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Design of a Tuned Mass Damper for a Suspension Bridge Model

Jose Marcos A. Silva Jr.[1], **Paulo R. Novak**[2], **Giovanni Bratti**[3], **and Francisco Augusto A. Gomes**[2]

[1] Civil Engineering Graduate Student, Federal University of Technology - Parana, Via do Conhecimento s/n, KM 01, 85503-390, Parana, Brazil

[2] Civil Engineering Graduate School, Federal University of Technology - Parana, Via do Conhecimento s/n, KM 01, 85503-390, Parana, Brazil

[3] Dept. of Mechanical Engineering, Federal University of Technology - Parana, Via do Conhecimento s/n, KM 01, 85503-390, Parana, Brazil

Abstract: Tuned Mass Dampers (TMDs) are very useful when aiming to mitigate vibration-related problems. This device, once attached to a structure, considerably attenuates the effects due to dynamic loads. Although TMDs exhibit a favorable performance over one specific frequency range, structures have multiple natural frequencies, which would sometimes demand a different TMD for each frequency. However, this kind of solution would often overload the primary structure and limit TMD's damping performance. The purpose of this work is to design and model a TMD capable of damping two vibration modes of a suspension bridge's deck. For this, the primary structure is modeled by the Finite Element Method, using ANSYS® combined with a MATLAB® routine, and applying the "Equal Peak Design" technique to estimate the optimum damping and frequency ratio for the system. The results show the design TMD can considerably reduce the structure's vibration levels.

Keywords: Tuned Mass Dampers, Finite Element Method, Vibration Control, Natural Frequencies, Numerical Analysis.

INTRODUCTION

Over the last century, due to the advance of the construction technology, a continuous increasing of computers capacity, and a refined control of the materials manufacturing process, the number of complex and long span structures have increased. In this scenario, many are the forms engineers found to adequately design those, and one of the most popular are the suspension bridges. Nonetheless, when responding to dynamic loading, this type of structure tend to present a very low natural damping, usually below 1%, which makes the energy dissipation one of the main issue when designing (Pacheco *et al.*, 1993).

Since Frahm (1911), Tuned Mass Dampers (TMDs) have been widely used when one aims to solve vibration related problems in many types of machinery. Their application in civil engineering structures, on the other hand, began to intensify after the important work of Den Hartog and Ormondroyd (1928). This device consists of a secondary vibrating mass attached to a primary structure and adequately tuned with its resonant frequency (Inman, 2017). In order to control its performance, the only parameter it must taken into account, once optimally tuned, is the mass ratio to the main structure's (Rao, 2017).

Many works have shown countless possible TMD applications. Yoon *et al.* (2021) developed and optimized a resonance-based mechanical dynamic absorber able to damp three natural frequencies of two cantilever beams. Zhu *et al.* (2018) studied and optimized distributed dynamic vibration absorbers for suppressing vibrations in plates, achieving optimal suppression effect of higher order modal vibration. Cieplok and Sikora (2015), and Pan and Zhang (2012), studied the effect of having more than one mass within the same dynamic absorber, showing how this type of solution is able to widen the device's bandwidth of operation. Yang and Dai (2014), and Ma *et al.* (2019), designed a two and three degree-of-freedom TMD, respectively, aiming only the main structure's fundamental mode. In both cases the result was a significant reduction of the vibration levels.

However, a structure has as many natural frequencies, i.e. vibration modes, as degrees-of-freedom (DOF). This would raise the necessity of designing a TMD for each of the critical ones. Nonetheless,this kind of solution usually overloads the main structure. For the annexation of the excessive mass, could easily compromise the performance of the devices appended thereon. This phenomenon is called "weight penalty" and it generally demands the limit for the TMD mass to be between 1-3% of the primary structure total mass (Mokrani *et al.*, 2017).

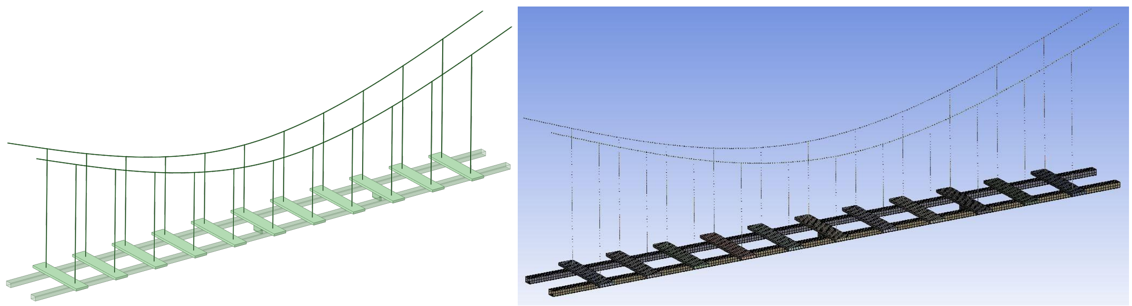
When aiming to control the many vibration modes of a bridge, some studies tried the use of multiple TMDs to control each mode separately, e.g. Tubino and Piccardo (2015) applied it to inhibit pedestrian induced vibration, whereas Wang *et al.* (2003) to highly speed trains induced excitation. Nevertheless, as the number of targeted modes increase, so does the weight penalty due to the extra masses attached to the primary structure. Proposing to prevent such phenomenon, the goal of this study is to design a TMD capable of damping two degrees-of-freedom at the same time (1st bending and torsional modes) for a Finite Element Suspension Bridge model applying Den Hartog (1985) technique. Other works pursued the same goal, while using different approaches. Lin *et al.* (2000), e.g., considered a coupled 2DOF TMD. Here, the followed procedure is similar to what was made by Mokrani *et al.* (2017) and Meng *et al.* (2020), in which the 2DOF TMD modes had their physical coordinates decoupled.

METHODOLOGY

Suspension Bridge Scale Model

The structure defined for this study as "primary" is exposed in Fig. 1. The deck of the bridge has two main longitudinal beams (girders) with a total length of 120 cm, its cross-section is 1 by 1.5 cm. Above them, there are 11 transverse beams (15x3 cm; and 0.5 cm thickness). These beams are equally spaced by 10 cm (axis to axis). Besides, there are 11 suspending cables (Ø0.2 mm) connecting the bridge's deck to a main cable (Ø0.5 mm). All suspenders are also equally spaced by 10 cm. The structure's total height is 31.5 cm, whereas the main cable's sag (ratio between height and total span) is 15 cm.

The boundary conditions for the model are as recommended by Serap *et al.* (2012) and Li *et al.* (2010). The girders are articulated, free to rotate about any axis, while not being able to translate. The main cable pair are fixed at both ends. All contacts between the parts were considered as bonded. Finally, aiming to simplify the model when designing the TMD, the cables' dynamics were ignored (Irvine and Caughey, 1974). Their interaction with the deck is restricted to tension only behavior, and linear-elastic assumptions are made. After the structure is modeled in ANSYS® SpaceClaim, a Static Structural Analysis is developed followed by a Modal Analysis to identify the targeted modes, both in ANSYS® Mechanical.



a) b)

Figure 1 – a) CAD presentation for the analyzed suspension bridge model; b) Meshing model for the structure

Two Degree-of-Freedom Tuned Mass Damper

In order to damp the first bending and torsional modes of the bridge model, a discrete 2DOF system is idealized to represent the suspension bridge's deck - see Fig. 2a. Since its translation and rotational movements appear to be fully decoupled in global coordinates, one could control it by using two individual TMDs, as shown in Fig. 2b. Those TMDs should then be adequately tuned to decrease the main body vibration response, which is governed by Eq. (1), from Rao (2017):

$$\frac{X_1 k}{F_0} = \left\{ \frac{(2\xi r)^2 + (r^2-\beta^2)^2}{(2\xi r)^2(r^2-1+\mu r^2)^2 + [\mu\beta^2 r^2 - (r^2-1)(r^2-\beta^2)]^2} \right\}^{1/2}, \tag{1}$$

where

$\mu = m_2/m_1$ – Mass ratio between the TMD and the primary structure
$r = \omega/\omega_n$ – Forced frequency ratio
$\xi = c_2/c_c$ – Damping ratio: TMD damping constant by its critical damping
$\beta = \omega_a/\omega_n$ – Frequency ratio: TMD frequency over the system's.

Other solution is connecting the main structure to a 2DOF TMD, in which both movements could be approximated as to be happening independently from one another, allowing the Eq. (1) to be solved separately for each mode. Such case is exposed in Fig. 2c. There, the primary structure is attached to a 2DOF TMD consisted of two point masses at both ends of a rigid bar, "pinned" at its center by a rotational spring-damper set (k_t and c_t), as well as "longitudinal" spring-damper set (k_2 and c_2).

Thus, what needs to be done in order to tune independently both vibration modes of the 2DOF TMD with the bridge's is solving both, the translational dynamic equation given by

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & k_2 \\ k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{2}$$

and the rotational dynamics equation for the system (2DOF TMD-Bridge)

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} ak_1+k_t & k_t \\ k_t & k_t \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} af \\ 0 \end{Bmatrix}. \tag{3}$$

Sequentially, to optimally define the parameters for each of the aforementioned equations, Den Hartog (1985) defined, using the "minimax" optimization problem-solving technique, minimizing the maximum value of the H_∞ norm (Eq. (1)), the optimum frequency ratio β, and optimal damping ratio ξ for a Structure-TMD system. This method is often called "Equal Peak Design". It allows the optimal dimensionless parameters presented in Eq. (1), to be estimated by Eqs. (4) and (5):

$$\beta_{optimal} = \frac{1}{1+\mu} \tag{4}$$

$$\xi_{optimal} = \sqrt{\frac{3\mu}{8(1+\mu)^3}}. \tag{5}$$

After solving Eqs. (2) and (3) in the form of Eq. (1), and using the conditions brought by Eqs. (4) and (5), the only parameter left to be estimated for an optimal 2DOF TMD design is the mass ratio μ. In this work, it was designed two sets of 2DOF TMDs, one with $\mu = 1/20$, labeled as Case 1, and other with $\mu = 1/40$, as Case 2.

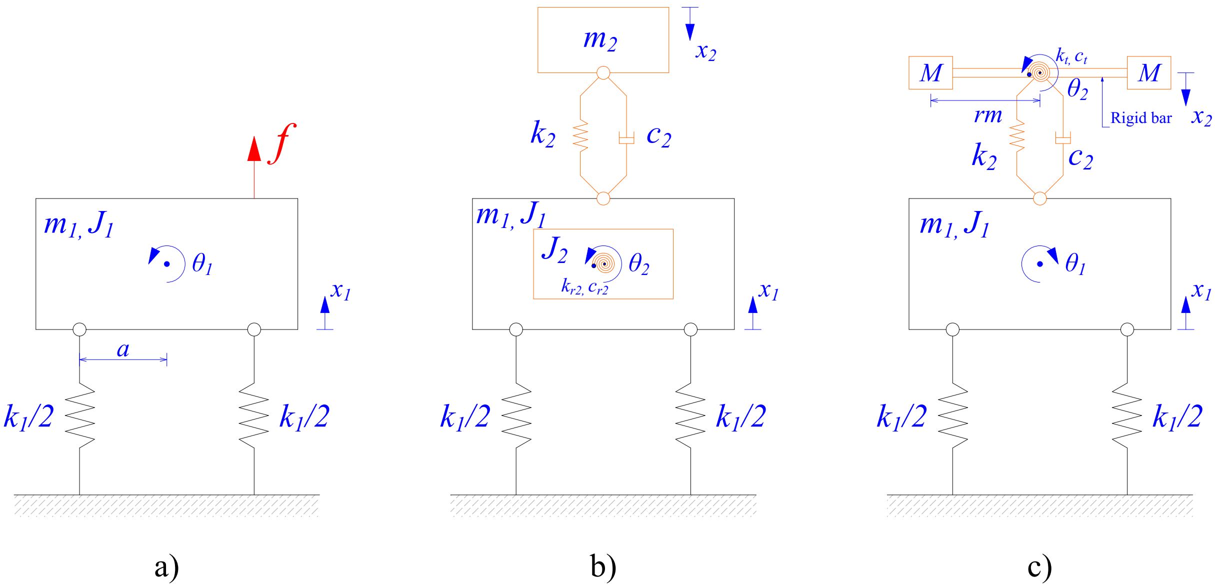


Figure 2 – a) Single DOF oscillating system representing the suspension bridge, b) Attachment of two individual TMD, one for each movement (bending and torsion), c) Fully decoupled 2DOF TMD controlling bending with k_2 and c_2 and torsion with k_t and c_t.

The final task is finding the physical parameters that allow both vibrating modes of the 2DOF TMD to be optimally tuned with the primary system's. In this study, the chosen model is presented in Fig. 3. The variables programmed to control the bending mode were the main beam's length "l" and the point masses "M", whereas the rotational movement is regulated by the arm "r_m", i.e. the distance between the main beam's x axis and each point mass.

The "springs" stiffness for the 2DOF TMD is given below (translation and torsional, respectively), according to Warren and Richard (2002):

$$k_2 = \frac{bt^3E}{2l}, \quad k_t = \frac{CG}{l} \quad \text{, where } C = bt^3\left[\frac{16}{3} - 3.36\frac{t}{b}\left(1 - \frac{t^4}{12b^4}\right)\right] \tag{6}$$

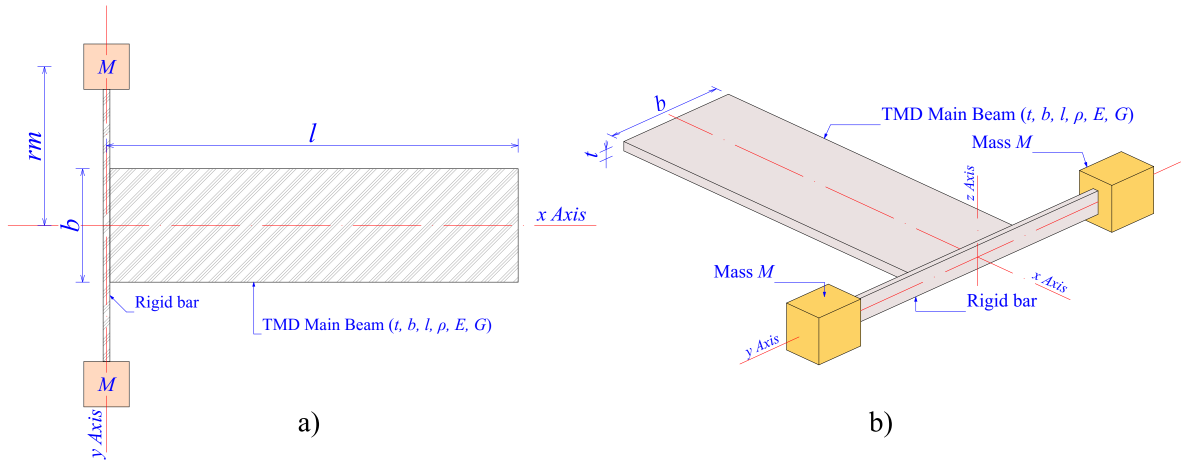


Figure 3 – a) Plan view of the 2DOF TMD, b) Isometric view of the 2DOF TMD.

The primary structure's physical variables, J_1 and m_1 are extracted directly from ANSYS®. Whereas J_2 and m_2 are functions of the first two (following the desired μ), considering the optimum design strategy (Eqs. (4) and (5)). Finally, it is recommended the 2DOF TMD to be placed where the biggest amplitude occurs, according to the targeting modes presented on Fig. 4, from the Modal Analysis developed in ANSYS® Mechanical.

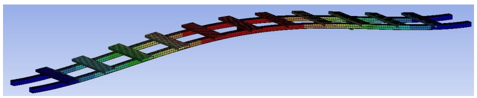
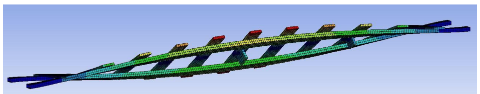
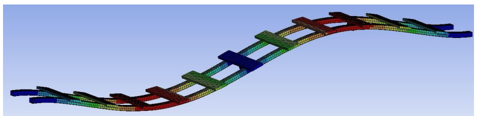
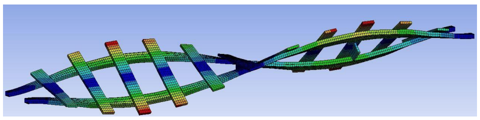
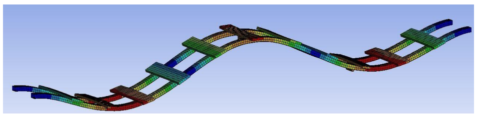
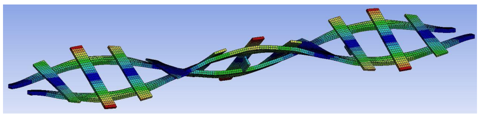
Numerical Mode Shape of the Bending Modes	Numerical (*Hz*)	Numerical Mode Shape of the Torsional Modes	Numerical (*Hz*)
 1st Bending (a)	23.16	 1st Torsional (b)	34.54
 2nd Bending (c)	26.91	 2nd Torsional (d)	58.38
 3rd Bending (e)	52.55	 3rd Torsional (f)	86.05

Figure 4 – a-f) Main estimated vibration modes. In the right side, the torsional modes, whereas in the left, the first three bending modes

RESULTS

Once all physical variables are adequately computed in a MATLAB® routine, the 2DOF TMDs are modeled in ANSYS® and placed in the middle of main span, according to the bridge's Modal Analysis previously presented. Table 1 exposes the estimated values for both proposed 2DOF TMD cases. The Frequency Response Function (FRF), as well as the Impulse Response Function (IRF) of the deck, for both considered cases, are presented hereafter. To the main beam and the rigid bar, aluminum alloy was assigned.

Table 1 – Geometric values for both 2DOF TMD cases considered

Physical Variables	l	t	b	M	r_m	$\xi_{optimal}$
TMD Case 1	8.87 cm	0.2 cm	2.5 cm	130.6 g	4.0 cm	0.1273
TMD Case 2	11.0 cm	0.2 cm	2.5 cm	64.2 g	5.1 cm	0.0933

After a Harmonic Analysis was developed in ANSYS® Mechanical, a FRF is presented in Fig. 5 representing the main structure's displacement behavior. It is seen the 2DOF TMD was able to damp both bending and torsional frequencies at the same time, as it was expected from the theoretical model (Mokrani *et al.*, 2017).

Besides, from Fig. 5, one may also notice that the 2DOF TMD, Case 1, exhibits a fairly better performance once attached to the bridge, reducing the maximum amplitude by 89% and 93% for the first bending and torsional modes, respectively. Whereas, in Case 2, the reduction of the vibration levels was of 86%, at the former, and 92% for the letter.

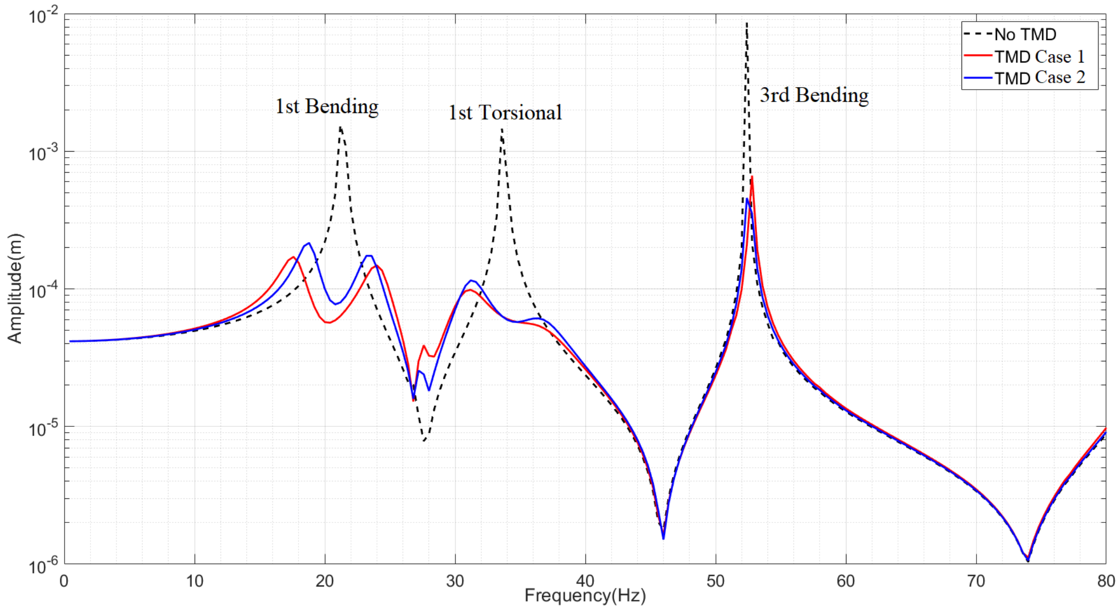


Figure 5 – Frequency Response Function comparing the behavior of the primary structure with no TMD attached in black, Structure-TMD Model 1 ($\mu = 1/20$) in red, and Structure-TMD Model 2 ($\mu = 1/40$) in blue.

The same result is perceived when observing the Impulse Function Response graph shared in Fig. 6. The 2DOF TMD, Case 1, showed a slightly superior performance at the steady-state phase of the response - 86% reduction, against 78% for case 2. This implicates that greater mass ratio would be a preferable choice. Notwithstanding, in practical applications, the 2DOF TMD, Case 2, would be more suitable, for it would decrease the weight penalty due to the additional mass, besides having a lower construction cost (Mokrani *et al.*, 2017).

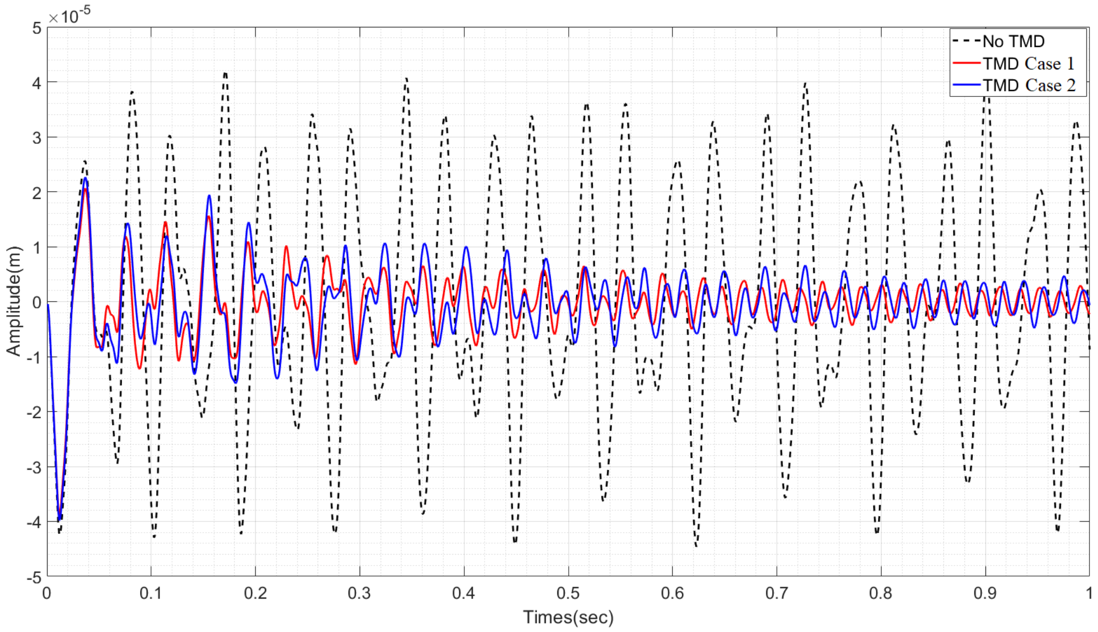


Figure 6 – Impulse Response Function plot comparing the behavior of the primary structure with no TMD attached in black, Structure-TMD Model 1 ($\mu = 1/20$) in red, and Structure-TMD Model 2 ($\mu = 1/40$) in blue.

CONCLUSIONS

The purpose of this work was to design a 2DOF TMD capable of damping the first two (bending and torsional) natural frequencies of a suspension bridge numerical model, using Equal Peak Design method by Den Hartog (1985). To accomplish that, at first, a simplified model of the primary structure was defined, modeled by the Finite Element Method using ANSYS®, and the target modes were chosen. Then, a MATLAB® routine was written to help estimate the optimal physical dimensions for the model. After finding those measures, the 2DOF TMD was also modeled by the FEM using the same tool, and attached to the main system.

It was observed that all 2DOF TMD cases were able to adequately damp both of the targeted frequencies at the same time, which is perceived by the significant vibration level's reduction at the analyzed structure. Thus, the methodology's effectiveness and the device's potential to damp both modes at the same time were numerically verified in this work. In the future, it is planed to expand the virtual model into a laboratory and experimentally replicate what was hereby achieved.

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