system's flexible mode shapes. Knowing a priori system's mode shapes and measuring the vibration response close to the critical speeds it is possible to identify the generalized modal unbalances. Then, generalized modal unbalances are used to obtain a set of orthogonal correction weights that can be installed in the correction planes to balance the rotor at the prescribed critical speeds. El-Shafei et al., (2004) evaluated a modal balancing method without trial weights that achieves satisfactory results if the rotor model is correctly identified. In Villafane Saldarriaga et al., (2011) the unbalance condition of the rotor system is obtained through the optimal solution of an inverse problem. Trial weights are not required by this approach and the balancing results are satisfactory even for nonlinear cases. However, such model-based methods require an accurate numerical model of the rotating system, which may not be an easy task to obtain for some industrial machinery.

Assuming the hypothesis that there is a relationship between the vibration amplitude and the correction mass, this paper presents the execution of flexible rotor balancing without test masses or the requirement of a representative model of the rotor, through the employment of neural networks. Here, based on available scarce data sets derived from rotor system vibration responses, artificial neural networks are trained to model the unknown relationship between unbalanced vibration responses and required correction masses. To illustrate the capabilities of the proposed methodology, a numerical case study will be discussed. The considered numerical study emulates the use of historical balancing data from the rotor system to train the proposed neural networks. To achieve this aim, an unbalanced rotor was modeled using the finite element method, and, from this, the coefficient of influence was applied to generated the balancing data. Based on the amplitude and phase of the signal (obtained by the finite element model) and the mass and correction position (coming from the coefficient of influence), data sets were derived and used to train and evaluated the proposed neural networks. Additionally, a convergence analysis will be discussed, evaluating the number of unbalanced responses required to train the artificial neural networks in order to achieve satisfactory vibration reduction. The proposed approach allows for the balancing of flexible rotors without the use of trial masses, the requirement of high-fidelity models, or the assumption of linearity between unbalance forces and measured vibrations (given that in principle artificial neural networks are more than capable of addressing non-linear behaviors in the data sets).

INFLUENCE COEFFICIENTS BALANCE METHOD

In this Section, the fundamentals of the influence coefficients balancing method will be discussed. It is important to reiterate that this balancing method is only used in this contribution to generate the data sets related to the considered numerical case study, and by no means is a requirement for the proposed balancing procedure based on artificial neural networks (i.e., any other established balancing procedure could be used to generate the numerical data). The influence coefficients balancing method uses known masses to experimentally ascertain the vibration response sensitivity of a rotor system concerning the trial weights, considering constant rotational speeds. From this sensitivity evaluation of rotation, a set of discrete correction masses are determined, aiming in minimizing the system's unbalance response. Conventionally, a trial mass is first applied to one of the balancing planes and the rotor responses are measured. This process is repeated for all the other balancing planes and all measurement speeds. From these evaluations, an influence coefficient matrix can be derived and used to obtain a set of potential correction weights. Amongst established balancing methods, the influence coefficients procedure is more commonly used in industrial applications, although, like many of the previously discussed methods, this technique requires a relatively large number of tests to achieve tolerable vibration amplitudes (Ibraheem et al., 2019), (Li et al., 2021). The coefficient of influence assumes a relationship between the rotor vibration response and the non-uniform distribution caused by the original unbalance. The procedure covers the acquisition of the original vibration response, definition of balancing planes, masses, and angular test positions. Then, the rotating machine is retested, measuring the vibration response under a known unbalance condition. Finally, the amplitudes and phrases derived from the original vibration responses, in addition to the amplitudes and phases of the vibration responses test conditions are redefined as complex numbers and related in vector form by means of a matrix called influence coefficients. Mathematically, the problem of unbalancing in flexible rotors is stated as following: as was describe early, the original unbalance and the vibration amplitudes are related in matrix form (influence coefficients matrix), as determined by Eq. (1).

$$\boldsymbol{V}_{vx1}^{j} = \boldsymbol{\alpha}_{vxn}^{jp} x \boldsymbol{U}_{nx1}^{p} \tag{1}$$

Where:

$\boldsymbol{U}^{p}$, represents the system's original unbalance condition;

$\boldsymbol{V}^{j}$, are vibration amplitudes;

$\boldsymbol{\alpha}^{jp}$, represents the system's original unbalance condition;

$\boldsymbol{v}$, described the number of measurement planes $(j = 1, ..., v)$;

$\boldsymbol{n}$, described the number of measurement planes $(p = 1, ..., n)$;

Adopting a specific speed condition $\boldsymbol{\Omega}$, Eq. (1) can be rewritten as follows:

$$V^0 = \begin{Bmatrix} V_0^1 \\ \vdots \\ V_0^v \end{Bmatrix} = \begin{bmatrix} \alpha^{11} & \cdots & \alpha^{1n} \\ \alpha^{21} & \cdots & \alpha^{2n} \\ \vdots & \ddots & \vdots \\ \alpha^{n1} & \cdots & \alpha^{nn} \end{bmatrix} \begin{Bmatrix} U_0^1 \\ \vdots \\ U_0^n \end{Bmatrix} = \alpha U_0 \tag{2}$$

Considering the same rotation speed Ω, a new unbalance condition can be determined by employing trail masses in a predefined plane and angular position.

$$V^1 = \begin{Bmatrix} V_1^1 \\ \vdots \\ V_1^v \end{Bmatrix} = \begin{bmatrix} \alpha^{11} & \cdots & \alpha^{1n} \\ \alpha^{21} & \cdots & \alpha^{2n} \\ \vdots & \ddots & \vdots \\ \alpha^{n1} & \cdots & \alpha^{nn} \end{bmatrix} \begin{Bmatrix} U_1^1 + W^1 \\ \vdots \\ U_1^n \end{Bmatrix} = \alpha U_1 \tag{3}$$

Assuming α matrix is constant, it is possible to determine its value by subtracting vectors V^j and dividing by the vector of trial masses, as shown in Eqs. (4) and (5), respectively.

$$V^1 - V^0 = \begin{Bmatrix} V_1^1 - V_0^1 \\ \vdots \\ V_1^v - V_0^v \end{Bmatrix} = \alpha \begin{Bmatrix} W^1 \\ \vdots \\ 0 \end{Bmatrix} \tag{4}$$

$$\alpha^{jp} = \frac{V_1^j - V_0^j}{W_p} \tag{5}$$

Lastly, the correction masses are obtained through the dot product between the inverse of matrix α and the original unbalance condition V_0.

ROTOR MODEL (FEM)

For the rotor modeling consider in this contribution, the procedure described by (Lalanne & Ferraris, 1998) was adopted, which proceeds with the calculation of the kinetic and potential energies of all rotor components, as well as the virtual work of external forces. Kinetic energy expressions are necessary to characterize discs, shaft, and mass of unbalancing, while deformation energy is also necessary to characterize the shaft. Then, the elementary equations are combined to get the overall equation of the rotating machine, so we can analyze the rotor behavior on each node (Lalanne & Ferraris, 1998). The general equation that describes the dynamic behavior of the rotor is obtained from the sum of the elementary disc, shaft, bearings, and unbalance matrices; this can be expressed as shown by Eq. (6).

$$M\ddot{q} + (C + \Omega G)\dot{q} + (K_1 + \dot{\Omega} K_2)q = W + F_u + F_m \tag{6}$$

Where M is the mass matrix, C is the damping matrix, G is the matrix associated with gyroscopic effects, K_1 is the stiffness matrix, K_2 is the stiffness matrix due to transient motion, and q is the vector of nodal displacements. Additionally, W, F_u and F_m represent the weight, unbalance, and reaction forces of the bearings, respectively. Table 1 presents the values of the geometric and physical properties of the simulated rotor.

Table 1: Geometric and physics properties of the shaft and discs.

Items	Properties	Dimensions
Shaft	Material = Steel	E = 205 GPa
	Length = 862 mm	ρ = 7850 kg/m^3
	Diameter = 17 mm	ν = 0.29
Disc D1	Material = Steel	$I_{DX} = I_{DZ} =$ 3.84•10^{-3} kgm^2
	Mass = 2.64 kg	$I_{DY} =$ 7.51•10^{-3} kgm^2
Disc D2	Material = Steel	$I_{DX} = I_{DZ} =$ 3.86•10^{-3} kgm^2
	Mass = 2.65 kg	$I_{DY} =$ 7.55•10^{-3} kgm^2

ARTIFICIAL NEURAL NETWORKS

As previously mentioned, balancing procedures can be divided into two main categories: signal-based methods and model-based approaches. In signal-based techniques, experimental data (i.e., the "signal") is used to estimate the unknown relationship between measured vibration amplitudes and corrections masses, usually assuming a linear correlation between unbalance excitation and measured amplitudes. Model-based approaches seek a representative model of the rotor system, by applying fundamental principles (such as the Lagrange equation discussed in the previous section) to evaluate the correlation of correction masses and the system's vibration amplitudes. In a way, the model-based methods can be interpreted as an approximation of the unknown correlations, usually derived by considering relatively small displacements (to reduce non-linearities in the analysis). Additionally, limitations in terms of computational cost or even incomplete knowledge of rotor system dynamics, often impose limitations on the fidelity of such representative models. While both methods have their merits, both seek to indirectly approximate the unknown relation between measured vibration and correction weights, usually by considering simplifying assumptions as linearity or small displacements, which may hinder their capabilities and applicability in more complex rotating systems. Hence, in balancing procedures, as in many engineering applications, the key issue is how to best approximate an unknown relationship.

Machine learning techniques are powerful tools to handle such complex analyses. Neural networks, for example, have been called "universal approximators" for their ability to handle a multi-dimensional space with a high degree of accuracy (Hornik et al., 1989). Multilayer Perceptrons (MLPs) are a class of feedforward artificial neural networks, in which systems of interconnected perceptrons, the basic unit in artificial neural networks emulating a neuron cell, apply nonlinear transformations over the linear combination of its inputs. Such a modeling approach aims at a nonlinear mapping between input and output vectors, and it is the superposition of many simple nonlinear transformations that enables MLPs to approximate extremely complex behaviors (Gardner & Dorling, 1998). Figure (1) brings a schematic representation of a conventional feedforward MLP.

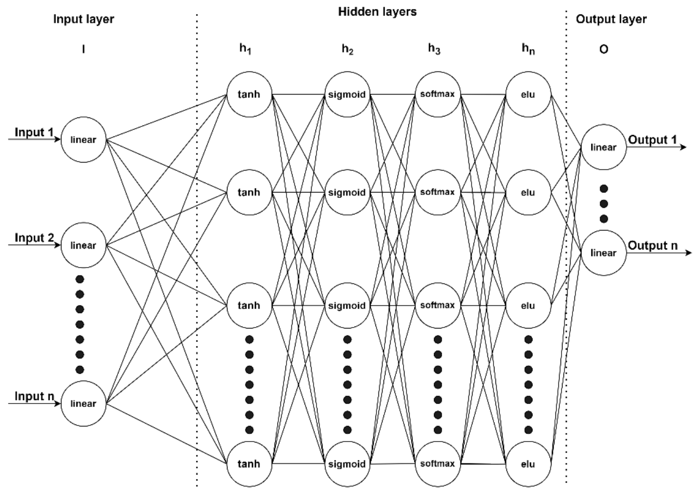

Figure 1: Multilayer Perceptron representation. Circles represent the nodes associated with the input, hidden, and output layers, while "linear", "tanh", "sigmoid", "softmax", and "elu", illustrates possible nonlinear transformations (also known as activation functions) applied at each node. Connecting lines represent the weights related to the linear combination of inputs on each node. Weights and node biases are the adjustable parameters, optimized during the MLP training.

Nodes implement complex transformations (commonly referred to as activation functions) on the linear combination of its inputs, returning the shifted weighted transformation (combined effects of inputs weights and nodes biases plus activation function) of such input's signals. Multilayer perceptrons consist of finitely combinations of parallel and sequential nodes. Parallel nodes define what is referred to as layers, commonly described as input layers (the layer directly handling the MLP overall input signals), hidden layers (a finite number of layers applying the complex transformations into the MLP signals), and an output layer (the layer that provides the MLP outputs). Traditionally, neural networks with a single hidden layer are referred to as "shallow" networks, while "deep" networks will be any neural network with two or more hidden layers. To achieve the desired approximation of the correlation between input and output signals, an optimization procedure must be carried on to find the best weights and biases in the neural network that minimizes the loss of information between inputs and outputs (for a regression problem such loss can be evaluated in terms of an error function). Such a step is commonly referred to as neural network training and usually relies on a backpropagation procedure. Essentially two steps are performed in every iteration of the optimization. First, training data is fed forward into the network, generating the corresponding outputs, and associated errors are evaluated in the loss function. Then, the loss function adjoint is propagated backward, through the chain rule, giving the gradient concerning the parameters to be optimized (network weights and biases). These steps are repeated until convergence is achieved.

A key advantage of artificial neural networks is their ability to learn through training examples, and generalize to unseen data, i.e., the ability to be tuned, through representative data sets, to represent an unknown correlation, without considering major assumptions, and provide coherent predictions concerning new data. Commonly, the tradeoff for such prediction capability is the requirement of relatively large training sets. In this contribution, a data generation procedure will be discussed aiming to reduce the data requirement of neural networks concerning the balancing of flexible rotors. For the application considered in this paper, the unknown correlation that the neural network is trained to approximate concerns the relation between measured unbalanced vibration amplitudes and related correction masses. Hence, the MLPs inputs are vibration amplitudes, and related phase angles for considered measuring planes, while its outputs are correction masses and related angular positions, for all considered balancing planes. Due to the relationship between the measured phase angle and correction mass angular position, a procedure can be derived to reduce the required amount of training data, as detailed in the following section. Table 2 presents the characteristics of the neural network used in this study.

Table 2: Geometric and physics properties of the shaft and discs.

Layer	Input	#1	#2	Output
#Neurons	4	8	8	4
Activation Functions	Linear	Sigmoid	Tanh	Elu

PROPOSED BALANCING PROCEDURE

The procedure proposed in this work aims to overcome one of the main limitations in the use of neural networks for balancing rotating machines, the requirement of a large amount of training data available to obtain an acceptable accuracy for the neural network predictions. This methodology is based on the existence of a relationship between the angular position of the correction mass (obtained by the **IC**) and the phase of the vibration response, that is, considering the unbalanced condition in a single plane, for a given amplitude of vibration and correction mass, it is known that the angular distance between the correction and unbalancing masses is 180 degrees. Thus, new unbalancing conditions can be generated from a $d\theta$ variation in the same direction, either in the correction mass or in the phase of the vibration response, in which can be used as training data, without the need to acquire new balancing data. This procedure consists of three steps, namely:

- Data acquisition by the finite element model and balancing by the coefficient of influence method;
- Iterative generation of hypothetical unbalance conditions from the mentioned correlation between phase and angular position;
- Use of the neural network in rotor balancing for unknown data. Evaluation of error, convergence, and balancing of new unknown conditions.

The first stage begins with the elaboration of the finite element model since it is possible to obtain the temporal signal for different unbalance conditions. Then, applying the discrete Fourier transform (DFT) on signals in time in the coordinates X and Z, the lateral vibration amplitudes of the rotor are obtained. Additionally, to better approximate the simulated responses to real conditions, white noise is added to the temporal signal. Figure (2) schematically presents the proposed procedure.

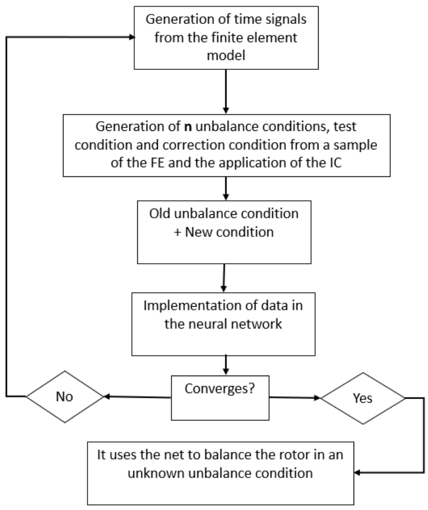

Figure 2: Proposed procedure.

For obtaining signal phases, a simulated encoder signal (reference signal) is created, and from this, the difference between measured and reference signals is determined. Figure (3) shows a rotor with an unbalance plane.

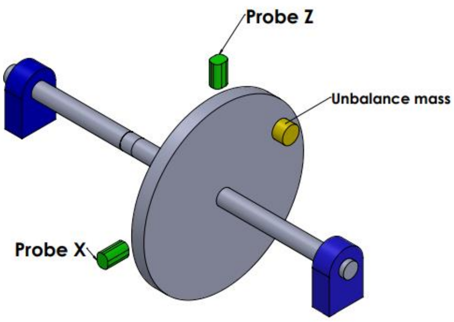

Figure 3: Unbalancing with one plane.

For simplicity, the FEM is used in the simulation of a rotor with unbalancing in a single plane. It was chosen to represent the unbalancing conditions in the phasor form, Amplitude∠ phase. The balancing of the rotor in question, proceeds with the application of the influence coefficient, determining the mass and test position (test mass∠ angular position), which will result in the correction mass and its angular position (correction mass∠ correction position). For the elaboration of the second stage, it is known that the unbalance and correction mass positions are directly related, that is, if the unbalance position varies by one degree, the correction position will also vary by only one degree, maintaining 180 degrees of difference between them. Thus, to build the neural network training data, several unbalance, test, and hypothetical correction conditions were generated, varying only the angular position from the original unbalance, keeping the amplitude constant, and applying the influence coefficient. Figure (4) schematically represents the methodology for generating new training and testing data.

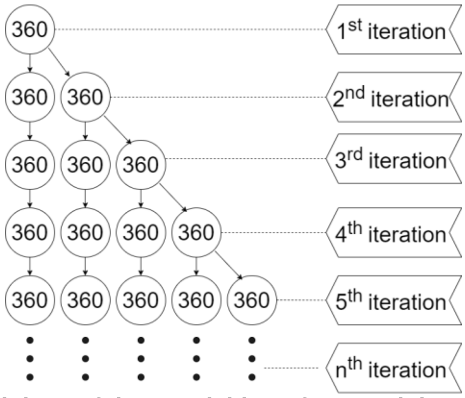

Figure 4: Methodology of the acquisition of new training and testing data.

Figure (5) features a rotor with unbalance on two planes.

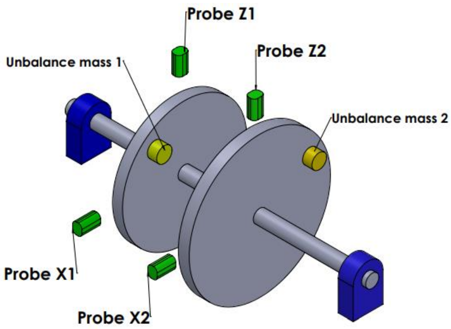

Figure 5: Unbalancing with two planes.

For the application of the method in $\boldsymbol{n}$ unbalance planes, a similar procedure is performed, that is, unbalance conditions are recorded in the planes in which one wishes to balance. Then, the **IC** is applied, obtaining the correction mass and its angular position (similar to the case with only one disk). The hypothetical unbalance conditions are created by varying the position of the correction mass, consequently resulting in the same variation in the phase of the vibration response. As for an unbalanced condition, this procedure will generate a $\boldsymbol{m}$ number of hypothetical conditions that can be added to neural network training data. For the third stage, an iterative procedure is used. The neural network receives the $\boldsymbol{m}$ hypothetical samples generated from the original $\boldsymbol{n}$ conditions and, after training, is evaluated by the error in an unseen unbalance condition. If the error is satisfactory, the process is interrupted and the network is considered ready to be used for balancing purposes. Otherwise, the finite element model associated with the influence coefficient method is used to generate new training samples to be added to the existing sample so the network can be retrained until convergence is achieved.

RESULTS AND DISCUSSIONS

This section is dedicated to the discussion of the results obtained from the application of the neural network in conjunction with generation and training data, as explained in the previous sections. The results were obtained from the modeling of a virtual machine (FEM), with a constant rotation speed of 1200 rpm. Starting the analysis of the results by the convergence curve of the absolute error in the condition of a balancing plane (as illustrated in Figure (6)), it is observed that there was a high reduction in the magnitude of the error as a function of the number of iterations. This phenomenon occurred due to the strong correlation between the vibration response and the unbalancing itself. As it was an analysis of only one unbalancing plane, there was no presence of force couplings, for example, facilitating the learning of the network.

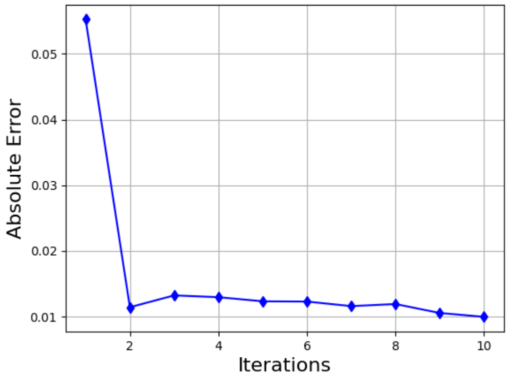

Figure 6: Error evolution as a function of interaction for a disk.

Through the predictability curves, it is perceived by evaluating the first unbalancing condition (see Figure (7)) that, despite the relationship cited between the vibration response and the unbalancing, the network was not successful in the training; given the lack of variability of the training data caused, mainly by the maintenance of the vibration amplitude (consequently, of the constant correction mass). For the third and fifth round of unbalancing conditions (as can be seen in Figures (8) and (9)), the network showed predictability in agreement with the trend line, which suggests its convergence.

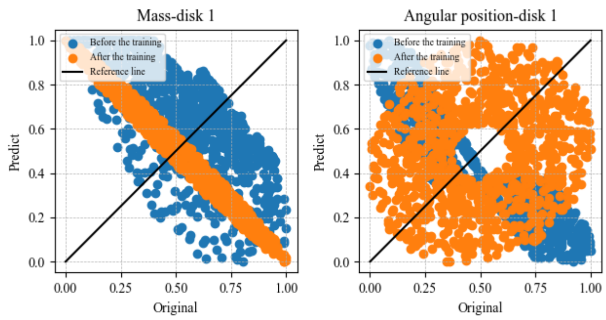

Figure 7: Comparison between original and predicted correction masses, before and after training, for the first iteration.

Figures (8) and (9) illustrate these comparative graphs of neural network predictability before and after training.

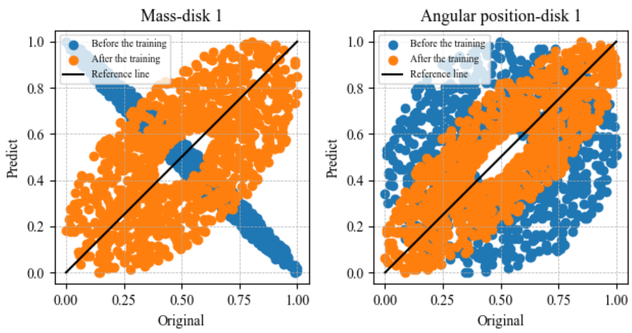

Figure 8: Comparison between original and predicted correction angular position, before and after training, for the third iteration.

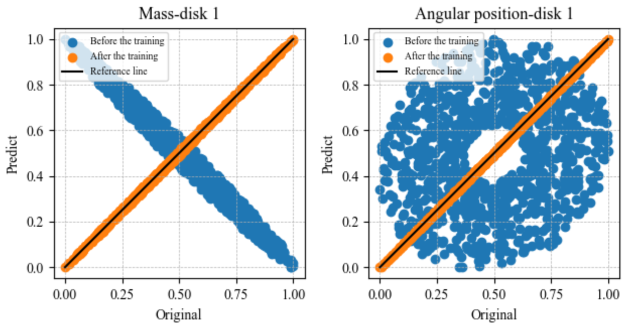

Figure 9: Comparison between original and predicted correction masses, before and after training, for the fifth iteration.

For the condition of two balancing planes, a convergence curve was also elaborated. As observed in Figure (10), a behavior similar to the previous case was obtained, that is, a rapid convergence in the first iterations, followed by a small positive variation.

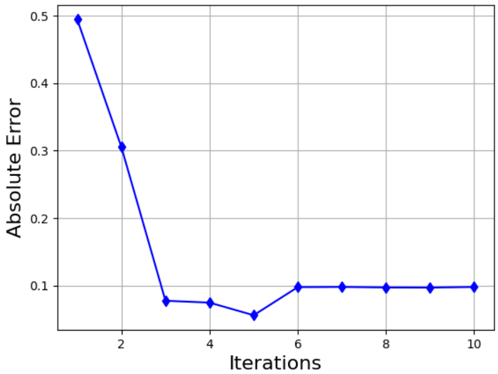

Figure 10: Error evolution as a function of interaction for two disks.

In the first iteration (seen in Figure (11)) it is noticed that despite a tendency to learn, the neural network did not present good predictability concerning the original test data. Again, as in trend analysis, this behavior is repeated due to the lack of variability in the network training data, caused by the constancy in the values of vibration amplitudes and correction masses.

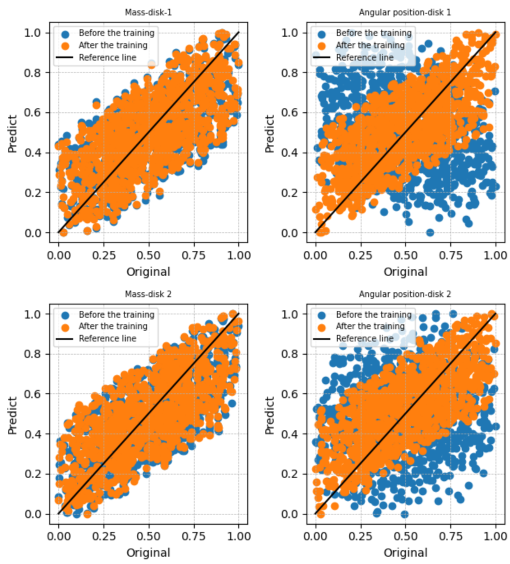

Figure 11: Comparison between original and predicted correction masses and angular positions, before and after training, for the first iteration.

In an intermediate condition of error convergence (Figure (12)), despite a noticeable improvement, the network is still inadequate for balancing purposes, see the comparison between the predicted (orange dots) and the original data (solid reference line).

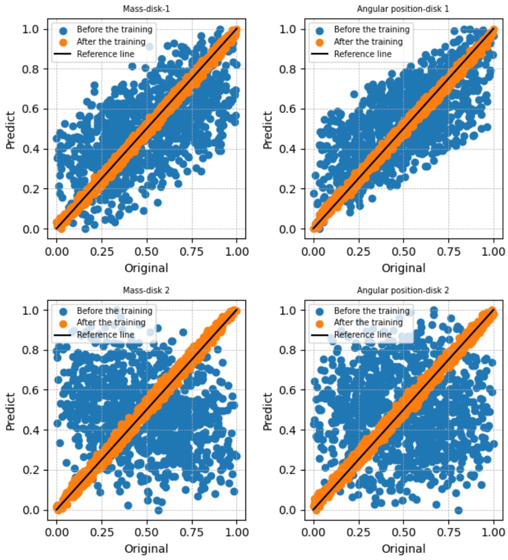

Figure 12: Comparison between original and predicted correction masses and angular positions, before and after training, for the third iteration.

As the network receives new data, greater variability in training data is generated and, consequently, better predictability is achieved, therefore, appropriate to the balancing of a new unknown condition, as illustrated by Figure (13).

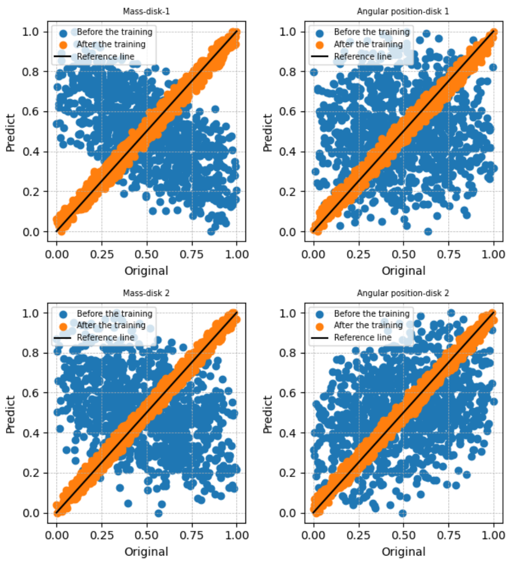

Figure 13: Comparison between original and predicted correction masses and angular positions, before and after training, for the fifth iteration.

CONCLUSION

A methodology for balancing flexible rotors without the use of test mass via Artificial Neural Networks was presented, in order to predict the mass and angular position of correction from the vibration signals. For this purpose, the FEM and the influence coefficient were used to provide the input parameters and the neural network response. In addition, it is known that among the main limitations of neural networks, there is the need for large amounts of samples to achieve satisfactory accuracy. With this in mind, a training data generation methodology was also proposed, which uses the relationship between the variation of the position of the unbalance mass and the correction mass to generate any number of hypothetical samples from a single real sample, thus dramatically decreasing the need for real samples.

Additionally, the methodology developed here was used to balance one and two planes. By analyzing the results of convergence and comparison between the data and the prediction from the trained and untrained network, it was noticed that in the first iteration, for both cases, the balancing did not have much effect, as all the hypothetical samples generated diverged, only in position and phase, with constant vibration amplitude and correction masses. However, as the model was fed with new real samples, the training data became more diversified, in this way the neural network began to understand the relationship between the unbalance condition and correction (mass and angular position). In addition, it is also observed that for one plan, this methodology was able to balance with only two real samples, a reduction of 82,45 %, while for two plans, five real samples were needed, a reduction of 86,0 % of the absolute error between the conditions predicted by the network and the original ones, indicating a greater learning difficulty due to the increase in the number of planes and coupling of unbalance forces.

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