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Parametric uncertainty applied to the robust control of a 2-DOF planar robotic arm

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Abstract: This work proposed an approach of input uncertain in the construction parameters in the robust control applied in a 2-DOF planar robotic arm. The robust control used in this paper rule the torque of the robotic arm due to the requested motion. Parameterization of the dynamic model is required, done in two different vectors. The vector Y is the vector only dependent on the angles, and the vector is the vector that depends only on the parameters of the system. The parameters analyzed are the length and mass of the link, moments of inertia, and the gear reduction ratio. The Passivity Based Robust Control in the dynamics of the robotic arm was applied by varying those parameters, building 100 samples in each case with variance by 1% first, after by 10%, and finally with 30% of the parameter set. It was observed that varying the gear reduction ratio motor of the first and second link exhibit better results than varying the length and mass of the second link. So an increased uncertainty in this parameter is more acceptable for the control applied.

Keywords: Uncertainty, Robust Control, 2-DOF Robotic arm.

INTRODUCTION

The control of robotic manipulators is highly approached in the area. Knowing and controlling the motion of a robot is necessary in a few cases, including robot-assisted surgery for example. One of the methods used for designing kinematics of robotics manipulators is inverse kinematics. For simulation of motion, analysis of manipulator structures, and design of control algorithms the dynamic model of the manipulator is required. Hence, Siciliano et al. (2009) presented detailed dynamics models for robotic manipulators. Several robust control techniques have been presented for tracking the motion of robotic arms. Dawson et al. (1990) presented a Proportional-derivative (PD) control, and Alvarez-Ramirez et al. (2000) focused on Proportional-Integral-Derivative (PID) control. Even neural networks were used for controlling robotic manipulators, as in Lewis (1996). For this work, the initial control applied was the Passivity Based Robust control presented in Spong et al. (2006) to track motion of a 2-DOF planar robotic arm.

The uncertainty is possible in different sources, such as process uncertainty; sensing uncertainty; and environmental uncertainty. In literature is hardly proposed a system with uncertainty in either the system or the constraint parameters but not both simultaneously, as proposed by Kolhe et al. (2011). Vitus et al. (2016) proposed a robust control that considered both of those uncertainties. Padmanabhan et al. (2022) designed a robust control based on the discrete-time formulation of the Uncertainty and Disturbance Estimator. Analysis of uncertainty in specific parameters of the system is not usual in the literature. In this work, the focus is on this case of uncertainty in specific parameters.

In simulations, the Monte Carlo method was applied, Rubinstein and Kroese (2011). The probabilistic model applied in these parameters was a normal distribution, where the mean was 0 and the variance 1%, 10%, and 30% with 100 samples, four parameters of the system were set to run with uncertainty one at a time. Hence, the error results was measured, then counted if was between the convergence range.

PROBLEM FORMULATION AND SOLUTION METHODOLOGY

Figure 1 is a simple diagram of the 2-DOF planar robotic manipulator, where a is the link length, l is the distance of the center of mass from the joint, m_l is the mass of the link, and I_l is the moment of inertia relative to the center of mass. Finally, the index 1 and 2 used in each parameter of the links are respectively referent to the first and second link. Each two joint has a motor that applies a torque τ. Controlling these two torques is possible to do the track motion of the robot through the angles θ_1 and θ_2. To describe the dynamics of a 2-DOF robot arm, with Euler-Lagrange formulation, the equations of motion are described by Equation 1.

$$\tau_0 = \mathbf{M}(\theta)\ddot{\theta}_r + \mathbf{C}(\theta,\dot{\theta})\dot{\theta}_r + \mathbf{G}(\theta) \tag{1}$$

Therefore, the proposal of the Passivity Based Robust control is represented in Equation 2.

$$\mathbf{M}(\theta)\ddot{\theta}_r + \mathbf{C}(\theta,\dot{\theta})\dot{\theta}_r + \mathbf{G}(\theta) - \mathbf{K}\sigma = \tau \tag{2}$$

Where $\mathbf{M}$ is the inertia matrix, $\mathbf{C}$ is the centrifugal and Coriolis effects torque matrix, using Christoffel symbols, $\mathbf{G}$ is

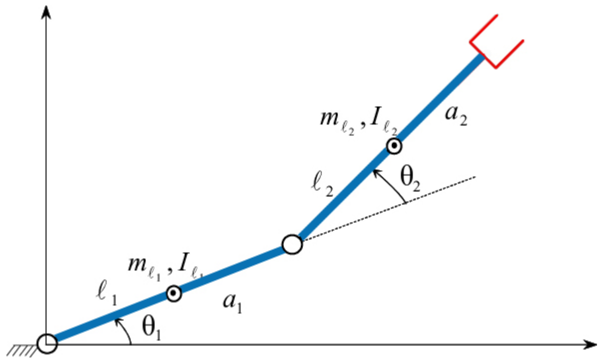


Figure 1: The 2-DOF planar robotic system.

the gravitational torques and $\mathbf{K}$ is the gain. σ is defined in Equation 3 and $\dot{\theta}_\mathbf{r}$ in Equation 4.

$$\sigma = (\dot{\theta} - \dot{\theta}_\mathbf{d}) + \Lambda(\theta - \theta_d) = \dot{\theta} - \dot{\theta}_\mathbf{r} \tag{3}$$

$$\dot{\theta}_\mathbf{r} = \dot{\theta}_\mathbf{d} - \Lambda(\theta - \theta_d) \tag{4}$$

Being τ_0 is a vector with the torque in each joint due to the matrix multiplication shown in Equation 1. For this work, where the interest is to look for the uncertainty in the parameters is clever to do a parameterization that separates the parameters from the angles contributions for the torques. Then, the parameterization $\mathbf{Y}\Pi_0 = \tau_0$ is the Equations 7 and 6, where $\mathbf{Y}$ is a vector that only depends on the angles, $\ddot{\theta}_r$, $\dot{\theta}_r$, $\dot{\theta}$, θ. Π_0 is a vector that only depends on the parameters of the links and motors, adding the gravity impact.

$$\Pi_0 = \begin{bmatrix} I_{l1} + m_{l1}l_1^2 + Kr_1^2 I_{m1} + I_{l2} + m_{l2}(a_1^2 + l_2^2 + I_{m2} + m_{m2}a_1^2) \\ m_{l2}2a_1l_2 \\ I_{l2} + m_{l2}(l_2^2 + Kr_2 I_{m2}) \\ m_{l2}a_1l_2 \\ -m_{l2}a_1l_2 \\ m_{l2}l_2g \\ m_{l1}l_1g + m_{m2}a_1g + m_{l2}a_1g \\ I_{l2} + m_{l2}l_2^2 + Kr_2^2 I_{m2} \end{bmatrix} \tag{5}$$

$$Y = \begin{bmatrix} \ddot{\theta}_{r1} & \ddot{\theta}_{r1}cos(\theta_2) & \ddot{\theta}_{r2} & \ddot{\theta}_{r2}cos(\theta_2) & sin(\theta_2)(\dot{\theta}_2\dot{\theta}_{r1} + \dot{\theta}_1\dot{\theta}_{r2} + \dot{\theta}_2\dot{\theta}_{r2}) & cos(\theta_1+\theta_2) & cos(\theta_1) & 0 \\ 0 & 0 & \ddot{\theta}_{r1} & \ddot{\theta}_{r1}cos(\theta_2) & -sin(\theta_2)\dot{\theta}_1\dot{\theta}_{r1} & cos(\theta_1+\theta_2) & 0 & \ddot{\theta}_{r2} \end{bmatrix} \tag{6}$$

where m_m is the mass of the rotor in the joint, I_m the moment of inertia of the rotor, K_r is the gear reduction ratio of the motor. Thereby, is possible to do the combination of Equation 2 and the parameterization applied. Although considering that the parameters have uncertainty, the term Π in Equation 7 is Π_0 plus a control term, Equation 8, that has the intention of ensuring robustness even with uncertainty.

$$\mathbf{Y}\Pi - \mathbf{K}\sigma = \tau \tag{7}$$

$$\mathbf{u} = -\frac{\rho \mathbf{Y^T}\sigma}{||\mathbf{Y^T}\sigma||} \tag{8}$$

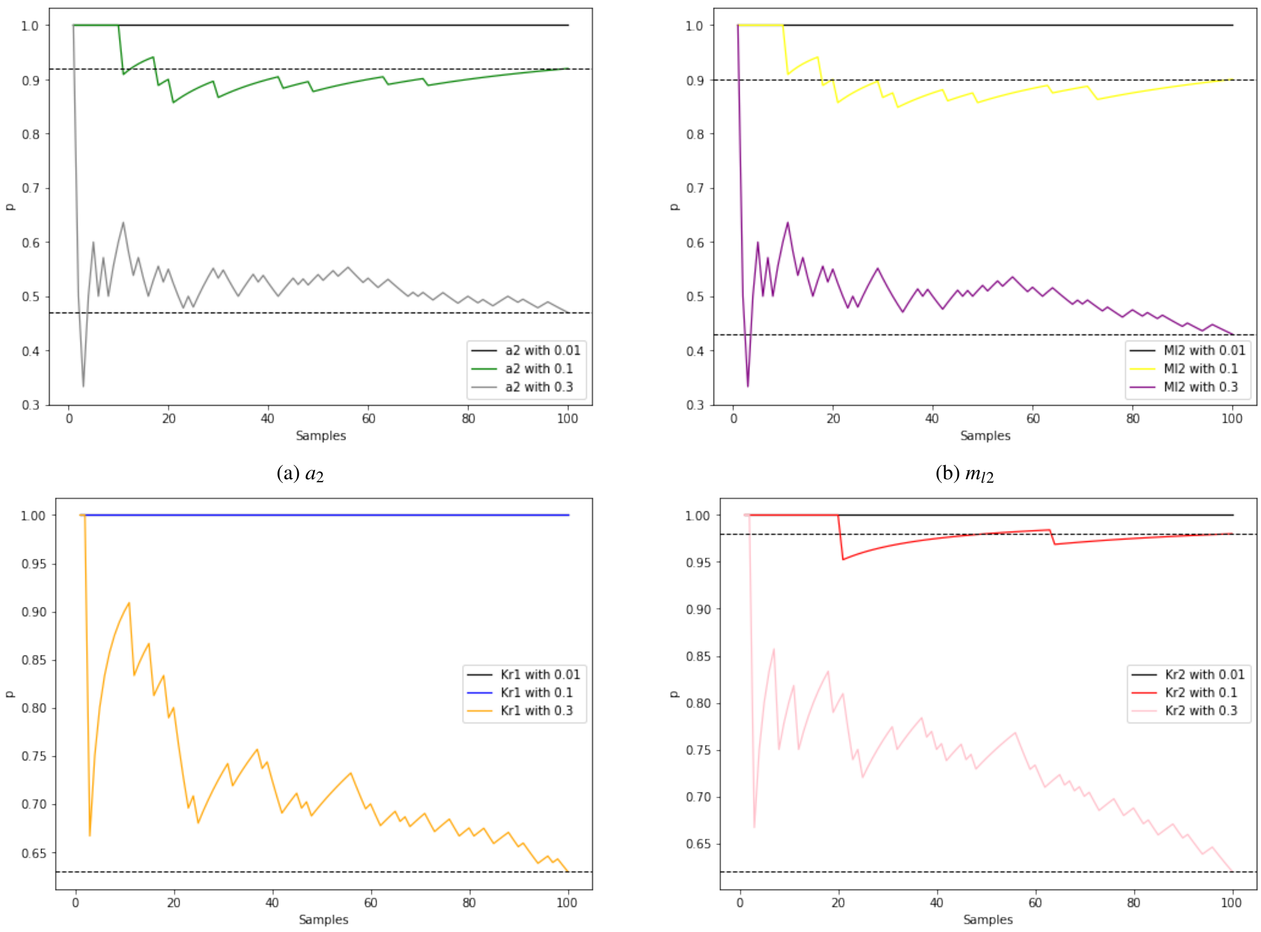
Where ρ is a term of the uncertainty of the vector Π_0. The combination of Equations 7 and 8 becomes Equation 9, which is the control applied in torques of 2-DOF planar robotic manipulator of this paper.

$$\mathbf{Y}\Pi_0 - \frac{\rho \mathbf{Y^T}\sigma}{||\mathbf{Y^T}\sigma||} - \mathbf{K}\sigma = \tau \tag{9}$$

RESULTS AND DISCUSSION

The parameters chosen to be analyzed initially are the second bar length, second bar mass, and the gear reduction ratio of the two motors. Taking 1%, 10%, and 30% of the variance around the valor initial as the mean, 100 samples were calculate randomly. Then, the system was simulated with each sample and verified if the error in the last 10 seconds of simulation is less or equal to 0.001. The total duration of each simulation was set as 30 seconds.

Figure 2 showed that with the increase in the variation fewer simulations reached the required condition. The vertical axis p is the percent of simulations that the error performed as required. Figure 2(a) is the figure that shows the results in simulation with variation in a_2. Figure 2(b) is the figure that exhibits the results with variation in m_{l2}. Figures 2(c) and 2(d) are the results of simulations varying the K_r of the first and second link respectively. In all the four parameters, with the variation of 1% the 100 simulations reached the error condition. With 10% of the variation, in K_{r1} all the 100 simulations attained the error condition. For K_{r2} 98% reached the condition, in a_2 92% reached the condition, and in m_{l2} the worst result, where only 90% achieve the error condition. With 30%, in a_2 47%, in m_{l2} 43%, in K_{r1} 63%, and in K_{r2} 62% of simulations reached the condition. Hence, it is possible to affirm that, in these conditions, is better to have an uncertainty in the K_r parameters in each link than in the second link mass and length. Although, the purpose is to understand the system behavior. The impact of the variation in each or some parameters of the system on the result of the control applied. Therefore, it was not considered if the variations are consistent at the moment.



(a) a_2

(b) m_{l2}

(c) K_{r1}

(d) K_{r2}

Figure 2: Figures of the percentage of the accurate control.

CONCLUSIONS

In literature, it is scarce to find an analysis by parameters to guarantee robustness in robot control. Thus, this paper proposed a parameter variation in the system, considering its uncertainty, for analysis of control behavior. The uncertainty in the K_r parameters in each link gave more robustness in the control that the uncertainty in the second link mass and length. Furthermore, the full paper will have more parameters analysis and cases where more parameters vary simultaneously, with more details about the solution methodology and an increase of the observations about the state-of-the-art review.

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