Balancing of a Rotating Machine using the Kriging approach

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Abstract: This work presents a new approach to balancing rotating machines using a technique based on vibration signals. The proposed methodology uses a Kriging surrogate model to perform the balancing of a rotating system. The Kriging formalism requires input and output samples. Therefore, the Influence Coefficient (IC) method is used to obtain the output samples, which are the correction mass and its corresponding angular position. The input samples are the unbalanced vibration amplitudes and associated phases. With these samples, the surrogate model is mounted, and hence for new unbalance conditions, is possible to predict the correction mass and its corresponding angular position, without the need for trial weights. Therefore a reduction of the time spent in the balancing procedure is obtained, whilst the vibration amplitude is also reduced. The obtained results using the proposed approach were compared to the IC method for validation purposes. The proposed approach demonstrated itself to be effective for balancing rotating machines, achieving a satisfactory reduction on the vibration amplitudes of the adopted rotor. The methodology was validated both numerically and experimentally.

Keywords: balancing, surrogate model, rotating machines, influence coefficients method, dynamics.

INTRODUCTION

Conventional balancing methods as signal-based techniques are widely used, nevertheless, they present some downsides. For instance, they consider the relationship between the vibration and the corresponding unbalance excitation to have linear behavior, which can affect the effectiveness of the balancing when the system presents some non-linearity. Additionally, these techniques consume considerable time due to the need for trial weights to determine the vibration's sensitivity to variations in the unbalance.

Different research has been published, on which the use of trial weights was not required (Zhang et al. (2021), El-Shafei, El-Kabbany, and Younan (2004)). Saldarriaga et al. (2011) studied a model-based balancing procedure, solving an inverse problem and finding the unbalance condition of their rotating machine. Carvalho et al. (2018) performed experimental validation of a finite element model-based approach, with the insertion of uncertainties in the unbalance distribution.

In the actual scenery where new technologies constantly arise, investing in new balancing techniques becomes relevant, following the evolution of the uncertainties analyses, optimization procedures, control, and other methods related to rotating machines. The proposed formalism does not require a representative model of a rotating machine, and does not require an excessive number of samples to obtain good results. For this reason, it is possible to reduce the time consumption, which is an important factor in a competitive industrial field. Through the past few years, new approaches earned visibility, and hence this work proposes a surrogate methodology for balancing.

The proposed approach relies on applying different unbalance conditions over a rotating machine with two planes and performing a balancing procedure with the Influence Coefficients (IC) method. Therefore, the input and output samples are obtained, which are, respectively: the unbalanced vibration amplitude with the associated phase angle; and the correction mass with the corresponding angular position. Once the samples are defined, it is possible to assemble the surrogate model. Afterward, the validation is performed, where, by using different samples, the results given by the IC method and surrogate are compared. Additionally, the vibration amplitudes are acquired to verify the reduction of each procedure. The proposed approach was performed numerically and experimentally.

KRIGING METHODOLOGY

According to Simpson et al. (2001), the Kriging methodology is composed of three steps: the first is the sampling, which consists of acquiring and selecting a group of data associated with the original model/system (the input and output samples cited earlier); the second is the formulation step, where the function to generate the surrogate model is selected; the third, and last, is the fitting step, where the adjustment of the selected function, to the sample data informed is done. After these three steps, the surrogate model is assembled. Finally, it requires a validation procedure, in which with new input fed to the surrogate, its results are compared to the original model/system.

The Kriging general formulation is shown in Eq. 1, where $f(\mathbf{x})$ is a polynomial function which provides a global approximation of the design space, $Z(\mathbf{x})$ is a spatial correlation function. The vector $\hat{y}(\mathbf{x})$ represents the predicted outputs related to an input vector given by $\mathbf{x}$.

$$\hat{y}(\mathbf{x}) = f(\mathbf{x}) + Z(\mathbf{x}) \tag{1}$$

The $Z(\mathbf{x})$ is shown in Eq. 2 which considers a Gaussian normal distribution process, with zero mean, and variance σ^2. In this equation, the spatial correlation function is given by $R(\theta, \mathbf{x}, \mathbf{w})$ where θ is the correlation parameters used to adjust the model. In Table 1 is presented the main possible correlation functions.

$$Cov[Z(\mathbf{x}), Z(\mathbf{w})] = \sigma^2 R(\theta, \mathbf{x}, \mathbf{w}) \tag{2}$$

Table 1: Correlation functions

Correlation Models	$R_j(\theta_j, x_j, w_j)$
Linear	$max(0, 1-\theta\lvert x_j - w_j\rvert)$
Gaussian	$exp(-\theta_j\lvert x_j - w_j\rvert^2)$
Exponential	$exp(-\theta_j\lvert x_j - w_j\rvert)$

In Eq. 3 is shown the complete formulation of this methodology, in which $g(\mathbf{x})$ represents the polynomial function (that can be constant, linear or quadratic), $\mathbf{Y}$ is the output samples vector, and $\mathbf{G}$ is a matrix containing the results of the function g applied with the input samples vectors. $\mathbf{R}$ is the correlation matrix, $\mathbf{r}(\mathbf{x})$ is the correlation vector between a new value $\mathbf{x}$ and the input sample points, and $\hat{\beta}$ are the polynomial function coefficients.

$$\hat{y}(\mathbf{x}) = g^T(\mathbf{x})\hat{\beta} + \mathbf{r}^T(\mathbf{x})\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{G}\hat{\beta}) \tag{3}$$

The optimum Kriging model is found by minimizing the Eq. 4, in which $|\mathbf{R}|$ is the determinant of $\mathbf{R}$, N_s is the number of samples and the variance is estimated by Eq. 5.

$$\underset{\theta}{min}\{\Psi(\theta)\} \equiv |\mathbf{R}|^{\frac{1}{N_s}}\hat{\sigma}^2 \tag{4}$$

$$\hat{\sigma}^2 = \frac{(\mathbf{Y} - \mathbf{G}\hat{\beta})^T\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{G}\hat{\beta})}{N_s} \tag{5}$$

INFLUENCE COEFFICIENTS METHOD

This method uses as input the unbalanced vibration amplitude and corresponding phase angle, and requires the definition of the balancing planes (where the correction masses are inserted), measuring planes (where the vibration sensors are located), and trial weights (used to apply a known unbalance force to the system). The output of this method are the correction masses and corresponding angular position, which are positioned at the balancing planes, aiming at reducing the vibration response (Wowk 1998).

In Eq. 6 is presented the IC method formulation, where the vibration amplitudes $\mathbf{V}^j$ is related to the original rotor unbalance distribution $\mathbf{U}^p$, by the influence coefficients α^{jp}. Where j is the number of measuring planes, p is the number of balancing planes, v is the number of measurement points ($j = 1, \ldots, v$) and n is the number of balancing planes ($p = 1, \ldots, n$). These variables contains complex information.

$$\mathbf{V}^j_{v\times 1} = \alpha^{jp}_{v\times n}\mathbf{U}^p_{n\times 1} \tag{6}$$

For a given speed Ω, Eq. 6 can be rewritten in Eq. 7, in which $\mathbf{V}_0$ is the vibration responses related to the original unbalance distribution $\mathbf{U}_0$ of the system.

$$\mathbf{V}^0 = \begin{Bmatrix} V_0^1 \\ V_0^2 \\ \vdots \\ V_0^v \end{Bmatrix} = \begin{bmatrix} \alpha^{11} & \alpha^{12} & \cdots & \alpha^{1n} \\ \alpha^{21} & \alpha^{21} & \cdots & \alpha^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{v1} & \alpha^{v2} & \cdots & \alpha^{vn} \end{bmatrix} \begin{Bmatrix} U_0^1 \\ U_0^2 \\ \vdots \\ U_0^n \end{Bmatrix} = \alpha\mathbf{U}_0 \tag{7}$$

By using a trial weight m_t attached to the first plane, i.e., $p = 1$, at the same speed Ω, the Eq. 8 is obtained, in which a known unbalance W^1 is added to the system.

$$\mathbf{V}^1 = \begin{Bmatrix} V_1^1 \\ V_1^2 \\ \vdots \\ V_1^v \end{Bmatrix} = \begin{bmatrix} \alpha^{11} & \alpha^{12} & \cdots & \alpha^{1n} \\ \alpha^{21} & \alpha^{21} & \cdots & \alpha^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{v1} & \alpha^{v2} & \cdots & \alpha^{vn} \end{bmatrix} \begin{Bmatrix} U_0^1 + W^1 \\ U_0^2 \\ \vdots \\ U_0^n \end{Bmatrix} = \alpha \mathbf{U}_1 \tag{8}$$

Once the trial mass is smaller than the system's mass, it is possible to consider the influence coefficients matrix constant. Therefore, by subtracting Eq. 7 from Eq. 8, the Eq. 9 is given.

$$\mathbf{V}^1 - \mathbf{V}^0 = \begin{Bmatrix} V_1^1 - V_0^1 \\ V_1^2 - V_0^2 \\ \vdots \\ V_1^v - V_0^v \end{Bmatrix} = \alpha \begin{Bmatrix} W^1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{9}$$

Afterwards, the trial weight is moved to the next balancing planes to determine the elements of the influence coefficients matrix. This process is generalized in Eq. 10.

$$\alpha^{jp} = \frac{\mathbf{V}_1^j - \mathbf{V}_0^j}{\mathbf{V}^p} \tag{10}$$

The correction masses m_c are obtained by the multiplication between the initial vibration response $\mathbf{V}_0$ and the inverted α. This procedure can be performed with different speeds Ω to obtain a broadband balancing efficiency. More information about the IC method can be found in Ehrich (1992), R. C. Eisenmann and R. C. J. Eisenmann (1998), Bently and Hatch (2002), Wowk (1998) and Muszynska (2005).

METHODOLOGY

Using a finite element model of a rotor with two discs, different unbalance conditions were applied to the model. These conditions were generated by the Latin hypercube method (Minasny and McBratney 2006), i.e., random conditions were applied on both planes to unbalance the system. Subsequently, a computational routine was used to apply the IC method and balance the rotor model. The final input and output samples used were a decomposition (on the X and Y axis) of the unbalanced amplitude and associated phase angle, for the input samples, and the mass and corresponding angular position, for the output samples. The generalization is shown in the Eq. 11, where $\mathbf{A}$ is the unbalanced amplitude or the correction mass, and the θ is the corresponding phase angle or angular position. With the samples defined, the Kriging surrogate model was built. Figure 1 presents the flowchart for the assembly and usage of the surrogate model.

$$\begin{aligned} \mathbf{X} &= \mathbf{A} \times cos(\theta) \\ \mathbf{Y} &= \mathbf{A} \times sin(\theta) \end{aligned} \tag{11}$$

The process shown in Fig. 1 is repeated using a descending number of samples. This is required to find the least number of samples necessary to produce an effective surrogate model. Additionally, it allows the reduction of the number of experimental tests to verify the methodology's efficiency.

Finally, to evaluate the effectiveness of the proposed approach the following error metrics were used: the Root Mean Square Error (RMSE), which was used as a global metric and is shown in Eq. 12; and the Maximum Error, used as a local metric and calculated by Eq. 13. In these equations, the $\hat{\mathbf{y}}$ is the surrogate's prediction, $\mathbf{y}$ is the output of the IC method, and $\mathbf{N}$ is the number of samples.

$$RMSE = \sqrt{\frac{\sum_{k=1}^{\mathbf{N}} (\hat{\mathbf{y}}_k - \mathbf{y}_k)^2}{\mathbf{N}}} \tag{12}$$

$$MaximumError = max(|\hat{\mathbf{y}} - \mathbf{y}|) \tag{13}$$

NUMERICAL RESULTS

The numerical procedure was performed with the use of 36 samples (input and output). Of these samples, 16 were used in the assembly of the surrogate model, and the remaining (20 samples) were used for validation. Numerically, it was possible to verify that the effectiveness of the approach, i.e., the surrogate model was capable of balancing the rotor

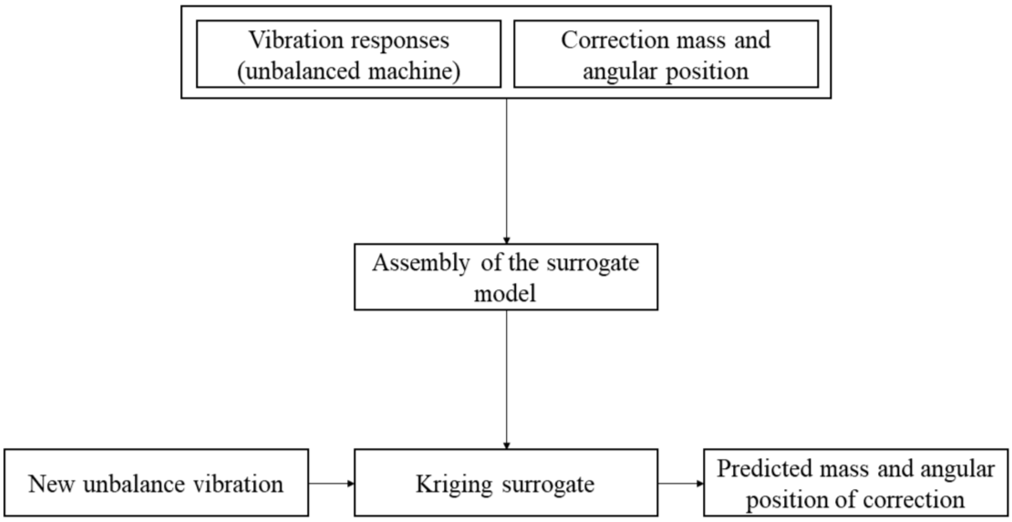

Figure 1: Flowchart of the surrogate model process.

model. Therefore the study for the minimum number of samples required to create the surrogate model was initiated. Table 2 and 3 present the results of this analysis, with the Maximum Error and Root Mean Square Error (RMSE) values, respectively.

Table 2: Number of samples and Maximum Error metric values between the IC method result and surrogate's.

	Maximum Error			
N° of Samples	Mass 1 (g)	Angular Position 1 (°)	Mass 2 (g)	Angular Position 2 (°)
16	7.145×10^{-14}	1.407×10^{-11}	7.448×10^{-14}	6.170×10^{-12}
8	8.197×10^{-14}	2.145×10^{-11}	7.936×10^{-14}	1.024×10^{-11}
5	5.052×10^{-14}	1.107×10^{-11}	5.595×10^{-14}	7.455×10^{-12}

Table 3: Number of samples and RMSE Error metric values between the IC method result and surrogate's.

	RMSE Error			
N° of Samples	Mass 1 (g)	Angular Position 1 (°)	Mass 2 (g)	Angular Position 2 (°)
16	1.709×10^{-14}	2.994×10^{-11}	1.525×10^{-14}	2.009×10^{-12}
8	2.445×10^{-14}	4.190×10^{-11}	1.958×10^{-14}	2.663×10^{-11}
5	1.836×10^{-14}	3.057×10^{-11}	1.878×10^{-14}	3.405×10^{-12}

Through the data exposed in Tab. 2 and 3, is possible to confirm that the effectiveness of the surrogate was not affected by the reduction of the number of samples. Therefore the minimum number of samples required to assemble the surrogate model is 5. The Kriging surrogate was built with the linear polynomial function and exponential correlation. These numerical results showed that the procedure is efficient, and hence the next step was to perform the experimental analysis.

EXPERIMENTAL RESULTS

The experimental procedure was performed over the test rig shown in Fig. 2. This rotating machine is composed of a steel flexible shaft with 840 *mm* length and 19.05 *mm* diameter, and three rigid discs, where D_1 has a mass of 0.658 *kg* at 408 *mm* from the shaft's coupling, D_2 has a mass of 0.658 *kg* at 250 *mm* from D_1, and D_3 has a mass of 5.013 *kg* at an intermediate position between D_1 and D_2. The rig has two hydrodynamic bearings B_1 and B_2, where each bearing has two proximity sensors, at horizontal and vertical positions (S_{1X}, S_{1Z}, S_{2X}, and S_{2Z}).

First, a balancing was performed on the test rig. This was necessary to apply the desired unbalanced conditions to the machine with reduced residual vibration. After this balancing, the process of creating the samples for the Kriging methodology began. Twelve different unbalance conditions were applied in the test rig, and twelve balancing procedures with the IC method occurred. From the 12 samples, 6 were used to assemble the surrogate model, and they are shown in Table 4. Unlike the numerical analysis, in the experimental, with the same amount of samples, the surrogate model could not perform the balancing. Therefore, one more sample was used to assemble it, and adjust the methodology. The

Figure 2: Test rig with flexible shaft and hydrodynamic bearings used in the experimental procedure.

probable reason behind this behavior is the presence of the hydrodynamic bearings, which inserts non-linearity that can mislead the surrogate model and its prediction. The same polynomial function and correlation model of the numerical methodology was used.

Table 4: Samples used in the assembly of the surrogate model.

	Unbalanced				Influence Coefficients			
Sample n°	Disk 1		Disk 2		Disk 1		Disk 2	
1	31.6793 μm	-97.883 °	12.2627 μm	-120.910 °	13.1241 g	15.4884 °	4.1043 g	-41.5192 °
2	16.0000 μm	-140.863 °	10.0995 μm	-171.318 °	1.6144 g	26.8963 °	13.2436 g	-55.9760 °
3	20.6376 μm	-154.440 °	12.9114 μm	-150.660 °	7.8661 g	20.5206 °	19.7116 g	-109.895 °
4	43.5832 μm	-174.551 °	23.9938 μm	-155.738 °	15.8162 g	-69.6882 °	14.5814 g	-83.5671 °
5	63.3791 μm	-174.551 °	31.6925 μm	163.942 °	29.8095 g	-63.226 °	12.8088 g	-81.612 °
6	30.6988 μm	179.47 °	10.8976 μm	148.707 °	13.7162 g	-54.1421 °	9.2373 g	-154.4421 °

With the surrogate model assembled, the validation of the proposed approach started. A total of six efficiency validation were made, and, for each, the machine was balanced twice: firstly with the proposed methodology (surrogate model) and secondly with the conventional methodology (IC method). The unbalanced scenarios used are shown in Table 5, and the results given by both procedures are presented in Table 6.

Table 5: Unbalance vibrations used in the validation procedure.

	Unbalanced			
Validation test	Disk 1		Disk 2	
1	17.604 μm	-134.999 °	8.7476 μm	-155.826 °
2	27.488 μm	163.055 °	11.479 μm	146.527 °
3	23.883 μm	-135.744 °	7.0789 μm	-121.305 °
4	31.958 μm	111.810 °	15.716 μm	-99.8600 °
5	55.265 μm	-107.326 °	24.261 μm	-104.404 °
6	42.153 μm	158.974 °	15.001 μm	136.442 °

Table 6: Correction masses and angular position given by the Surrogate model and IC method.

	Surrogate model				IC method			
Validation test	Disk 1		Disk 2		Disk 1		Disk 2	
1	4.1080 g	-17.5832 °	7.9319 g	-47.2216 °	5.9602 g	-25.4231 °	6.0404 g	-37.7941 °
2	8.8428 g	-89.9046 °	6.8967 g	-92.9847 °	10.835 g	-91.8912 °	3.3569 g	-90.5041 °
3	14.164 g	-32.8578 °	7.1534 g	125.308 °	13.224 g	-40.7958 °	8.6037 g	102.626 °
4	17.391 g	-30.5575 °	14.524 g	78.7892 °	17.232 g	-42.7466 °	24.012 g	70.6323 °
5	30.342 g	-24.5657 °	19.873 g	84.7534 °	27.226 g	-4.55230 °	12.140 g	74.9559 °
6	17.782 g	-81.6779 °	7.1132 g	-155.726 °	22.273 g	-85.3806 °	7.3023 g	141.578 °

Table 7 presents the percentage of reduction of the vibration amplitude for each validation test for both methodologies, the proposed and the conventional. Furthermore, Fig. 3 and Fig. 4 show the Fast Fourier Transform of the validation test 2 and 3, which are the best and worse performance of the surrogate model respectively, for both sensors.

Table 7: Vibration amplitude percentage of reduction for the Surrogate model and IC method.

	Surrogate model		IC method	
Validation test	Reduction S_{1X} (%)	Reduction S_{2X} (%)	Reduction S_{1X} (%)	Reduction S_{2X} (%)
1	92.68	85.78	88.98	91.48
2	87.57	96.19	71.46	53.72
3	88.99	74.55	88.46	93.21
4	87.35	87.50	75.93	76.21
5	81.98	88.31	87.59	89.55
6	90.31	90.61	94.48	88.60
Average Reduction (%)	88.15	87.16	84.48	82.13

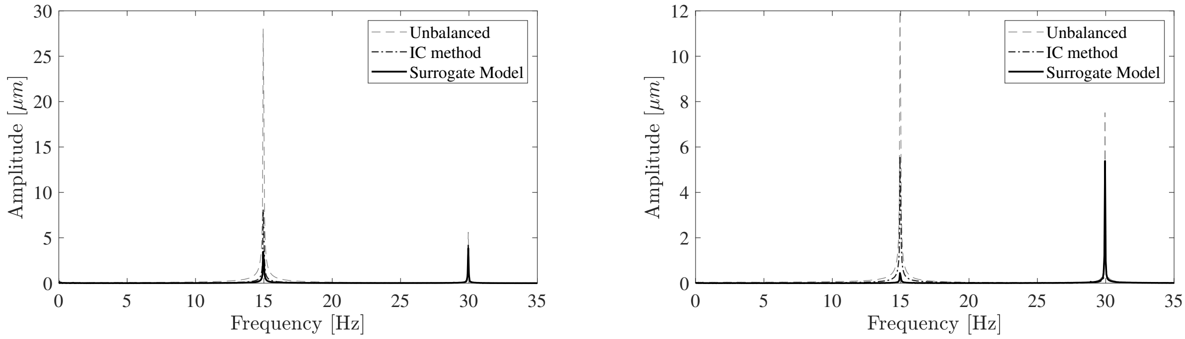

Figure 3: Validation test 2 frequency spectrum for sensors S_{1X} and S_{2X}, respectively.

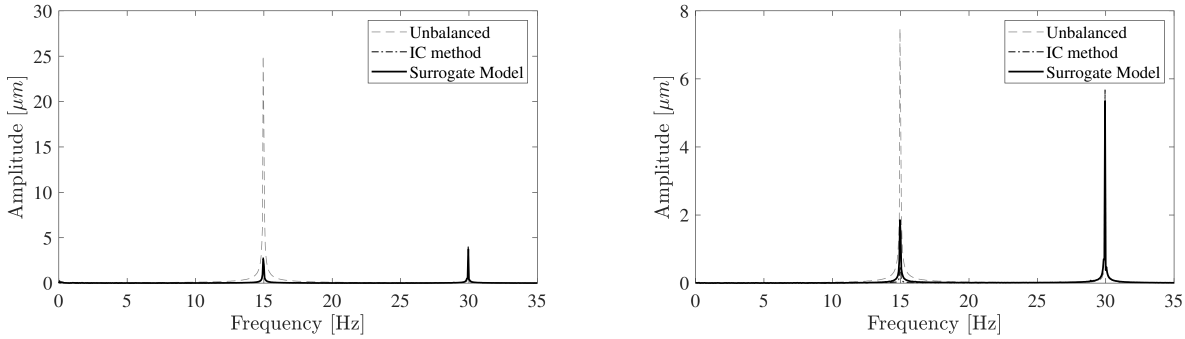

Figure 4: Validation test 3 frequency spectrum for sensors S_{1X} and S_{2X}, respectively.

CONCLUSION

Although a few modifications and adjustments were necessary to enable the experimental efficiency of the surrogate model, the obtained results show that the proposed technique is capable of balancing rotating machines. The average reduction for the proposed approach is close to the IC method, and it has presented itself even more superior during the validation tests. However, it is led to believe that with more tests the metrics would become closer. This approximation is a good indicator of the proposed approach's reliability, once it would present a behavior similar to the IC method. Nevertheless, once the surrogate model is finished it does not require the usage of trial weights.

The test rig used requires a more detailed study to understand the effects of the hydrodynamic bearings on the surrogate model and its prediction capability. In addition, further investigation is needed to understand the behavior of the experimental surrogate model, seeking to explore new possibilities, such as feedback loops.

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