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# General power-flow method for vibro-acoustic mid-frequency problems

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*Abstract: Most of the main production and transportation industries work with complex systems that present important vibro-acoustic mechanics, such as automotive vehicles, airplanes, marine and trains. Generally, these systems are composed by a set of distinct coupled components, e.g., pillar/column connected to a flexible plate or an acoustic cavity. At a specific frequency range of interest, these components vibrate in very distinct wavelengths simultaneously, resulting in a scenario denoted as mid-frequency problem. One of the methods used to simulate such systems is the Hybrid FE-SEA Method, which couples the Finite Element (FE) Method and Statistical Energy Analysis (SEA) by means of a robust formulation. The method has been used for more than a decade since developed and presents superior performance, when compared to other approaches in this scenario. There is, however, a limitation in the method to components vibrating in small wavelengths (SEA subsystems): their vibrational response needs to be approximated from elementary structural components, resulting in less robust results for irregular component configurations. In this sense, a novel method has been proposed, allowing for irregular configurations to be fully described in a power-flow framework. The method has been denoted Generalized Hybrid FE-SEAS, but it has been evaluated only for high-frequency problems and has yet to be applied to mid-frequency problems. The main aim of the present work is to evaluate the Generalized Hybrid FE-SEA. In this sense, the formulation will be reviewed and the method will be applied to two numerical cases. The results obtained show that the novel method is capable of deriving superior results to the established Hybrid FE-SEA when compared to a reference Monte Carlo simulation.*

***Keywords: Vibro-acoustics, Mid-frequency problems, Hybrid FE-SEA, Statistical Energy Analysis, Finite Element Method***

## INTRODUCTION

Vibro-acoustic systems in engineering industries (such as automotive, aerospace and marine) are analyzed by the use of multiple numerical methods. The application being explored mainly guides the choice of approach used for analysis. The system's configuration or the degree of complexity are also aspects that may underline the use of a specific method or a combination of multiple at same time. Lastly, although a specific approach is capable of optimal results, the demanded processing cost may be unfeasible for the context and, therefore, is not suited for the task.

Usually, these mentioned aspects are related to the frequency spectrum adopted for the application: for low frequency analysis, less modes are being represented, resulting in less processing costs and a more coherent deformation. Methods like Finite Element Method (FEM) (Meirovitch, 2010) for structural and Boundary Element Method (BEM) for acoustical applications exhibit optimal performance, as coarser meshes are required at these frequency regions. These methods are also capable of describing the highest level of detail for the system's configuration. For the case of high frequency analysis, the highly incoherent deformation and concentration of modes demand the model for expressive detailing in the system. Approaches like the Statistical Energy Analysis (SEA) (Lyon and Dejong, 1995) became an important alternative for this type of scenario, as the space and ensemble averaged descriptions made in the framework serve as a superior approximation for the uncertain diffuse field produced in the system. Moreover, these descriptions are obtained by analytical formulations, resulting in reduced processing costs.

If mid-frequency problems are considered (when small and large wavelength deformations are present at the same time at different components of the system), the established method that presents the optimal performance is the Hybrid FE-SEA (Shorter and Langley, 2005a), which connects the system's dynamic equilibrium with a power-flow model. This is elegantly idealized with the use of the reciprocity relationship between the direct-field radiation and the diffuse reverberant loading (Shorter and Langley, 2005b), which allows for the diffuse reverberant field of the components to be directly derived from the direct-field's impedance from their boundary. Moreover, most of the implementations of the method in commercial softwares assume that deformations in a diffuse field can be considered to be spatially incoherent, resulting in enormous computational cost reductions, as the power-flow model can then be computed by the already robust and established analytical formulations of SEA.

Although these analytical formulations proved to be excellent approximations to diffuse field descriptions, their range of possible configurations is limited only to the elementary scope. This is a consequence of these formulations begin analytical, which are incapable of obtaining known solutions to reasonably complex deformations. For this reason, the irregular components that assume small wavelength deformations in Hybrid FE-SEA (and SEA as well) are sectioned into multiple elementary sub-components/subsystems. Usually, the commercial softwares offer a set of possible subsystem's configurations: symmetric acoustic cavity, flat and curved plates are common examples (ESI Group, 2022).

Generally, these irregular components are not only sectioned, but considerably simplified to ensure a minimum amount of modes is guaranteed in most of subsystems. This simplification may imply loss in important information about the subsystem's deformation and, therefore, a generic description of the subsystem becomes appealing. This could be obtained with the support of a FE model of these components. In the case of a periodic FE model, the direct field impedance is captured by the dispersion curves (Cotoni, Langley and Shorter, 2008). Whereas, if a standard FE model is used, ensemble averaging techniques can be applied to the derive the direct field impedance (Devriendt et al., 2015). These direct-field impedances are used to model the exchange of energy between the components of system, however the power dissipated by these components is still analytically computed in both SEA and Hybrid FE-SEA. A former idea and formulation for a generic description for the dissipated power is presented in Alimonti et al. (2019) and an alternative is presented by Hinz (2021), where, in the latter, the dissipation is related to the intrinsic mechanic damping and the ensemble average internal dynamic mechanics.

These discussed generic descriptions were also evaluated in a power-flow model in Hinz (2021), where high frequency problems were considered in the analysis. In this context, this more generic approach is denoted as Numerical SEA, presenting a direct comparison to the established SEA. The results obtained exhibited the versatility of the method to different configurations (elementary, orthotropic materials and complex junctions) and superiority to SEA for irregular geometries. In case there are also components vibrating in large wavelengths in the system, a hybrid formulation is employed by the novel method (in this context, in order to reduce confusion, the method is denoted as Generalized Hybrid FE-SEA). The goal of the present work is to evaluate the performance of the novel method in a mid frequency problem, which had not been done yet, and compare it with Hybrid FE-SEA. A brief description of the two hybrid methods is present in the following. The implementation required for the novel method to be applied in the hybrid context and obtained results are also presented.

## Hybrid FE-SEA Method

The established method for mid-frequency problems idealizes the system into a set of two types of subsystems: the deterministic subsystems, the ones vibrating in large wavelengths, containing a small number of modes in the analyzed frequency spectrum. And the statistical subsystems, representing the rest of subsystems, presenting small wavelength deformations and high concentration of modes. Due to the highly coherent behavior of the deterministic subsystems, a coarse mesh is reasonable to represent their deformation, therefore they are described using a FE model, i.e., a dynamic stiffness matrix $\mathbf{D_d}$. In the case of statistical subsystems, their deformations are modeled by diffuse reverberant fields, which can be fully described by direct field impedances $\mathbf{D_{dir}}$ from the boundary/connections and a intensity parameter $C$ (this is the definition of the reciprocity relationship presented by Shorter and Langley, 2005b).

The cross-spectral response of the coupled system is obtained as (Shorter and Langley, 2005a)

$$\mathbf{S_{qq}} = \mathbf{D_{tot}}^{-1}\left[\mathbf{f_{ext}}\mathbf{f_{ext}}^{H} + \sum_i C_i \,\mathrm{Im}\left\{\mathbf{D_{dir,i}}\right\}\right]\mathbf{D_{tot}}^{-H}, \tag{1}$$

where the total dynamic stiffness matrix is the summation of the deterministic dynamic stiffness matrix and statistical subsystems' direct-field dynamic stiffness matrices ($\mathbf{D_{tot}} = \mathbf{D_d} + \sum_i \mathbf{D_{dir,i}}$). The $i$ subscript corresponds to the $ith$ statistical subsystem. The vector $\mathbf{f_{ext}}$ represents the external loading. The parameter $C_i$ represents the intensity of the $ith$ diffuse-field and is derived by idealizing a power-flow model between the statistical subsystem's (diffuse) wavefields. Usually in Hybrid FE-SEA's literature, the ensemble average modal energy $\langle \mathscr{E} \rangle$ is used in the place of the diffuse intensity parameter $C_i$ due to their defined relationship ($\langle \mathscr{E}_i \rangle = \pi\omega C_i$) (Shorter and Langley, 2005b), but here the $C_i$ is retained for generalization. Similar to SEA, the power-flow balance equation for the $ith$ subsystem is defined as

$$\langle \Pi_i^{\mathrm{in,dir}} \rangle = \langle \Pi_i^{\mathrm{out,rev}} \rangle + \langle \Pi_i^{\mathrm{diss}} \rangle, \tag{2}$$

where $\langle \Pi_i^{\mathrm{in,dir}} \rangle$, $\langle \Pi_i^{\mathrm{out,rev}} \rangle$ and $\langle \Pi_i^{\mathrm{diss}} \rangle$ are, respectively, the $ith$ statistical subsystem's ensemble average power being injected to its diffuse-field (by the radiation of the direct-field), the ensemble average power being ejected by its diffuse-field to other subsystems and the ensemble average power dissipated by the subsystem. The two first power-flow contributions ($\langle \Pi_i^{\mathrm{in,dir}} \rangle$ and $\langle \Pi_i^{\mathrm{out,rev}} \rangle$) are derived in accordance with the dynamic equilibrium of the system, therefore taking into account the subsystems' impedances ($\mathbf{D_{dir,i}}$ and $\mathbf{D_d}$). Their full derivation can be found in (Shorter and Langley, 2005a). Again, similar to SEA, the dissipated power $\langle \Pi_i^{\mathrm{diss}} \rangle$ is defined as,

$$\langle \Pi_i^{\mathrm{diss}} \rangle = \pi\eta_i \langle E_i \rangle = \pi\eta_i n_i \langle \mathscr{E}_i \rangle = \pi\omega \mathscr{M}_i C_i \tag{3}$$

where $\eta_i$, $n_i$ and $\mathscr{M}_i$ are, respectively, the damping loss factor, modal density and the dissipation coefficient of the $ith$ statistical subsystem. Here, the latter coefficient becomes equivalent to the modal overlap factor ($\mathscr{M}_i = \pi\eta_i n_i$). Finally, the power-flow balance equation can then be expressed as,

$$\begin{bmatrix} \mathscr{M}_1 + \mathscr{M}_{\mathrm{d},1} + \sum_{j \neq 1} h_{\mathrm{j},1} & \dots & -h_{1,\mathrm{N}} \\ \vdots & \ddots & \vdots \\ -h_{\mathrm{N},1} & \dots & \mathscr{M}_{\mathrm{N}} + \mathscr{M}_{\mathrm{d},\mathrm{N}} + \sum_{j \neq N} h_{\mathrm{N},\mathrm{j}} \end{bmatrix} \begin{bmatrix} C_1 \\ \vdots \\ C_{\mathrm{N}} \end{bmatrix} = \frac{1}{\pi \omega} \begin{bmatrix} \Pi^{\mathrm{ext}}_{\mathrm{in},1} \\ \vdots \\ \Pi^{\mathrm{ext}}_{\mathrm{in},\mathrm{N}} \end{bmatrix}, \quad (4)$$

where $\mathscr{M}_{\mathrm{d,i}}$, $h_{i,j}$ and $\Pi^{\mathrm{ext}}_{\mathrm{in,i}}$ represent, respectively, the dissipation coefficient from the deterministic subsystems, the transfer coefficient from the $ith$ to the $jth$ statistical subsystem and the input power from external loads to the $ith$ statistical subsystem. All these coefficients were obtained from the derivation subsequent of Eq. 2. Each of these coefficients are derived in detail in Shorter and Langley, (2005a). The obtained diffuse-field amplitudes $C_i$ can then be related to their subsystem's energies, and, therefore, to the statistical subsystem's engineering units. The response of the deterministic subsystems can be directly obtained by solving Eq. 1, as the diffuse amplitudes were already determined.

## GENERALIZATION OF THE METHOD

As mentioned, the power-flow balance of the ith statistical subsystem is fully described according to their direct field dynamic stiffness and dissipation coefficient. The generalization of the method consists in deriving these same parameters for subsystems with generic configurations, not limited to elementary scope. Here in present work, these generic derivations were obtained by the use of standard FE models of the statistical subsystems. Their FE matrices were averaged in an ensemble to derive proper diffuse contributions for the power-flow derivation. The efficient averaging technique employed by Devriendt (2015) is used here, as its analytical calculation allows for large processing costs reduction. Lastly, the derivation of $\mathbf{D_{dir,i}}$ is achieved by the use of these ensemble averaged FE matrices in a mathematical procedure denoted Schur complement (Devriendt, 2015). As for $\mathscr{M}_i$, the same averaged matrices were used (allowing for a single implementation) in combination with the FE formulation for intrinsic mechanic damping (Mace, 2000). Their full derivations in this generalized context were presented in Hinz (2021) and in Morhy (2022).

The implementation of these generic derivations requires expressive matrices multiplications, due to the use of FE models. In order to achieve feasible computational efficiency when modeling complex systems in mid and high frequency problems, a reduction of models becomes necessary. Therefore, projections into modal basis are employed to the interior of the statistical subsystems, where most of the degrees of freedom are concentrated, and to the deterministic subsystems (Morhy, 2022). The only remaining nodal degrees of freedom after reduction of model are the ones contained at junctions/excitation points connected solely to statistical subsystems, where they were retained to enforce compatibility. Another important aspect in the implementation of these generalized descriptions is regarding the energetics of the diffuse wavefields. If no post-process is applied to the FE matrices, a single lumped generic wavefield is defined for each statistical subsystem. This condition actually is susceptible to overestimation of energy dissipation, as it assumes equipartition of energy between all possible wavefields inside the subsystem. For example, in the case of a flat plate, this assumption is not reliable, as the in-plane modes dissipates and exchanges far less energy than out-of-plane ones. But for irregular geometries, the wavefields tend to become coupled and this overestimation tends to cease. A detailed process to partition the out-of-plane and in-plane wavefields from a lumped wavefield of a flat plate is presented in Morhy (2022).

## NUMERICAL EVALUATION

The novel method was evaluated in two numerical cases and was compared to the results of the established Hybrid FE-SEA method and a reference curve. This reference curve was obtained from a FE Monte Carlo model, which generates an ensemble of similar randomized systems, their ensemble average is the reference curve. The closer the hybrid methods converge to the reference curve, the better the respective method is assumed to be modeling the system. In the presented work, the randomization of the FE Monte Carlo samples was then employed by the application of constrains (either clamped or pinned) over random portions of the domain of the statistical subsystems. A convergence analysis was employed to determine the required number of samples for each numerical case in the FE Monte Carlo ensemble. The FE matrices and 3D information used in the novel method and FE Monte Carlo were extracted from VAOne software (ESI Group, 2022) and post-processed in Matlab (Mathworks, 2020). In the case of the established Hybrid FE-SEA method, its results were directly obtained from VAOne. Furthermore, it is assumed a damping loss factor of 1% (0.01) to every analyzed subsystem, six element per wavelength in FE models and the extraction of, at least, modes with natural frequencies up to twice the maximum frequency analyzed. The material properties of the analyzed subsystems are listed on Tab. 1 (they are either composed of aluminum or steel). Lastly, at each case, a single subsystem is excited by a transverse point force of 1N (represented by a purple arrow). In the FE Monte Carlo model, the point force is randomly placed at the excited subsystem for each sample (with the condition of being far from discontinuities to avoid coupling of wavefields).

### Co-planar flat plates coupled by a beam

The first numerical case consist of two aluminum co-planar plates coupled at four specific points (two for each plate) to a stiff steel beam with rectangular cross section. These connections are exhibited in the figure as yellow circles (one is hidden behind the beam). Regarding their geometry, both plates have and area of $0.723m^2$ with the excited (the green one) and receiver plate (the orange one) admitting, respectively, a thickness of $1mm$ and $2mm$. The beam of $1.1m$ length

Table 1: Aluminum and Steel material properties

| Properties | Aluminum | Steel |
|---|---|---|
| **Density** $\rho$ [kg/m³] | 2700 | 7800 |
| **Young's Modulus** $E$ [GPa] | 71 | 210 |
| **Shear Modulus** $G$ | 26.7 | 80 |
| **Poisson's Ratio** $\nu$ | 0.329 | 0.3125 |

has a cross section shape of $0.1m$ x $0.08m$ with $10mm$ of thickness. The plates were modeled as statistical subsystems with partitioned wavefields (in-plane and out-of-plane), whereas the beam was identified as deterministic.

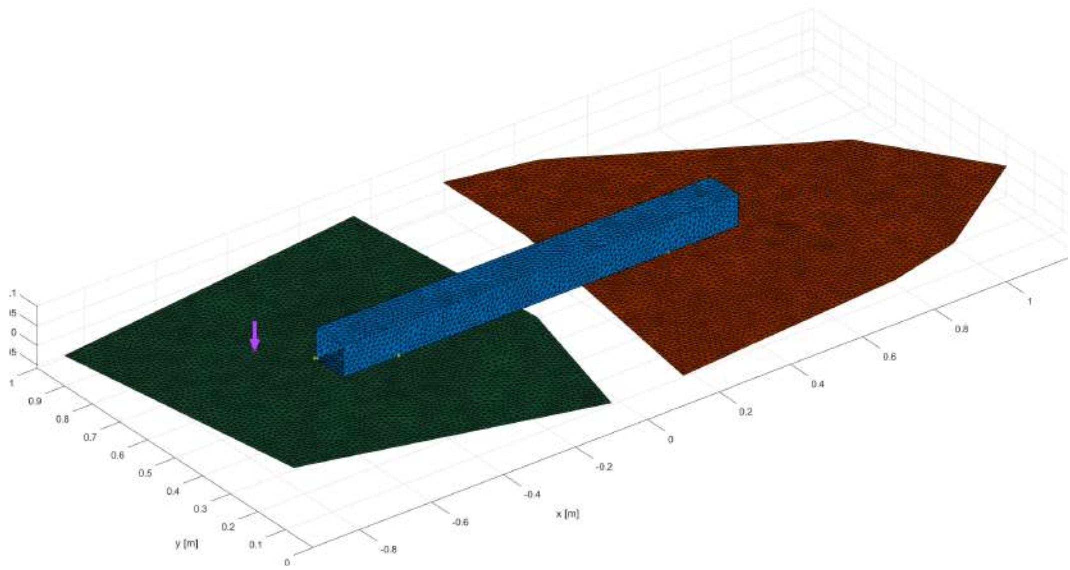
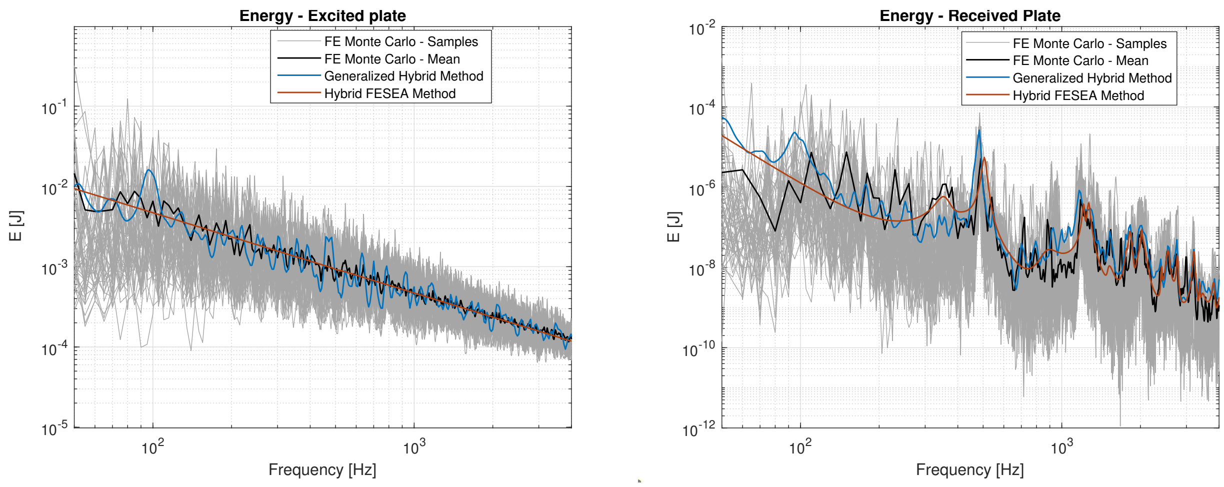



Figure 1: Numerical case 1 - 3D visualization

Results for the excited and receiver plates are exhibited, respectively, in Fig. 2a and Fig. 2b. In the graphs, we clearly see the results from each sample of the FE Monte Carlo (gray curves) and its mean (black curve), which is the reference result. For the excited plate's results, the response is predominantly influenced by the internal mechanics of the respective subsystem and by the external loading, resulting in a simple decay over frequency for all methods, due to the high modal overlapping presented for such flat plate. In the case of the receiver plate, specific predominant modes become present in results, which is a consequence of the modal behavior of the steel beam that connects both plates. The stiff beam admits a strong spatially coherent deformation, due its low modal density, therefore the receiver plate's response is highly influenced by these predominant modes. Here, again, both hybrid methods were capable of obtaining similar results compared to the reference, showing that the novel method is capable of deriving robust descriptions for the statistical subsystem's wavefields. In case a lumped wavefield was assumed for the statistical subsystems, an overestimation of in-plane wavefield's dissipation of energy is obtained, resulting in the receiver plate's response exhibited in Fig. 3 (with FE Monte Carlo sample's results omitted). In terms of computational cost, the established Hybrid FE-SEA presented the best performance between the compared methods (Fig. 4), due to the analytical formulations used to model the statistical subsystems' wavefields.




(a) Excited plate

(b) Receiver plate

Figure 2: Numerical case 1 - vibrational energy results

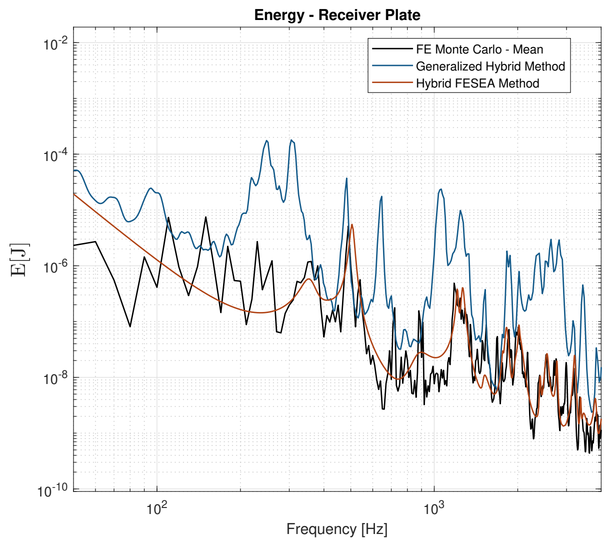



Figure 3: Numerical case 1 - vibrational energy results for a receiver plate with lumped wavefield

## Cube beam framework

The second case is exhibited in Fig. 5 and consists of four flat plates (green) connected by their edges to a single beam framework (orange), resulting into a similar cube shaped system (with its top and bottom sides opened). It is assumed that these plates have their whole edges connected to the framework and one of the plates is excited. The beam framework has a hollow structure with a squared cross section of 2.52cm side, 3mm of thickness and is made of steel. As for the plates, they are identical, having a squared area of 0.4724m², thickness of 2mm and are made of aluminum. Moreover, a clamped boundary condition is applied to two bottom outer edges of the beam framework (Figure 6). The plates were modeled as statistical subsystems and the beam framework as deterministic. The plates' wavefields were partitioned into out-of-plane and in-plane ones.

The vibrational energy results for the front excited plate and a side plate are exhibited, respectively, in Fig. 7a and Fig. 7b. For the excited front plate, both methods were capable of obtaining comparable results to the reference. However, for the side plate's result, a divergence is presented between the established hybrid method and the reference in higher frequencies, which is mostly caused by the simplifications made by the analytical formulations to the junctions between the plates and the beam framework (in established hybrid, it is assumed that line junctions are always straight lines due to analytical solutions. Here in the second case, a squared junction is assumed). The novel method, however, presented great convergence to the reference's results, showing that the generalized descriptions were capable of improving the analysis in such complex vibro-acoustic problems. Furthermore, in terms of computational costs, the novel method demanded far

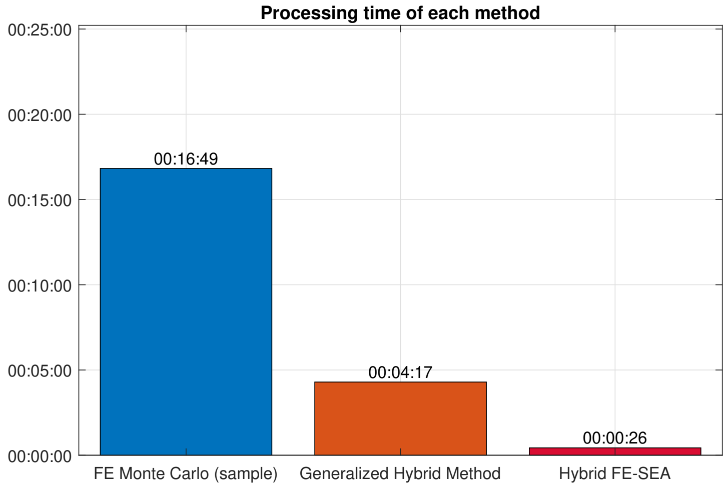



Figure 4: Numerical case 1 - Computational costs

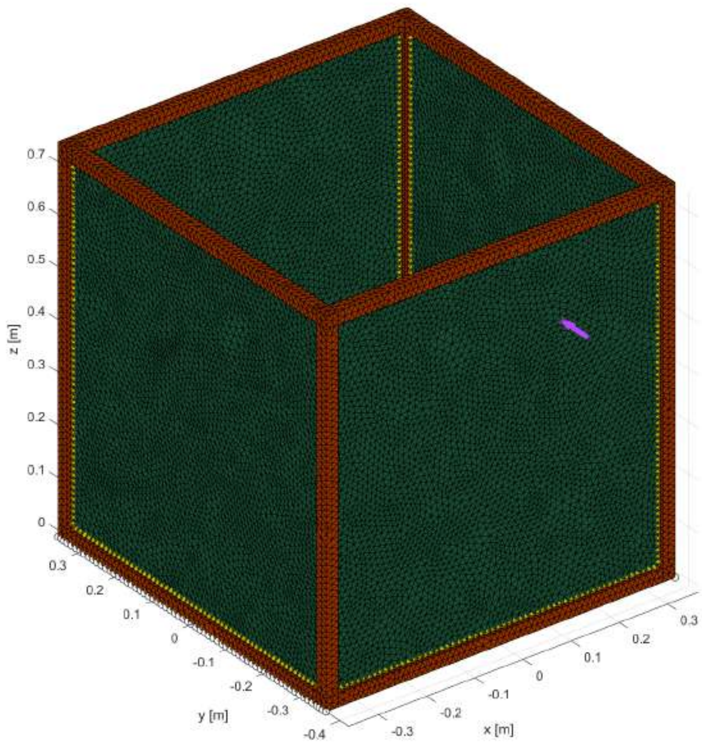



Figure 5: Numerical case 2 (3D visualization)

less compared to a single sample of the FE Monte Carlo ensemble and the established Hybrid FE-SEA method (Fig. 8), the latter difference is mostly caused by the software's simplifications to the junction, as probably four separated straight line junctions were assumed of each plate, instead of just a generic one.

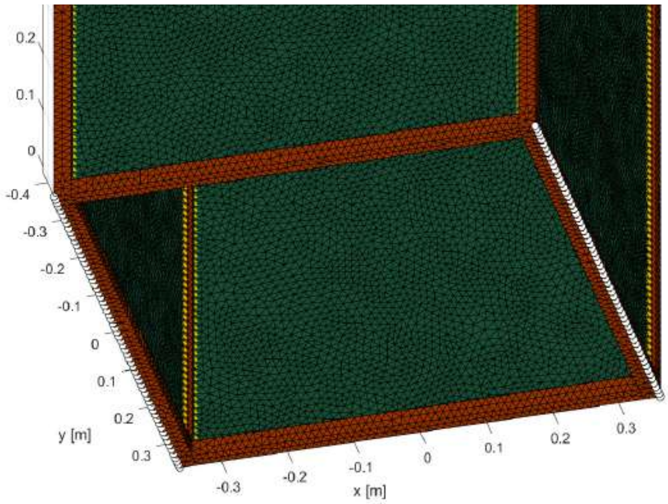
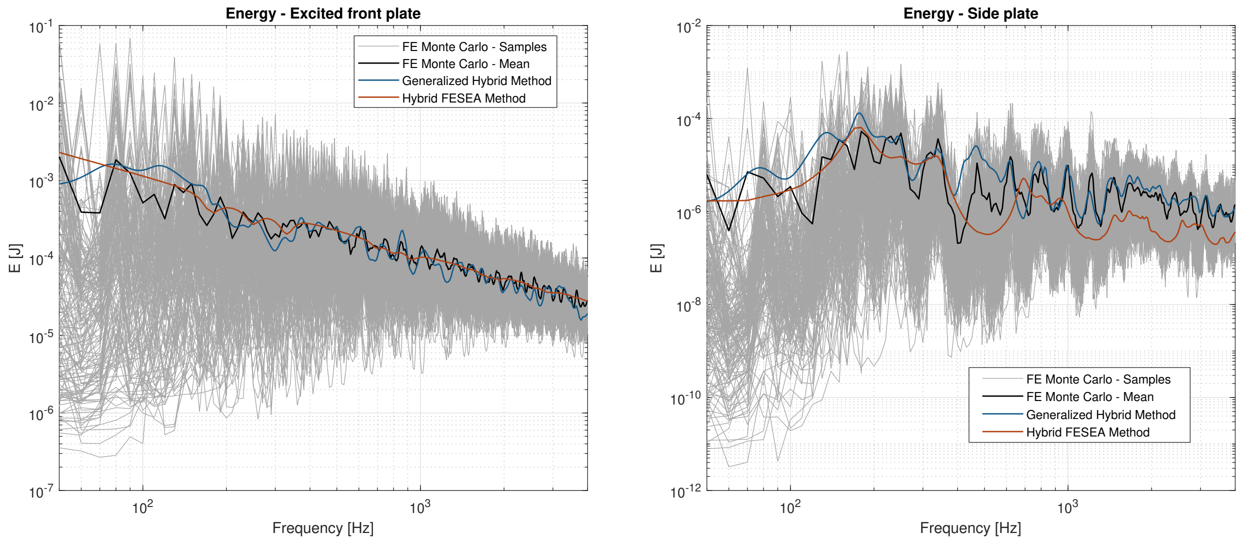



Figure 6: Numerical case 2 - Constrained nodes (white balls)




(a) Excited front plate

(b) Side plate

Figure 7: Numerical case 2 - vibrational energy results

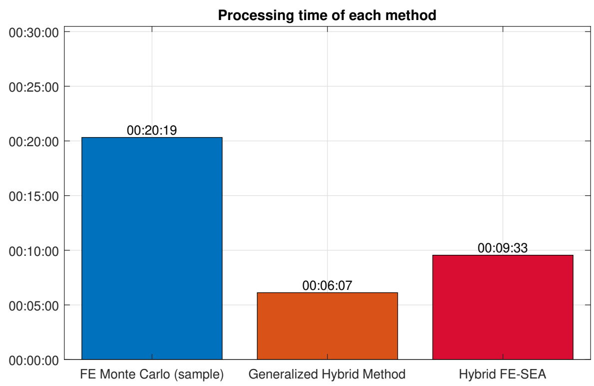



Figure 8: Numerical case 2 - Computational cost

## CONCLUSIONS

As an alternative for modeling of complex vibro-acoustics systems, the Generalized Hybrid FE-SEA exhibited promising performance when evaluated in mid-frequency problems against established methods. For cases where elementary configurations predominantly compose the system (first numerical case), the novel method presented equivalent results to the established hybrid one, however demanded larger computational costs. Whereas for cases with complex configurations (second numerical case), the analytical formulations used in established Hybrid FE-SEA method presented difficulties in generating a proper wavefield definition for the statistical subsystems. In the other hand, the generic descriptions successfully modeled the power-flow between the diffuse wavefields of the statistical subsystems in the cost of less computational processing, representing a powerful tool for vibro-acoustic modeling.

There are still more evaluations and developments to be made regarding the generic descriptions. No acoustic cavity was explored in the presented work, which is essential to vibro-acoustic scenarios, therefore, no area connection between structural and acoustic domains was also explored. A deeper and detailed analysis of the partitioned wavefield is still to be made, deriving a generic process to identify and partition wavefields on irregular configurations. Plenty of future works are possible for the development of complex vibro-acoustic modeling.

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