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Sensitivity Analysis of Discrete Wavelet (DWT) Based Damage Localization on a Beam-Like Structure

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Abstract: A numerical campaign is conducted on a beam-like structure to investigate damage localization and the influence of boundary conditions on the efficacy of the method. The objective is to define a unimodal function near the damaged region, to assess the potential for the solution of the damage localization inverse problem. Discrete wavelet transform (DWT) data treatment is performed onto displacement data obtained in beams submitted to transverse static loads applied at selected points over its length. Three boundary condition cases are investigated: Simply supported, cantilever, and double-clamped. The damage localization goal is to determine a region of probable damage. Investigating diverse beam aim at understanding how damage localization is impacted by the boundary conditions of each case. Additionally, the influence of decomposition level, wavelet type, and damage severity, were also studied. The simulated damage is applied to the structure via local reduction of the elasticity module of one element. The method then treats displacement data and reveals the probable region with damage with good precision, The successful preliminary results show good potential for the proposed DWT-based damage localization method.

Keywords: Damage Localization, Structural Health Monitoring, Wavelet Transform.

INTRODUCTION

Structural health issues are a great concern for most of the structures in critical applications, from small to larger scales. A beam is naturally induced by many forces and tensions. This type of structure is the focus of the investigation by many authors such as Yang and Huang (2021) and Nick et al. (2021) since it is found in several engineering applications ranging from light structures in aerospace systems to heavy and robust structures in civil construction, navy ports, and many others.

A beam-like structural element is designed to operate under certain service and environmental constraints that may induce its degradation. Once its performance fails to meet the design requirements, either by natural wear or unpredicted stresses, it usually indicates that damage might have developed on the structure. Even apparently insignificant damage can cause great trouble if not predicted or quickly repaired.

Engineers need to design well-calculated parts but need to develop even greater methods to monitor their projects after birth. From this need comes the Structural Health Monitoring concept, Farrar (2007) defines it as a method to observe and monitor a certain structure, obtain readings from it through whatever sensors used, and by these readings, detect, localize, and identify the damage, so a solution can be done before trouble comes in. A review under this idea in made by Sohn et al. (2003).

To observe a signal, treat and read it, a recently developed technique is of great use: wavelet transform. This transform is like the Short-Time Fourier Transform (STFT) but without the loss of time information. Wavelets are composed of a family of basic functions that can describe a signal in a localized time (or space) and frequency (or scale) domain. Important authors on this field are Chui (1997), Daubechies (1992), Mallat (1999) and Misiti (1996).

In other words, Wavelet Transform analysis is based on the idea that any signal can be divided into a series of base functions called “waves’’. This signal treatment is of great use for Probability of Detection (P.O.D.) problems when damage needs to be detected. Authors such as Katunin (2021) and Zhou and Chen (2018) use this approach to beam structures. Wavelets have two main branches: continuous (CWT) and discrete (DWT); this study is based solely on the latter type.

This paper presents numerical results for the sensitivity analysis of a DWT-based damage detection method, which will hopefully lead to the definition of a unimodal function near the damage region, which value decreases as it moves away. The idea is to define, through a test of hypothesis, a method to find a region of the beam which is most likely to be damaged. In that sense, the tools need to “see” damage on three levels: detection, localization, and identification., and each step will involve an increased degree of complexity. This work focuses on the first two steps of the process. Thus, a numerical campaign conducted on a beam-like structure, modeled by the Finite Element Method (FEM), aims to investigate damage detection and localization and the influence of the boundary conditions on the results. For all the analyses presented, the damage is included in the numerical model by means of a controlled reduction of the elasticity module of one element. Discrete wavelet transform (DWT) data treatment is performed onto displacement data obtained from beams submitted to transverse static loads applied at selected points over its length. Three boundary condition cases are investigated: simply supported, cantilever, and double-clamped. aiming at understanding how damage localization is impacted by the boundary conditions of each case. Additionally, the influence of decomposition level, wavelet type, and damage severity, were also studied.

Model Description

A uniform, isotropic beam is numerically modeled using the finite element routines available in the Matlab® Vibration Toolbox. At this stage of the research, only static responses are used for the preliminary investigations of the method's potential and limitations.

The element mass ($\boldsymbol{M_e}$) and stiffness ($\boldsymbol{K_e}$) matrices used in the FE models of the beam are given by:

$$M_e = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad (1)$$

$$K_e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (2)$$

Where ρ is the mass density of the material, $\boldsymbol{A}$ is the cross-section area, $\boldsymbol{l}$ is the element length, $\boldsymbol{I}$ is the cross-section moment of inertia and $\boldsymbol{E}$ is the elastic (or Young's) module of the material. Matrices in Equations (1) and (2) correspond to a two-node element with two degrees of freedom per node, namely, the transverse displacement and the in-plane rotation.

The beam is discretized using a regular mesh of 100 elements. Different sets of boundary conditions are imposed in order to investigate the method´s ability to identify damage in positions close to physical joints and connections in more complex structures composed of beam-like parts, such as trusses, periodic arrangements, etc. Transverse static loads are applied at different locations on the beam to evaluate possible effects of force location relative to the damaged sites.

Discrete Wavelet Transform

Discrete Wavelet Transform enables us to compute a set of coefficients connected to either lower or higher frequencies from an original signal "S". The frequencies (or scales) associated to the coefficients are computed by passing the original signal through a low-pass filter, h, and high-pass filter, *g*, (LPF and HPF). The LPF (*h*) gives a set of coefficients called approximation, "A", while the details, set of coefficients which contains the higher frequencies of the signal, are so-called details, "d" and give rise from the HPF (*g*). The process of filtering downsamples the original signal "S" whenever a new filtering step is performed in the approximation coefficients. The process can be repeated by providing different levels of decomposition, "J". For each J level of decomposition, J+1 subbands of frequencies will be provided. For instance, for J = 3, the signal "S" will be split into A3, D3, D2, and D1 levels for frequency. A sketch of the DWT decomposition, at J=2, is shown in Fig. 1.

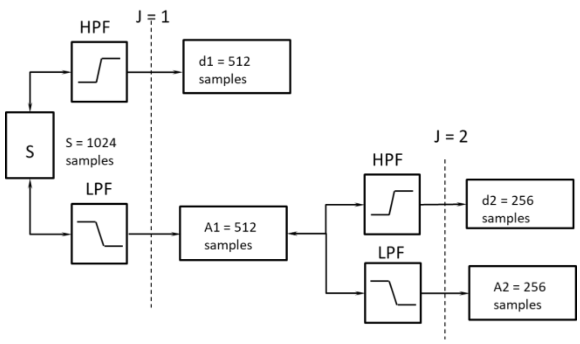


Figure 1 – Sketch of DWT transform.

The filters *h* and g, come from the wavelet mother, whose functions are carefully chosen, taking into account some necessary features such as compactness and orthogonality. Each wavelet mother possesses its very own low and high decomposition filters coefficients, *h* and *g*, respectively, and the approximation, “A” and the details, “d”, coefficients arise from the convolution of the signal “S” and the filters, as follow in Equations (3) and (4).

$$A_{[n]} = \sum_{k=0}^{k} S_{[n]} \cdot h_{[k-n]} \tag{3}$$

$$d_{[n]} = \sum_{k=0}^{k} S_{[n]} \cdot g_{[k-n]} \tag{4}$$

It is important to mention that this paper only deals with static problems and the details, *d*, from the first level of DWT decomposition were considered for the results. During the very first stage of this research J=1, 2, and 3, were observed. For J >1 the results were very scattered, leading the authors to adopt J = 1 for any analysis.

Boxplot Statistical Index

This work was aimed to try to localize the damage, previously imposed to a certain part of the beam, through the Discrete Wavelet Transform – DWT applied to the beam deflection submitted to a certain kind of load and supports. All the wavelet approximation coefficients of DWT were generated at J=1 using as the wavelet mother SYM4. According to Katunin (2010) and (2011), DWT is more suitable to observe damages in 1D and 2D geometries thanks to its good sensitivity and better computational efficiency in comparison with other transforms. To perform such an analysis, we first found the details vector of DWT coefficients for a non-damaged beam, producing a d_0-vector. In sequence, the same steps were followed for the damaged beam, yielding also a d_d-vector with the same length of d_0.

To perform the final evaluation a thirty vector, d^2 is then created, by subtracting d_d, and d_0. To eliminate negative values of this vector, the resulting vector was then squared and normalized by its maximum value, as follows in Equation (5).

$$d^2_{index} = \frac{(d_d - d_0)^2}{\max\left[(d_d - d_0)^2\right]} \tag{5}$$

The vector is then plotted along the beam. The idea behind this is to localize the damage through the higher coefficients at the vicinity of the damage.

At the first moment, the coefficients were a bit scattered along the beam, turning the damage location into a difficult task. To face such a problem a box plot toll was invoked. The box considers an interval of points centered to a given point (node) along the beam. To show the d^2-index on point 3, for instance, the values of the index from points 1,2, 4, and 5, are computed. So, the statistical values of that set of points *i.e* average mean, median, standard deviation, and the outliers data point are stored in the box associated with position 2. In short, a boxplot enables us to see these signals through statistical eyes, in such a way we can define precise indexes for a probable damage region, Velleman (1981).

Figure 2 shows a boxplot representation and the information contained on it. On each box, the central mark indicates the median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. Besides, the maximum and minimum values of the coefficients set are shown. The outlier’s data points, not included in the computations, are also identified through the markers `'+'`.

Therefore, the variability of the values is indicated by the height of the box, along with other statistical data. This tool brings up several types of possible indicators which might be adequate to damage localization.

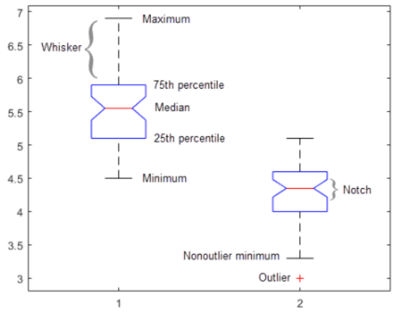


Figure 2 – Boxplot statistical representations.

Five possible indicators were studied: (1) median; (2) standard deviation; (3) 1º quartile; (4) 3º quartile; (5) box height, which is the difference between the 3º and 1º quartiles. All indicators tested were able to detect the damage. The median (1) and the box height (5) played an important role to localize the damage. In fact, they were seen as strong enough, since both, tested separately, were able to indicate the damage at the same location. Figure 3 shows damage localization for a 1000 mm long simply supported beam discretized with 101 nodes and 100 elements. The damage was simulated through the 50% of reduction in the elasticity module at the element 25. The concentered load "F" was applied to node 25.

(a) (b)

Figure 3 – Damage localization for a simply supported beam. (a) d²-index: median. (b) d²-index: box height.

From Fig. 3 (A-b) the reader can straightforwardly see that both indexes (median and the height of box plot) were able to identify the jump of the DWT coefficients in the damaged neighborhood. However, it does not matter which index was applied, the coefficients nearby the supports were seen to bulge, underlining the support effects. Further, the support effects will be treated in this paper.

DWT Transform for different parameters and beam conditions

So far, we presented the technique to be used to investigate the damage localization through the wavelet transform using a certain kind of mother wavelet. However, the investigation on the effects of the wavelet mother, type of the beam supports and the severity of the damage were not taken into account. So, the effectiveness of the d^2-index in identifying the damage was tested for a simply supported beam facing: (a) the load location; (b) wavelet type; (c) damage severity; and (d) contour conditions.

All analyses, throughout this paper, are done based on the beam displacement.

Initially, for load location, Figure 4 shows the non-damage beam displacement for different positions of the load. The load was applied to nodes 25 and 51, 75, which contain, in total, 101 nodes and 1 meter long.

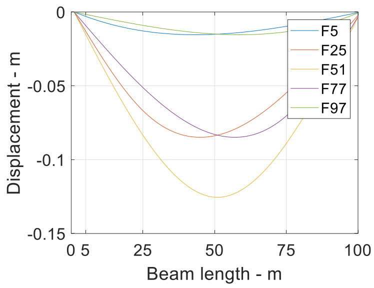
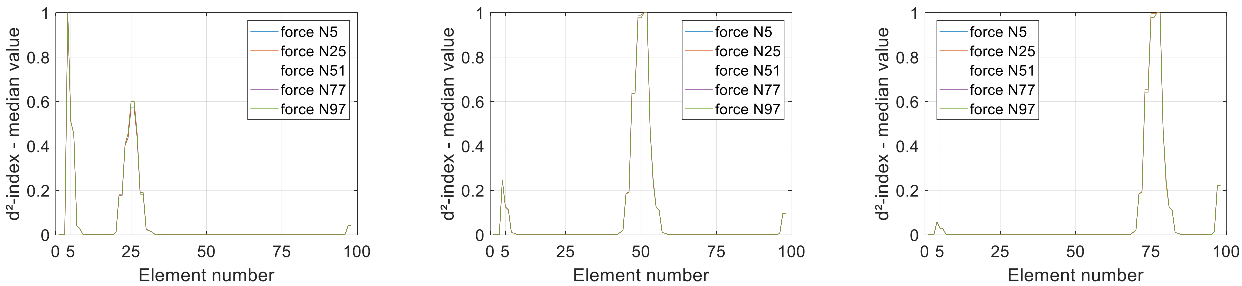
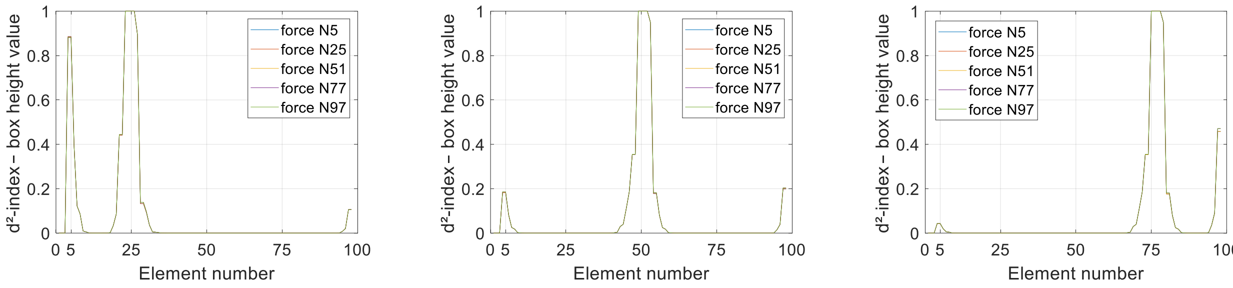


Figure 4 – Beam displacement for the same load applied to different nodes.

The damage was produced by reducing the elasticity module of one beam element (about 10 mm length) by 50% of the regular value (E = 200 GPa). Analyses were done for damaged elements: 25, 50 and 76. Figure 5 (a-f) shows the d^2-index as a function of the beam length.



(a) (b) (c)

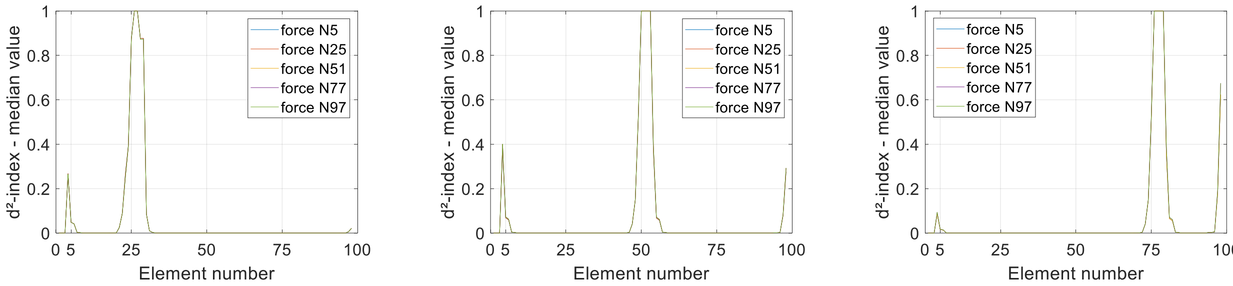


(d) (e) (f)

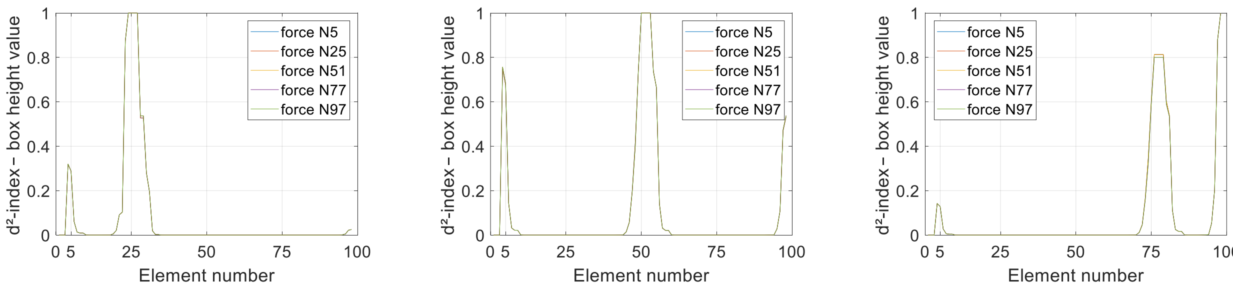
Figure 5 – *d²*-index for the simply supported beam for different load application points using SYM8. (a-c) *d²*-index taking into account the median values. (d-f) *d²*-index taking into account the box height.

From the set of Figure 5 (a-f) one can see that the damage was successfully localized by both, the median and the box height. Further, the point where the concentrated load was applied did not cause any effect in terms of the location of the damage.

In a sequence, this time for Wavelet Types, a new type of wavelet mother was tested, facing the same boundary conditions, load, and damage location. This time “DB5” wavelet mother was employed to yield DWT- details coefficients at J=1. Figure 6 (a-f) shows the same information on the damage, but instead, using “DB5” as wavelet mother. From these figures (Fig. 6 a-f), the reader can see the same information basically. The index created through the DWT-details coefficients was able to localize the damage.



(a) (b) (c)



(d) (e) (f)

Figure 6 – *d²*-index for the simply supported beam for different load application points using DB5. (a-c) *d²*-index considering the median values. (d-f) *d²*-index considering the box height.

Despite the good localization of the damage, the point that deserves attention is nearby the supports. It is noteworthy that the DWT- details coefficients seem to be affected by the presence of the beam supports for both wavelet mothers. However, it is worthy to mention that the coefficients “jump” at the supports seems to decrease in case of DB5 is applied. Even so, due to the nature of the transform at the supports, it is hard to identify any damage nearby them. So, in the sequence of this paper, only SYM8 will be used for the *DWT* transform.

For Damage Severity, the set of figures in Fig. 7 (a-d) shows the d^2-index for a different level of damage severity and point of load application for a simply supported beam. The damaged element, *e*, was kept the same, e = 25, while the damage severity ranged from 10% to 50% of the elasticity module. The reader can straightforwardly see that the *DWT* was able to localize the damaged element, using either the median of d^2-index or the height of the boxplot (meaning the 25th and 75th percentiles). Peaks of d^2-index coefficient were seen at e = 25. Further, it is worth noting that curves of the d^2-index distribution were completely overlapped, no matter the severity of the damage (10% up to 50% of the elasticity module). However, again, the border condition plays an important role in the d^2-index distribution. At the first support the coefficient jumps, most likely, due to the rotation that this kind of support enables.

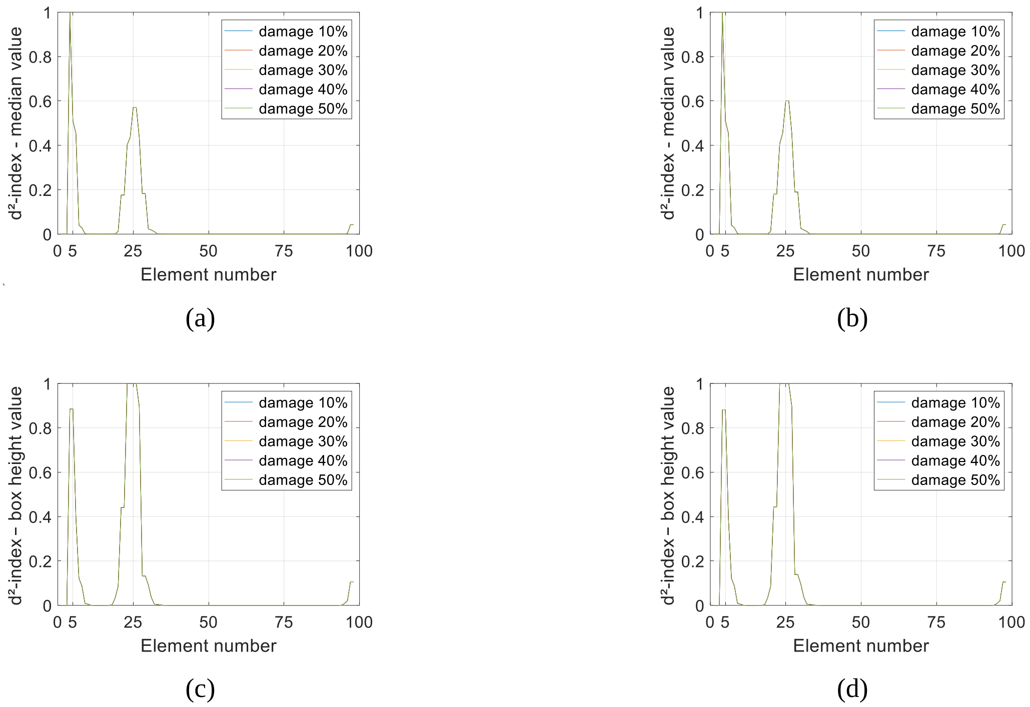


Figure 7 – *d²*-index distribution for the simply supported beam at different damage severity (10% up to 50%). (a-b) Median values of *d²*-index for load application at nodes 25 and 77. (c-d) Box height values of *d²*-index for load application at nodes 25 and 77.

Further, we investigated damage localization for for a beam under different kinds of supports. A simply supported beam has already been studied facing different wavelet mother, damage severity, and load application.

We had been speculating that the rotation at the support might be the cause of the d^2-index coefficient jump at the support. Now two different cases of degrees of freedom at the support will be studied. At first, the cantilever beam is studied using the same wavelet mother and level studied before. To promote a thorough understanding of the problem we also applied the same load at different nodes, at once. In Fig. 8, the deflection of an undamaged cantilever beam is shown. Curves are displayed according to the point where the load is applied. For instance, curve named F25 is addressed to the load applied at node 25. Despite the different points of application, the load was kept constant.

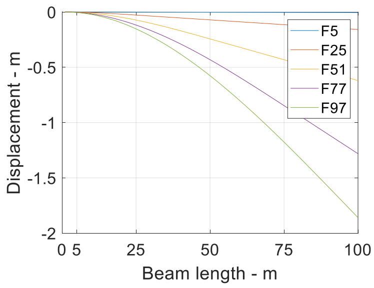


Figure 8 – Cantilever beam displacement for different *locus* of the load application.

The deflection of a cantilever beam subject to a point transverse load show two parts along its length with very distinct characteristics. The stretch from the fixed edge to the load application point deflects in accordance with the resulting bending moment, while the stretch from the load application point to the free edge has zero shear force and bending moment, resulting in an undeformed pattern. Therefore, it is expected that damage in this second stretch would not cause any differences in the total deflection curve of the damaged beam when compared to the undamaged one. Hence, only when the load is applied to the tip of the cantilever beam all cases of damaged regions can be investigated. For all cases of load application, we only simulated damage in the part of the beam subjected to non-zero bending moment and shear force.

For the cantilever beam, damage localization was performed using the same d^2-index from the DWT wavelet transformed from the damaged and the undamaged beams, eq. 8. As for the simply supported beam, SYM8 at j=1 level of decomposition produced the clearest readings. Figure 9 (a-e) shows the median values of d^2-index distribution throughout the beam. For these results, the damage severity was kept constant at 10 % of the elasticity module, at elements e = 5, 25, 50, 76, and 96. For each damage value, the same load was applied at nodes 5, 25, 51, 77, and 97 at once.

The damage was successfully localized by the *DTW*, through the d^2-index does not matter where the load was applied. Damages in the vicinity of the clamping are very well detected, as in the case of Fig. 9 (a), whose damaged element was the fifth one. Recalling the reader, for a simply supported beam we could not localize damaged elements nearby the supports.

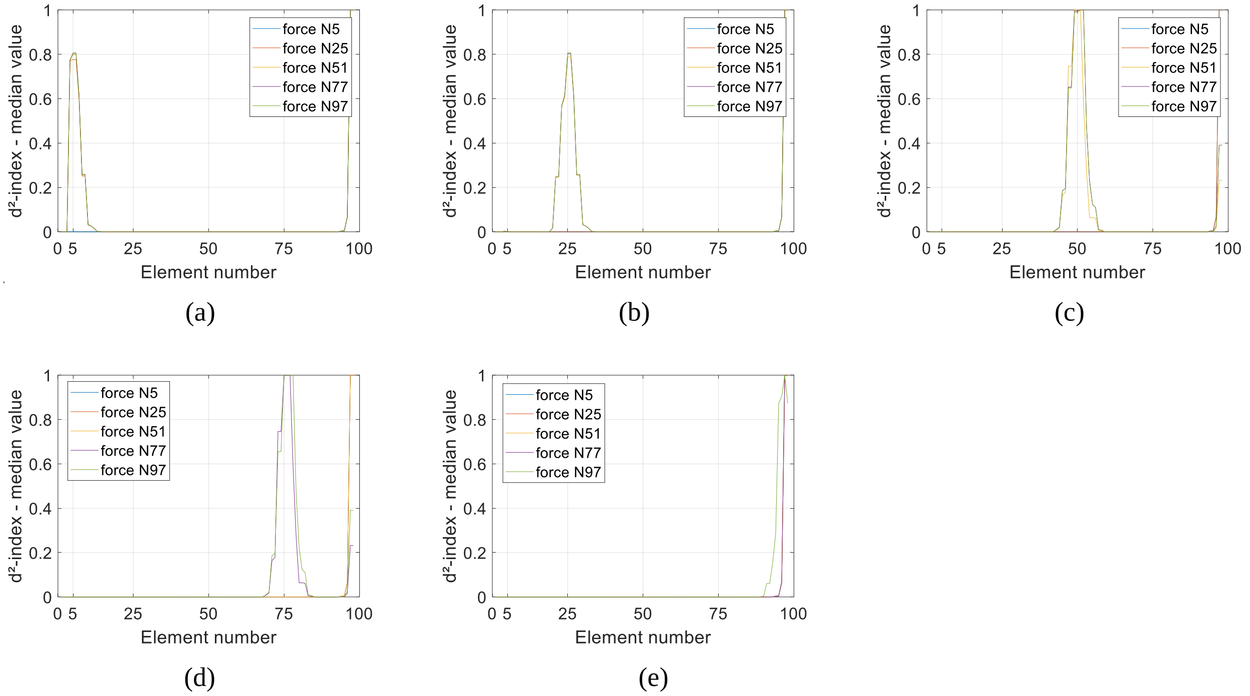


Figure 9 – d^2-index distribution in a cantilever beam for fixed damage and different nodes load application a-e) damage at e=5, 25, 50, 76, and 96, respectively.

However, it is worth noting the higher coefficients of d^2-index nearby the free border of the beam Fig. 9 (a-e). Again, the rotation at the free border causes higher coefficients in its vicinity, affecting the damage localization at this zone. It can be seen that, in the case of Fig. 9 (e), in which the damaged element is the 97^{th} one, it was not possible to localize through the *DWT* transform.

Meanwhile, the double clamped beam has a deflection pattern similar to the simply supported case, but now with zero rotation at the supports (rot_{xy} = 0 at x =0 and x= L). Mathematically, this corresponds to a continuous and smooth function with null first derivatives at the boundary points of its domain. Figure 10 illustrates the deflection curves for an undamaged double-clamped beam for different load application points.

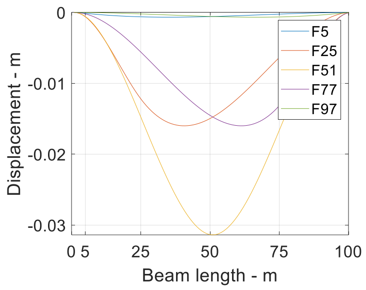


Figure 10 – Double clamped beam displacement for distinct load application node.

As in the previous cases, DWT transform and d^2-index were tested to localize damaged elements throughout the clamped beam. The 10% of elasticity module damaged elements were placed very near the supports (e=5, 25, 50, 76 and 96). Figure 11 (a-e) show the median of d^2-index distribution for different points of load application and damaged element.

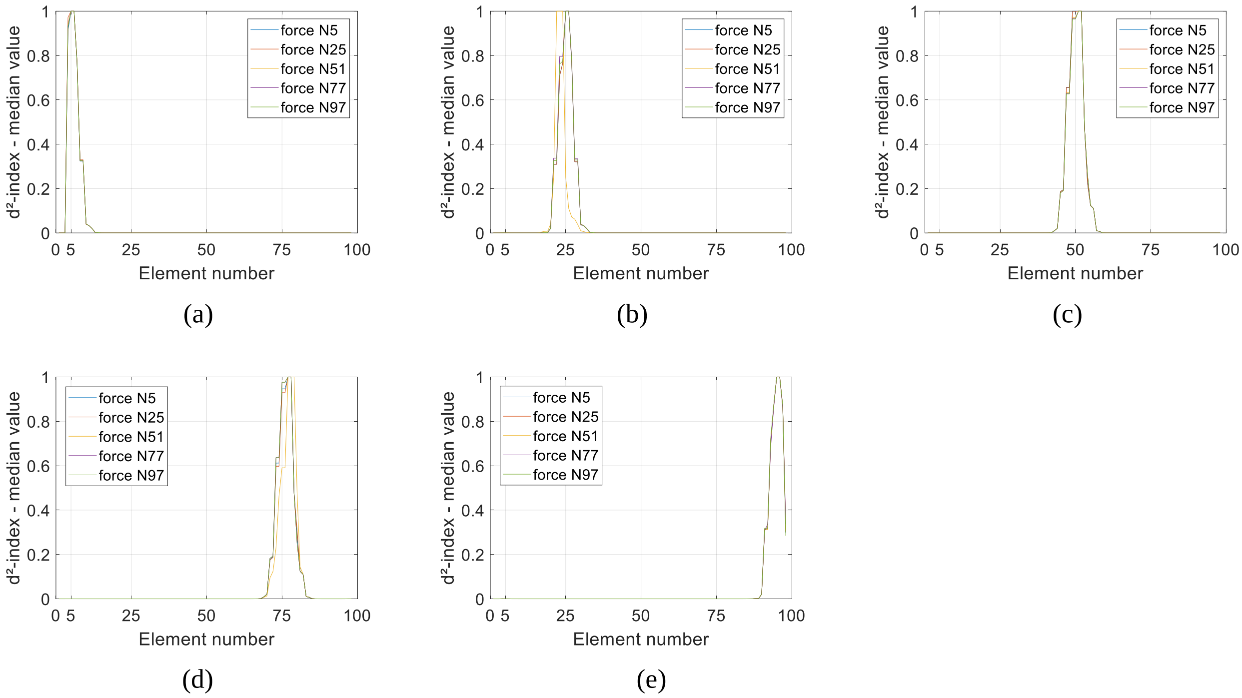


Figure 11 – d^2-index distribution in a double-clamped beam for fixed damage and different nodes load application a-e) damage at e=5, 25, 50, 76, and 96, respectively.

Again, the d^2-index successfully localized the damaged element, no matter where the load or the damaged element was placed. This time no jumps of the coefficients were seen and the supports, due to the zero rotation, did not affect the coefficient behavior.

Synthesis of the Results

Table 1 presents a condensed view of the damage localization results, based on the two proposed indicators for damage detection *i.e.* the maximum median and the maximum "box height". Here we summarized all cases investigated. It can be noticed that, regardless of either the type of beam or the levels of damage severity, the proposed indexes are able to accurately detect damage, for both sides of the structure, with exception of the damages nearby the border condition in which the rotation is not avoided.

Table 1 – Damage localization for each beam type with damage indexes defined in section 4, for three points of force on nodes 5, 51 and 77.

Beam type	Damaged Element	Max median node (*)		
		F5	F51	F77
Simply Supported		25-26	25-26	25-26
Cantilever	25	-(**)	25-26	25-26
Double Clamped		25-26	22-24	25-26
Simply Supported		75-78	75-78	75-78
Cantilever	76	-(**)	-(**)	75-78
Double Clamped		75-78	77-79	75-78
Beam type	**Damaged Element**	**Max box height node (*)**		
		F5	F51	F77
Simply Supported		23-26	23-26	23-26
Cantilever	25	-(**)	23-26	23-26
Double Clamped		23-26	22-26	23-26
Simply Supported		75-78	75-78	75-78
Cantilever	76	-(**)	-(**)	75-78
Double Clamped		75-78	77-79	75-78

* probable region of damage detection is expressed by beam nodes.

** for the cantilever beam the method is not able to localize the damage in case of the damaged element is forwarded related to the load.

Concluding Remarks

This paper investigated the potential of a *DWT*-based damage localization procedure. The proposed approach consists in examining two independent statistical indicators obtained from the *DWT* coefficients of damaged and undamaged deflections of beams subject to transverse static loads. To achieve such a goal an indicator level, named *d²*-index, was created and suggested by the authors.

A study on how such an index is affected by the boundary conditions, the point of the load application, the type of the wavelet mother, and the damage severity, was also carried out.

Different structures develop different structural displacement behavior, although the method was able to localize damage on all with great precision. Regarding the boundary conditions type, the study was able to show that the supports, in which the rotation is free, affect the damage localization the most. Jumps of d²-index were seen in the vicinity of the border, without any relationship with the damage. On the other hand, for the doubled–clamped beam, damages nearby the supports were very well localized by the technique. Further, regardless of the damage severity, the method was able to detect the damaged element, being insensitive to the point of the load application.

Lastly, the index, figured out by the authors, seemed to be sensitive to the problem and was successfully applied throughout the paper. However, despite the great potential is severely affected by the rotation at the supports.

Damage severity tests showed good potential for damage localization even for small levels of damage independent of the point of the force application. As for future works, the main goal is to apply this DWT-based damage localization method on the same beams and conditions but for a dynamic load, when transience will be considered. And further, develop an automated localization procedure based on the proposed indicators.

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