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On the formation of a super attenuation band in a beam

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Abstract: This paper investigates the formation of a super attenuation band in a beam structure with a vibration absorber attached. Typically, the interaction between the host structure and the vibration absorber creates a scattering mechanism and adds an anti-resonance component to the structure dynamic. When the system is correctly tuned, the two effects can be potentially combined and form a single but wide attenuation band. Using the dynamic stiffness method, the transversal and angular displacement transmissibilities of the structure are determined and the effect of changing the natural frequency of the vibration absorber is investigated numerically. It is shown that there are two tuned conditions in which a smooth super attenuation is formed. One is when the system is tuned to a frequency which corresponds to the anti-resonance of the point and transfer receptances with respect to the source. The second condition is when the system is tuned to the free-free natural frequency of the system. In both conditions, the correct tuning creates a smooth super attenuation band capable to reduce the transversal vibration in the beam over a larger frequency range.

Keywords: super attenuation band, beam structure, vibration absorber

INTRODUCTION

The use of vibration absorbers to mitigate or reduce the amplitude level of vibration has been a hot research topic in the area of structure dynamics (Kela and Vähäoja, 2009). Typically, due to the interaction between the vibration absorber and the host structure, these devices create a scattering mechanism and add an anti-resonances component to the structure dynamics (Gonçalves, Brennan and Cleante, 2021), acting as a filter and reducing the vibration amplitude by several orders of magnitude. The main advantage is that the frequency at which this anti-resonance occurs is controlled by the vibration absorber and can be adjusted to occur at any particular frequency. However, its bandwidth is narrow, and the performance of the device depends upon the absorber mass. Different approaches have been investigated to overcome this limitation, e.g., using arrays of vibration absorbers with the same (Silva et al., 2020) or different (Brennan, 1997; Hu et al., 2021) tuned frequency or using a combined force and moment vibration absorber (Toit et al., 2021).

An alternative is to form a smooth super attenuation band. This phenomenon has been investigated in mono-coupled structures such as rod and lumped systems (Cleante et al., 2022; Gonçalves, Brennan and Cleante , 2021) and consists of combining the scattering mechanism with the local resonance attenuation band to form a single, but wide attenuation band. This phenomenon has been also exploited in infinite metamaterial beams. Xiao et al. (2013) showed that using an array of vibration absorbers, due to the multi-coupled characteristic of the structure, only a quasi-super attenuation band is formed. This paper investigates the use of a single force vibration absorber as a passive vibration control measure to form a super attenuation band in a beam structure. Using the dynamic stiffness method, the transverse and angular displacement transmissibilities are determined, and the formation of super attenuation band in a single beam span is investigated numerically. To achieve this, the required natural frequency of the vibration absorber is determined and hence the optimal tuned frequency is determined. This extended abstract is organised as follows. Following the introduction, the methodology to investigate the problem is described. A numerical investigation into the formation of the super attenuation band is carried out before closing with some conclusions.

PROBLEM STATEMENT

A one-dimensional beam structure is shown in Fig. 1. It consists of a uniform homogeneous beam with a single-degree-of-freedom vibration absorber attached to the centre of the beam. The beam is subjected to bending vibration which involves transverse and rotational displacements.

The system shown in Fig. 1 consists of a single beam of length l, with forces F, moments M, transverse displacement W and angular displacement θ, at each end of the beam, where the subindex L and R denote left and right respectively. The vibration absorber consists of a mass m_a connected by a spring of stiffness k_a and material loss factor of η_a to the centre of the beam, i.e., at $l/2$. The system is modelled in the frequency domain using the dynamic stiffness method. The vector of normalised forces and moments is given by $\mathbf{f} = \{\hat{F}_L \;\; \hat{M}_L \;\; \hat{F}_I \;\; \hat{M}_I \;\; \hat{F}_R \;\; \hat{M}_R\}^{\mathrm{T}}$, where $\hat{F} = F/EI\beta^3$ and $\hat{M} = M/EI\beta^2$. It is related to the vector of displacements and rotations $\mathbf{w} = \{W_L \;\; \theta_L/\beta \;\; W_I \;\; \theta_I/\beta \;\; W_R \;\; \theta_R/\beta\}^{\mathrm{T}}$ by the

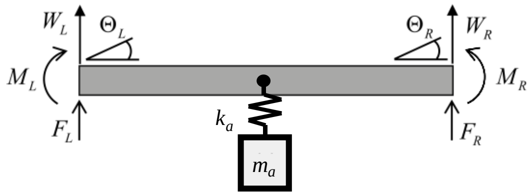

Figure 1 – Illustration of a beam with a vibration absorber attached to its centre.

dynamic stiffness matrix $\mathbf{D}$, such that (Gardonio and Brennan, 2004)

$$\mathbf{D}=\begin{bmatrix} -K_{11} & -P & K_{12} & V & 0 & 0 \\ -P & Q_{11} & -V & Q_{12} & 0 & 0 \\ K_{12} & -V & -2K_{11}+D_{\text{att}} & 0 & K_{12} & V \\ V & Q_{12} & 0 & 2Q_{11} & -V & Q_{12} \\ 0 & 0 & K_{12} & -V & -K_{11} & P \\ 0 & 0 & V & Q_{12} & P & Q_{11} \end{bmatrix}, \quad (1)$$

in which

$$K_{11}=[\cos(\beta l/2)\sinh(\beta l/2)+\sin(\beta l/2)\cosh(\beta l/2)]/R$$
$$K_{12}=[\sin(\beta l/2)+\sinh(\beta l/2)]/R$$
$$P=\sin(\beta l/2)\sinh(\beta l/2)/R$$
$$V=[\cos(\beta l/2)-\cosh(\beta l/2)]/R$$
$$Q_{11}=[\cos(\beta l/2)\sinh(\beta l/2)-\sin(\beta l/2)\cosh(\beta l/2)]/R$$
$$Q_{12}=\left[\sin\left(\beta l/2\right)-\sinh\left(\beta l/2\right)\right]/R$$
$$R=\cos\left(\beta l/2\right)\cosh\left(\beta l/2\right)-1$$

and $\beta=(\omega^2\rho A/EI)^{1/4}$ is the flexural wavenumber of the beam, ω is the angular frequency, ρ, E, A and I are, the density, the Young's modulus, the cross-sectional area and the second moment of area of the beam; the non-dimensional dynamic stiffness of the force vibration absorber is given by (Brennan, 1997)

$$D_{\text{att}}=\frac{\mu\beta l\left(1+j\eta_a\right)}{1+j\eta_a-\left(\beta l/\beta_a l\right)^4}, \quad (2)$$

where $\mu = m_a/\rho Al$ is the ratio between the mass of the vibration absorber and the mass of the beam, and β_a is the flexural wavenumber of the beam at the natural frequency of the vibration absorber, i.e., when $\omega=\omega_a=\sqrt{k_a/m_a}$. The displacement vector at each node, i.e., at each cell connection, is given by

$$\mathbf{w}=\mathbf{R}\mathbf{f}, \quad (3)$$

where $\mathbf{R} = \mathbf{D}^{-1}$ and the transverse and angular displacement transmissibilities are determined, respectively, by

$$T_W=\frac{W_R}{W_L} \text{ and } T_\theta=\frac{\theta_R}{\theta_L}, \quad (4a,b)$$

where $W_L = \mathbf{R}(1,1)$ and $W_R = \mathbf{R}(2,1)$ are the transverse and angular point receptances, respectively, and $\theta_L = \mathbf{R}(5,1)$ and $\theta_R = \mathbf{R}(6,1)$ are the transverse and angular transfer receptances, respectively, at the opposite side of the structure where the external force is applied.

SUPER ATTENUATION BAND FORMATION

In this section, the effect of correctly tuning the vibration absorber to improve the attenuation properties in a single beam is investigated numerically. In order to investigate the dynamic effect of attaching a vibration absorber to a single beam span, the transverse displacement transmissibility given by Eq. (4a), is calculated numerically as a function of l/λ, where λ is a flexural wavelength of the beam and l/λ_a, where λ_a is the flexural wavelength at the natural frequency of the vibration absorber, for a mass ratio of $\mu = 1$; l/λ is set to vary from 0.2 to 2 and l/λ_a is set to vary from 0.75 to 1.25, both with an increment of 0.01. The material loss factors η and η_a are set to 0. The top-view of the modulus of the transverse displacement transmissibility is plotted in Fig. 2. The frequencies limiting the attenuation bands in -3 dB as a function of the natural frequency of the vibration absorber are determined numerically and are plotted in Fig. 2 as a black solid line. Also plotted as a blue dashed line is the natural frequency of the vibration absorber and as a thin blue solid line is the tuned frequency, l/λ_t, which is flexural wavelength at the frequency where the interaction between the host structure and the vibration absorber creates an anti-resonance in the system transmissibility (Salleh and Brennan, 2007).

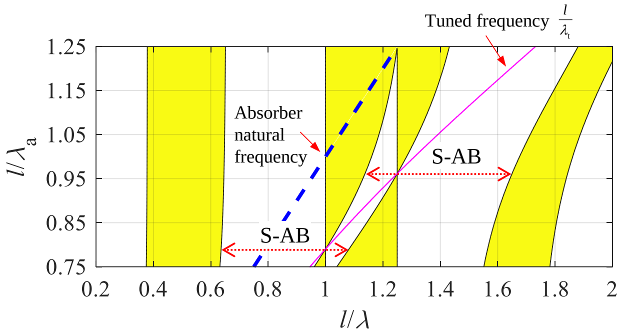

Figure 2 – Top view of the transverse displacement transmissibility for $\mu = 1$. The colour is for $|T_W| > -3$ dB. The black lines indicate the frequencies limiting attenuation bands in -3 dB. S-AB denotes super attenuation band.

It is seen in Fig. 2 that due to the dispersive characteristic of the beam and the dynamic interaction with the vibration absorber, the tuned frequency of the system occurs at a frequency higher than the natural frequency of the vibration absorber. Moreover, note that with the evolution of the vibration absorber natural frequency there are two tuning conditions at which the super attenuation band is potentially formed. These frequencies correspond to $l/\lambda_t = 1$ and $l/\lambda_t \approx 1.25$. The dynamics of the structure at these two tuned conditions are examined in Fig. 3(a) for $l/\lambda_t = 1$, and in Fig. 3(b) for $l/\lambda_t \approx 1.25$, which shows the transversal and angular displacement transmissibilities with their corresponding point and transfer receptances. Also plotted is the transmissibility for a bare beam (without vibration absorber).

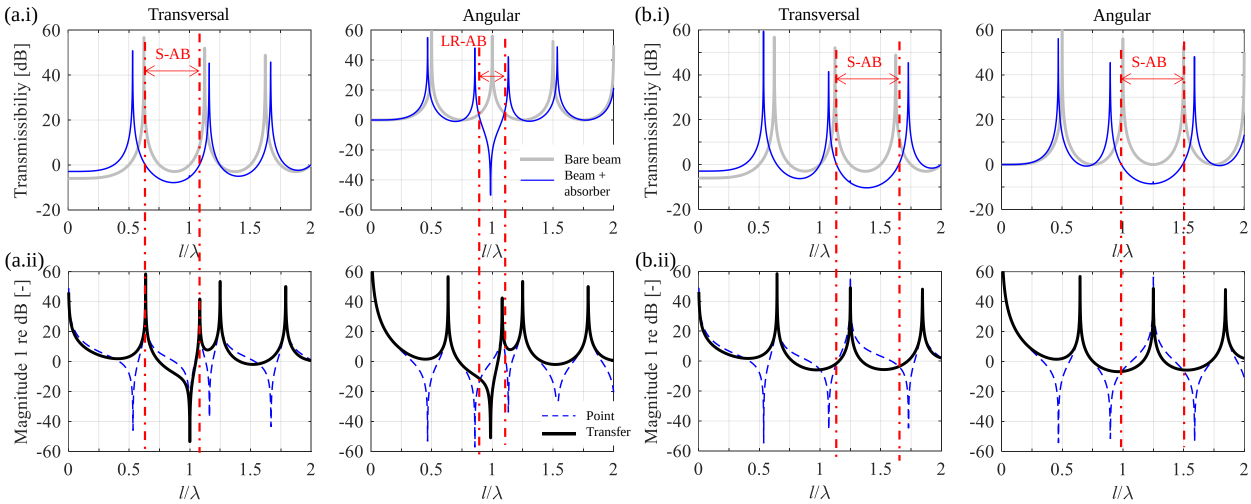

Figure 3 – Frequency response plots of the transversal displacement for $m = 1$ and the vibration absorber tuned to: (a) $l/\lambda_t = 1$; and (b) $l/\lambda_t \approx 1.25$. (.i) Displacement transmissibility; and (.ii) Point receptance and transfer receptance. LR-AB and S-AB denote the local resonance attenuation band and super attenuation band, respectively.

It is clear in Fig. 3 that for both conditions a smooth super attenuation band is formed, with a considerable improvement in their attenuation properties such as the bandwidth of the attenuation band and the magnitude of attenuation within the band. However, in each of the cases, their mechanisms of formation are different. When the vibration absorber is tuned so that $l/\lambda_t = 1$, the super attenuation band occurs in the transversal displacement transmissibility but not in the angular transmissibility, which presents only a local resonance attenuation band. In this condition, the point receptance and the transfer receptance of the transverse and angular displacements correspond to the anti-resonances of the structure, as seen in Fig. 3(a.ii). The vibration absorber forces the beam to be motionless at its ends so that the beam behaves analogously to pinning the left-end and to fixing the right-end of the beam. When the vibration absorber is tuned so that $l/\lambda_t \approx 1.25$, the super attenuation band occurs in both transverse and angular displacement transmissibilities. In this condition, the tuned frequency of the system corresponded to the free-free natural frequency of the beam and the vibration absorber is attached to a frequency in which the attachment point is motionless.

CONCLUSIONS

This paper has investigated the use of a linear mass-spring vibration absorber as a passive vibration control measure to improve the attenuation band properties in a beam structure. Using the dynamic stiffness method, the behaviour of the structure in terms of the transversal and angular displacement transmissibilities have been evaluated by changing the natural frequency of the vibration absorber. Two tuned conditions in which a super attenuation band is formed have been determined. In one condition, the system is tuned to the anti-resonance frequency of the structure resulting in a super attenuation band in the transversal displacement transmissibility and a local resonance attenuation band in the angular displacement transmissibility. In the second condition, the system is tuned to the free-free natural frequency of the beam and the vibration absorber is attached to a nodal point of the beam, for that specific frequency, resulting in a super attenuation band in both transverse and angular displacement transmissibilities. In both conditions, the correct tuning creates a smooth super attenuation band capable of reducing the transverse vibration of the beam over a broad frequency range.

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