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Time-Delayed Feedback Control with Adaptive Gain for Impact Oscillator

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Abstract: Tuning nonlinear controllers parameters is one of the most challenging and crucial steps in their design, and time-delayed feedback controllers serve as a prominent example of such systems. One of the most promising recent advancements is the adaptive gain time-delayed feedback control that eliminates the need to set control gains and makes it robust to slow changes of the system parameters. However, the properties of this controller are not yet fully understood, especially with respect to the gains convergence. This work investigates the local properties of the adaptive time-delayed feedback control applied to an impact oscillator. the stability analysis based on the Floquet theory provides some insights into the convergence of the adaptive gains and how it can affect the stability of the targeted orbits.

Keywords: Impact oscillator, time-delayed systems, nonlinear control

INTRODUCTION

Identification of control properties such as the robustness to noise and how local stability is changed with control gains, in particular when related to delayed feedback methods, has been considered a crucial step in understanding these methods' limitations and setting control parameters. This knowledge is important for controllers in applications such as in energy harvesting, smart structures, high power lasers, cardiac rhythms, atomic force microscopes, amongst others.

The time-delayed feedback method (TDF) proposed by Pyragas (1991) and its variations have been applied in engineering systems e.g. (De Paula *et al.*, 2017). Various aspects were investigated including the global stability of the controlled orbit (Höhne *et al.*, 2007), analysis and optimization of the control parameters by Floquet theory (Pyragas, 2002) and limitations for orbits with an odd number of positive Floquet exponents (Nakajima and Ueda, 1998). These studies ultimately lead to improvements of the original TDF method. For example, several modifications were proposed to avoid the odd-number limitation as in the unstable time-delayed feedback control (UTDF) (Pyragas and Novičenko, 2013) and the periodic gain time-delayed feedback control (Leonov, 2014).

At the beginning of the last decade, there has been a focus on the adaptability of the controller parameters such as the time delay itself (Pyrgas and Pyragas, 2011) or the feedback gains (Lehnert *et al.*, 2011), aiming to reduce the knowledge required to stabilize an orbit and also improve robustness to slow changes in the system parameters. However, effects of these modifications on TDF have not been tested, especially concerning the robustness to noise as the introduction of parameter dynamics can potentially increase the sensitivity of the system.

This work investigates the adaptive gain time-delayed feedback control method (ATDF) proposed by Lehnert *et al.* (2011) by analysing how the adaptation of the feedback gain modifies the Floquet exponents of the target orbit. The controlled system is a piecewise linear impact oscillator for which an unstable period-1 orbit (UPO) embedded in a chaotic attractor is targeted. We also compare our results to the ones obtained by the classical TDF method to develop a better understanding of how the controller locally stabilizes the target orbit. Results indicate that ATDF converges to different gains depending on the initial conditions which can potentially affect its robustness to noise.

IMPACT OSCILLATOR MODEL

The impact oscillator model investigated in this paper is based on the experimental impact oscillator rig developed by Wiercigroch *et al.* (2020). It is composed of a leaf spring, with elastic stiffness k_1 and an equivalent damping coefficient c connected to a lumped mass m. A secondary support holds an impact beam with elastic stiffness k_2 at a gap g from the lumped mass, as shown in Figure 1(a). A magnet-coil coupling is used to directly excite the mass and also is used to provide any actuation to the system. The evolution of the system dynamics can be followed by solving its equation of motion which can be represented as a set of the first order ODEs,

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{X} \\ -\frac{k_1}{m}x - \frac{k_2}{m}(X-g)H(X-g) - \frac{c}{m}\dot{X} + \frac{F_{\text{coil}}}{m} \end{bmatrix}, \tag{1}$$

where X is the mass displacement, $\mathbf{x}$ is the state vector, and dot represents derivative with respect to time.

The force applied to the mass by the coil is considered to be linearly proportional to the current I running through it.

Thus F_{coil} can be written as:

$$F_{\text{coil}} = a(I_0 \cos(\omega t) + u), \tag{2}$$

where u is the control signal, a is a mechanical-electrical constant determined experimentally, I_0 is the amplitude of the excitation current and ω is the frequency of excitation. Fig. 1(b) presents a comparison between the predicted (black) and experimentally measured (red) coil forces, while trajectory of the calibrated impact oscillator model (in black) is compared with experimentally recorded one (in red) in Fig. 1(c). As it can be seen there is an excellent match between the modelling and experiments.

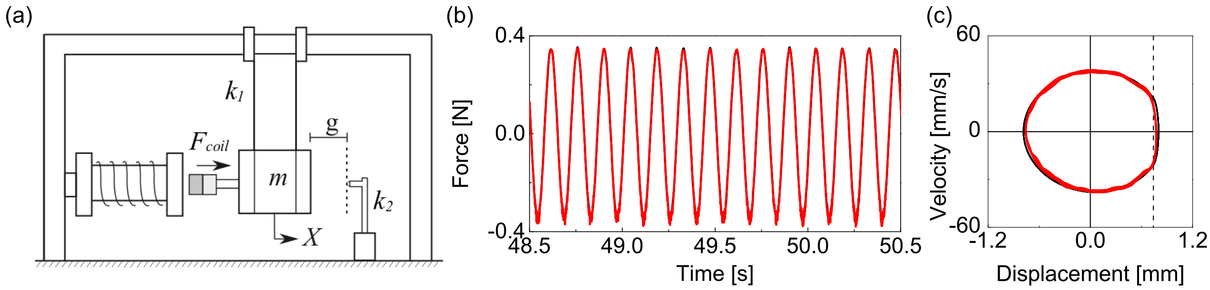

Figure 1 – (a) Schematic of the impact oscillator model; (b) measured (black) and predicted (red) force with a free vibrating mass at 7 Hz; (c) numerical (black) and experimental (red) phase portraits for a period-1 response showing an excellent agreement.

ADAPTIVE GAIN TIME-DELAYED FEEDBACK CONTROL

Typical equations for a dynamical system with TDF control are:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, t) + \mathbf{u}(\mathbf{y}(t), \mathbf{y}(t-\tau)), \\ \mathbf{y}(t) &= \mathbf{C}(\mathbf{x}), \end{aligned} \tag{3}$$

where $\mathbf{x}$ is the internal state vector, $\mathbf{y}$ is the output vector obtained using an observation function $\mathbf{C}$, and $\mathbf{u} = \mathbf{K}(\mathbf{y}(t-\tau) - \mathbf{y}(t))$ is the TDF control signal, τ is a delay and $\mathbf{K}$ is a gain matrix.

The idea behind ATDF is adapt the gain matrix $\mathbf{K}$ by minimizing the cost function G formulated based on the control signal. The entire argument hinges on the fact that TDF's control signal tends to zero if the target orbit is stabilized, thus, the minimization of the cost function $G > 0$ of the control signal can also generate gains that would stabilize such orbit.

Hence, let the cost function G be defined as:

$$G = \frac{1}{2}(\mathbf{y}(t) - \mathbf{y}(t-\tau)) \cdot (\mathbf{y}(t) - \mathbf{y}(t-\tau)) \tag{4}$$

The speed-gradient method can be applied to minimize the cost function in relation to the matrix gains, which defines the derivative of $\mathbf{K}$ in respect to time as:

$$\dot{\mathbf{K}} = -\gamma \nabla_{\mathbf{K}} \dot{G} \tag{5}$$

where γ is a convergence parameter that defines how quickly $\mathbf{K}$ varies with time, and $\nabla_{\mathbf{K}}$ indicates the gradient in respect to $\mathbf{K}$.

For the sake of simplicity and without loss of generality it is assumed that $\mathbf{y} = \mathbf{x}$ and only one coordinate x_i is directly actuated by the control. In this situation, by considering the TDF control signal, $u = \kappa(x_i(t-\tau) - x_i(t))$, and using Eq. 5, the evolution for the gain κ can be calculated:

$$\dot{\kappa} = \gamma(x_i(t) - x_i(t-\tau))(x_i(t) - 2x_i(t-\tau) - x_i(t-2\tau)) \tag{6}$$

Finally, we can describe the complete dynamical system with control as:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\kappa} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{x}) + \mathbf{u}(\mathbf{y}(t), \mathbf{y}(t-\tau)) \\ \gamma(x_i(t) - x_i(t-\tau))(x_i(t) - 2x_i(t-\tau) - x_i(t-2\tau)) \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 0 \\ \vdots \\ \kappa(x_i((t-\tau) - x_i(t))) \\ \vdots \\ 0 \end{bmatrix}. \tag{7}$$

SIMULATION OF IMPACT OSCILLATOR WITH ATDF CONTROL

This section aims to investigate the ATDF method applied to an impact oscillator. The main aim is to evaluate the Floquet exponents at different values of κ for a target period-1 UPO and then correlate them with the adapted control gains through time. This enables us to analyse how the evolution of gains can help to reach the target orbit's stability, which reveals insights into the ATDF convergence.

A fourth-order Runge Kutta method is used to numerically integrate the system described by Eq. 7 ensuring adequate for error sensitive solutions. The system presents a chaotic response at an excitation frequency of $\omega = 8.18$ Hz with parameters listed in Tab. 1. We target a period-1 UPO embedded in the chaotic attractor and analyse how the gain κ and the stability of the controlled orbit evolve. The local stability of the period-1 UPO is obtained by the calculation of its maximum Floquet exponent real part, $\mathrm{Re}(\mu_{\max})$, as described in (Costa *et al.*, 2019). In short, if $\mathrm{Re}(\mu_{\max}) > 0$, the orbit is unstable and the controller cannot stabilize the system, but if $\mathrm{Re}(\mu_{\max}) \leq 0$, the orbit is stable. Also, lower values of $\mathrm{Re}(\mu_{\max})$ indicate a greater stability and faster convergence to the target orbit.

Table 1 – Model and control parameters utilized in simulations.

Symbol	Value	Unit	Symbol	Value	Unit
m	1.325	kg	k_1	4331	N/m
k_2	87125	N/m	c	0.27	kg/s^2
g	$0.74\ 10^{-3}$	m	a	0.799	N/A
I_0	1.45	A	γ	1	-

We apply the control after the uncontrolled is simulated for at least two excitation periods so the required delayed states are generated before the controller is turned on. Results are shown in Fig. 2. As depicted, the applied control successfully stabilizes the target orbit, which is shown in Fig. 2(f), in three different stages. During the first stage, the gain κ slowly grows, as shown in Fig. 2(b), up to a critical point where a period-2 UPO, shown in Fig. 2(g), is stabilized. In this first stage (10 s to 30 s), the control signal, shown in Fig. 2(c), and the cost function Q, shown in Fig. 2(e), do not present a defined envelope. After the period-2 response is reached, the gain grows almost linearly. During this second stage (30 s to 50 s), the control signal presents an envelope as the gain grows with time and the system response does not change from the period-2 orbit. The evolution of the target period-1 orbit Floquet exponents, shown in Fig. 2(d), reveals that the target orbit is unstable in the first two stages of the control process. Finally, when control gains reach a critical point very close to the threshold of stability of the target orbit, a third stage is initiated where the system slowly moves away from the period-2 response and converges to desired periodic orbit, depicted in red. During this last stage, the control gains stabilize to a value κ_∞, shown by a magenta line, which can stabilize the target orbit. The control signal and cost functions also tend to zero as the stabilization of the target orbit is reached. By analysing the final Floquet exponents of the orbit, we can confirm that as the evolution of κ is slow when compared to the system dynamics, the gain stabilizes close to the threshold of stability ($\mathrm{Re}(\mu_{\max}) = 0$).

The test performed indicates that ATDF can successfully set the control gains and stabilize the target orbit. For small values of γ, where the evolution of the gain κ is much slower when compared to the evolution of the system, the control parameters converge to values near the threshold of stability. This is disadvantageous if noise is considered or the system is subjected to fast changes in its parameters, as the target orbit stability can easily change in these conditions by small variations of $\mathrm{Re}(\mu_{\max})$.

It is important to mention that higher values of γ make the control gain converge to higher values, farther from the threshold of stability, which improves the control robustness to noise as observed in (Lehnert *et al.*, 2011). However, this technique is not reliable as there is no procedure to set γ and it would defeat the purpose of an adaptive control that eliminates the requirement of setting control parameters. Finally, if the Floquet exponents considering the gain κ as part of the state space are calculated, the Floquet exponent related to κ presents a zero real value, indicating that there is no preferable value of convergence κ_∞ as long as it is in the region that stabilizes the target orbit. This also indicates that, depending on initial conditions, different values of κ can be reached for the same value of γ.

CONCLUSION

This work analyses the ATDF method for an impact oscillator by evaluating how the gain of the controller and the stability of the target orbit evolve over time. It was observed that for a slow dynamic of the gains, the control manages to stabilize a target period-1 orbit near the threshold of stability. It was also noted that, for fast dynamics of gains, κ_∞ may fall inside the region of stability, but the value of κ_∞ is not reliable and depends on initial conditions even when considering the same value of γ. Thus, if there is a requirement for greater stability of the orbit due to rapid changes in the system parameters, or noise, a different control strategy or a modification of ATDF will be required.

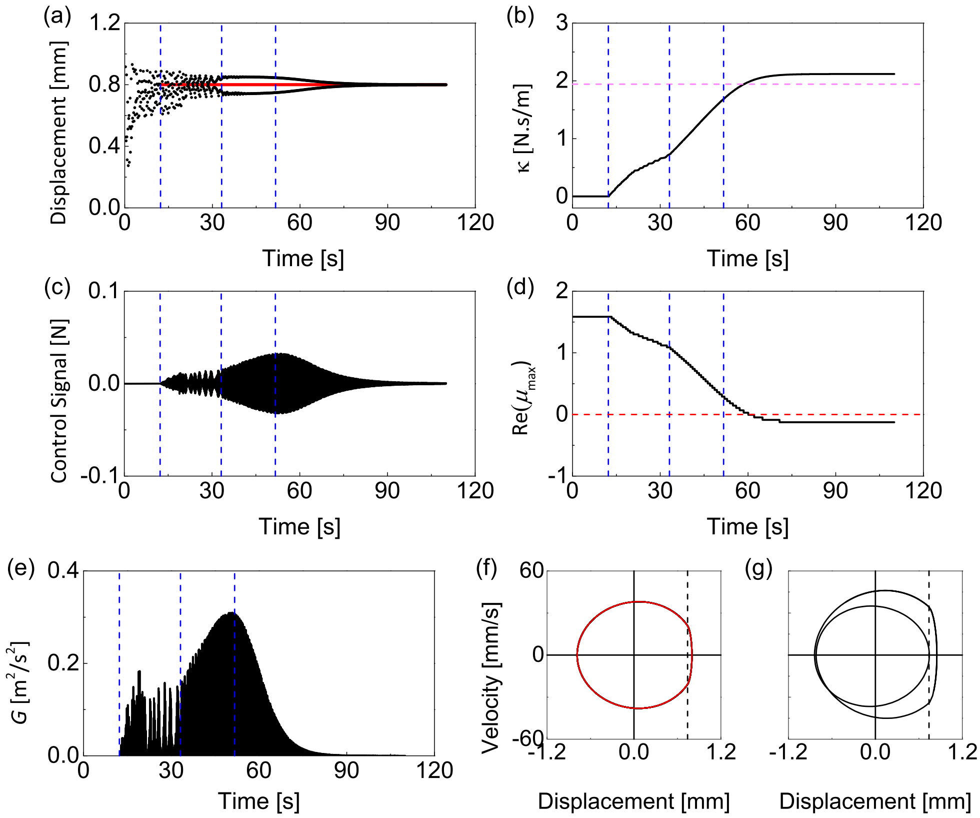

Figure 2 – Adaptive control of the period-1 UPO for small convergence parameter γ; (a) stroboscopic time history of system displacement: the target orbit is marked in red while the system response is marked in black; (b) time history of the adaptive gain κ. The converged value of gains κ_∞ is depicted by a horizontal dashed magenta line; (c) time history of control signal; (d) correspondent Floquet exponent for each value of κ through time. Horizontal red dashed line indicates the stability limit; (e) value of the cost function G through time; (f) phase plane of the target (red) and stabilized (black) orbits. (g) Period-2 response generated by the system prior to stabilization of the period-1 UPO. The critical points of control stages are depicted in dashed vertical blue line and dashed vertical lines in phase portraits indicate the impact boundary.

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