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Transformers Surrogates for Vortex-Induced Vibrations Computational Simulations

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Abstract: The accurate prediction of structural instability caused by vortex shedding behind bodies or by nonlinear unsteady aerodynamics is fundamental to avoiding the degradation of structural performance or even failure of the system. Numerous approaches can represent analytical models to model both the structure and fluid. The CFD (Computational Fluid Dynamics) approaches consists of solving the Navier-Stokes equations directly, mostly limited by heavily computational costs that, many times, are tough to satisfy in practical engineering. To increase the expectations of solving practical problems, surrogate models are an alternative approach to the underlying physics. Such models have become an essential tool to simplify the analysis and can be a very useful tool in broad industrial applications. In this work, we propose the self-attention transformers model to act as a surrogate for vortex-induced vibrations (VIV) dynamics. We show by numerical experimentation that the surrogate model can accurately predict the VIV dynamics, and more importantly, it can be a suitable tool for many-query applications like sensitivity analysis, design, optimization, or uncertainty quantification.

Keywords: Vortex-induced vibrations, Transformers, Deep learning, Surrogate modeling

INTRODUCTION

The vortex-induced vibrations (VIV) of floating structures play an essential role in the design of offshore engineering. The accurate prediction of structural instability is critical due that the vortex shedding behind bluff bodies may lead to degradation of structural performance or even structural failure. Typically, the description of vortex-induced vibrations requires a thorough model of fluid-structure interaction, which comprises an oscillator for the structure coupled to a flow formulation based on Computational Fluid Dynamics (CFD) approaches. The CFD approaches consists of solving the Navier-Stokes equations directly, mostly limited by heavily computational costs (CPU time and memory) which limits the advance in the field. Such time-consuming tasks often hamper the use of high-fidelity codes constructed upon physics-based models. That becomes a more critical issue whenever one faces many-query applications like sensitivity analysis, design, optimization, or uncertainty quantification.

In order to cope with these challenges, reduced-order models (ROMs), also called surrogate models, can be an interesting approach to help to obtain predictions with lower computational cost than CFD and become a useful tool with broad industrial applications. ROMs have become popular within the field of turbulent flows due to their success in being a proxy for high-fidelity models (Brunton *et al.*, 2020). ROMs allow modeling the main flow dynamics with a reduced computational cost. Furthermore, the utilization of data provided by experiments and high-fidelity simulations for a better understanding of the underlying VIV phenomena has become a new challenge and research opportunity.

Recently, data-driven machine learning (ML) models have gained prominence due to the potential to enhance the capability of computational simulations to describe complex physical systems (Oishi and Yagawa, 2017). Several works have been dedicating the construction of predictive data-driven machine learning models able to return accurate predictions at a low cost (Zhu and Zabaras, 2018; Freitas *et al.*, 2021, 2022). Here, the focus relies on the transformer model (Vaswani *et al.*, 2017), built on self-attention, which has become the state-of-the-art machine learning approach for a large set of natural language processing (NLP) tasks. In the recent work proposed by (Geneva and Zabaras, 2022) transformers were applied to model dynamical systems that can replace otherwise expensive computational models. Such a model has been proven able to accurately predict various dynamical systems and outperforms classical methods that are commonly used in the scientific machine learning literature.

The performance of the self-attention transformers model for modeling physical dynamics makes it a suitable tool for the analysis of VIV dynamics and fluid flows. The aim of this work is to apply the self-attention transformers model for modeling VIV dynamics. Here, we demonstrate the applicability of the present approach using data provided by a phenomenological model proposed in (Qu and Metrikine, 2020) which captures essential features of the VIV dynamics. To the authors' best knowledge, this is the first work to extend the self-attention transformers model for the surrogate modeling of fluid-structure interaction. Also, that might be expected that such an approach can be extended for the analysis of fluid flows that include a large number of variables and DOFs (*degrees of freedom*), which is beyond the scope of the present work and further research efforts are needed.

The remainder of this paper is organized as follows. The next section details the governing equations of the phenomenological model adopted for generating the synthetic data to train the surrogate model. Section 3 presents preliminary results. The paper ends with a summary of our main findings.

COMPUTATIONAL FRAMEWORK

The proposed computational framework aims to model the dynamics of the physical systems described by an ordinary or partial differential equation using modern machine learning techniques. More specifically, assuming a dynamical system given by the form

$$\frac{d\psi}{dt} = f(\psi, t, \mathbf{x}, \theta) \qquad \mathbf{x} \in \Omega \subset \mathbb{R}^m \tag{1}$$

, where $\psi \in \mathbb{R}^n$ is the solution of the dynamical system of n state variables with parameters θ, in the time interval $t \in \mathscr{T} \subset \mathbb{R}^+$. The solution of this general form can characterize a vast range of physical phenomena embodying high-dimensional problems such as fluid flow and transport processes, multi-physics and multi-scale systems such as chemical kinetics and molecular dynamics, among others.

Specifically, it is assumed that the continuous solution of the system in the time interval $\mathscr{T}$ is discretized by T time steps with a time-step size Δt. Thus, the modeling of a dynamical system is posed as a time-series problem propagating from the initial state ψ_0, making the modeling of the dynamical system applicable to the use of machine learning architectures.

Here, the machine learning model has two core components. The transformer model which modeling dynamics and the embedding model for projecting physical states into a latent space. The embedding model is trained prior to the transformer. This embedding model is then frozen and the entire dataset is projected to the hidden latent space in which the transformer is then trained. After training, the embedding decoder is used to reconstruct the solution from the latent space. Further details regarding the machine learning model can be found in (Geneva and Zabaras, 2022), and a brief introduction of the transformer and embedding models are given in the next subsections.

Transformer model

In the present approach, the transformer model is posed as a time integrator for the dynamical system. The model is based on NLP with the primary input a word vector embeddings of a text. Hence, considering that any dynamical problem can be posed as a sequence of vectors, the primary input of the transformer model is the embedded latent space of the dynamics, $\mathscr{Z} = (\mathbf{z}_0, \mathbf{z}_1, \cdots, \mathbf{z}_T)$, where $\mathbf{z}_j \in \mathbb{R}^e$ is the embedded state at time-step j.

The motivation for the usage of a language modeling architecture to predict physical dynamics relies on this model is designed for the sequential prediction of words in a text (Radford and Narasimhan, 2018; Wolf *et al.*, 2020). This suggests that transformer models may be a viable route to construct ROMs for dynamical systems (Chen *et al.*, 2020; Geneva and Zabaras, 2022). Considering a dataset with time-series of the embedded dynamical system $\mathscr{D} = \{\mathbf{z}^i\}_{i=1}^{D}$, the transformer model is trained using the standard time-series Markov model log-likelihood,

$$L_z = \sum_{i}^{D} \sum_{j}^{T} -\log p(\mathbf{z}_j^i | \mathbf{z}_{j-k}^i, \ldots, \mathbf{z}_{j-1}^i, \eta) \tag{2}$$

, where η are the transformer's parameters, like fully-connected neural networks. Here, the likelihood is assumed as a standard Gaussian between the target latent space and the transformer prediction, resulting in a L_2 loss. It is worth remarking that for problems suffering from the curse of dimensionality, training the transformer with a low-dimensional latent space allows the diminishing of the training costs significantly.

Embedding model

The second component of the computational framework is the embedding model for projecting physical states into a latent space, $\mathscr{F} : \mathbb{R}^n \to \mathbb{R}^e$ and $\mathscr{G} : \mathbb{R}^e \to \mathbb{R}^n$. In the present approach, the Koopman dynamics methodology is used to develop the embedding model. The Koopman theory assumes that any dynamical system can be described as an infinite dimensional linear operator acting on an infinite set of state observable functions, $g(\psi_j)$,

$$Kg(\psi_j) \triangleq g \circ \mathbb{F}(\psi_j) \tag{3}$$

, where $\mathbb{F}$ is the dynamic map from one time-step to the next and K is the infinite-dimensional linear operator referred to as the Koopman operator (Koopman, 1931). Thus, the states can be evolved in time through repeated application of the Koopman operator,

$$g(\psi_{j+1}) = Kg(\psi_j), \;\; g(\psi_{j+2}) = K^2 g(\psi_j), \;\; g(\psi_{j+3}) = K^3 g(\psi_j), \; \ldots \tag{4}$$

, where K^p represents a p-fold composition, i.e., $K^3(g) = K(K(K(g)))$.

In the present approach, a data-driven machine learning model is used for learning the Koopman operator. More specifically, the Koopman operator assumes the form of a learnable banded matrix that is learned with the embedding model. Given a dataset with time-series of the dynamical system $\mathscr{D}_\psi = \{\psi^i\}_{i=1}^{D}$, the embedding model is training using the loss function given by,

$$L_E = \sum_{i}^{D}\sum_{j}^{T} MSE(\psi^i_j, \mathscr{G}\circ\mathscr{F}) + MSE(\psi^i_j, \mathscr{G}\circ K^j\mathscr{F}(\psi^i_0)) + \|K\|^2_2. \tag{5}$$

The embedding loss function consists of three components. The first is reconstruction loss which ensures a consistent mapping to and from the latent space. The second component is the Koopman dynamics loss which enforces $\mathbf{z}_j$ to follow linear dynamics. Finally, the last term is a L_2-regularization of the Koopman parameters which helps the model to discover the underlying dynamical modes and also avoid overfitting.

NUMERICAL DATASET

This section briefly introduces the numerical simulation of a phenomenological model used to generate the database used in the study. Here, the phenomenological model consists of an elastically supported 1DOF cylinder along with a wake oscillator that replaces the vortex-shedding mechanisms of the flow. The results of (Qu and Metrikine, 2020) show that this model can be used to simulate both the free and forced vibration experiments. More specifically, the system dynamics is described by a cross-flow displacement variable y and by a wake phenomenological variable q that are both dependent on time τ. Based on this formulation, the equations of motion are given as

$$\ddot{y} + \left[\frac{2\zeta}{S_t v} + \frac{C_{D_m}}{(\mu + C_A)\pi^2 S_t}\right]\dot{y} + \frac{1}{S_t^2 v^2} y = \frac{C_{L_0}}{4(\mu + C_A)\pi^3 S_t^2} q \tag{6}$$

$$\ddot{q} + \varepsilon(1 - q^2)\dot{q} + q = \sum_{j=0}^{3}(A_j|y|^j\dot{y} + B_j|y|^j\ddot{y}) \tag{7}$$

, where v is the reduced velocity, μ is the mass ratio, ζ is the damping ratio, C_A is the potential added mass coefficient, S_t is Strouhal number, C_{D_m} is the mean drag, C_{L_0} is the oscillating lift, ε is a nonlinearity parameter and A_0, B_0, A_1, B_1, A_2, B_2, A_3 and B_3 are coupling parameters. All model parameters, time, and dependent variables are dimensionless.

The parameters v, μ, and ζ are considered to specify a scenario, whereas the remaining parameters are frozen. By designating the scenario and the initial conditions for Eqs. (6) and (7), the system turns suitable to undergo numerical integration. In this regard, the assigned parameter values of the phenomenological model are presented in Tab. 1.

Table 1: Parameters of the phenomenological model.

Parameters	Values
Damping ratio, ζ	1.5×10^{-3}
Added mass, C_A	1.0
Strouhal number, S_t	1.93×10^{-1}
Mean drag, C_{D_m}	1.19
Oscillating lift, C_{L0}	3.84×10^{-1}
Nonlinearity parameter, ε	5×10^{-2}
Coupling parameter, A_0	4.20
Coupling parameter, A_1	11.3
Coupling parameter, A_2	−68.7
Coupling parameter, A_3	50.5
Coupling parameter, B_0	1.50
Coupling parameter, B_1	8.50
Coupling parameter, B_2	−11.1
Coupling parameter, B_3	2.60

RESULTS AND DISCUSSIONS

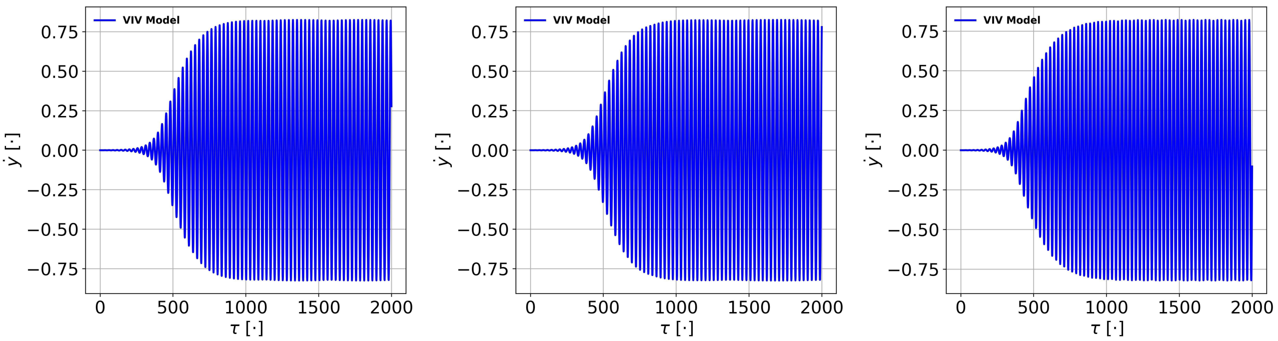
In the present section, we demonstrate the performance of the proposed methodology. Here, we are interested in learning the set of solutions of different operation conditions given by the equation parameters θ. However, the methodology can be extended for more critical issues such as faces many-query applications like sensitivity analysis, design, optimization, and uncertainty quantification.

For the training process, synthetic data is generated using the phenomenological model given by Eqs. (6) and (7) with the parameters in the Tab. 1. More specifically, the first dataset is constructed using a constant mass ratio $\mu = 8.6$ and varying the reduced velocity by assuming the following probabilistic model,

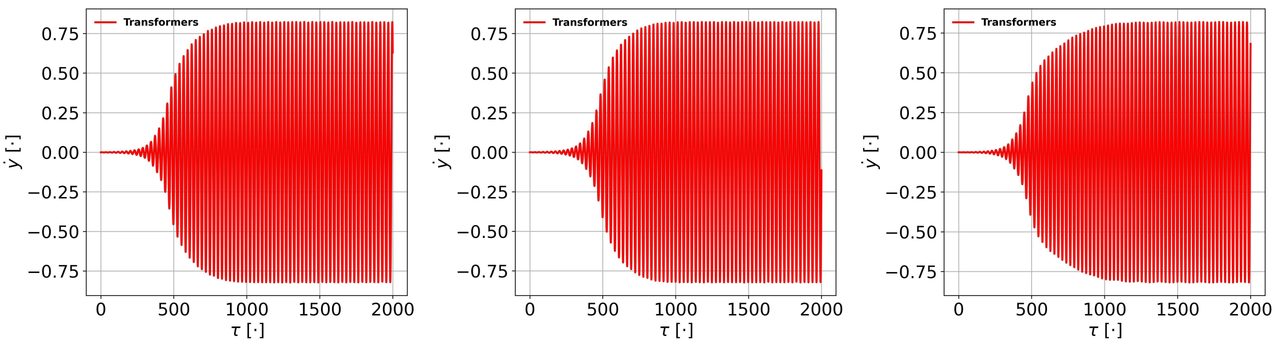
$$v = v_l + (v_u - v_l)\xi_v \tag{8}$$

, where $v_l = 5.5$ and $v_u = 6.5$ denote the lower and upper bounds of the probabilistic model, and ξ is a independent and identically distributed uniform random variable taking values into $\xi_v \in U[0,1]$. Here, the training, validation, and testing datasets containing 2048, 64 and 256 time-series at a time-step size of $\Delta t = 0.25$ solved using a Runge-Kutta method, respectively. The datasets have time series of 2000 time-steps containing the solution during unsteady and steady states, allowing provides the whole dynamics of the system. The validation dataset is used for evaluating the model during the training process. Also, that can be used to search for the best hyperparameters during the training such as reducing the learning rate when an accuracy metric has stopped improving. Finally, the testing dataset is used to evaluate the trained transformer model.

Figure 1 shows three randomly selected test samples for which only the initial state is provided and the machine learning model predicts 2000 time steps forward. As we can the transformer model is able to accurately predict the cross-flow displacement. Moreover, it is verified that the transformer model and the phenomenological model have the same structure Fig. 2, which qualitatively indicates that the surrogate model learned the underlying physical dynamics.



(a) Phenomenological model



(b) Transformers model

Figure 1: Displacement in the cross-flow y direction predicted by the transformer model and provided by the phenomenological model for a randomly selected reduced velocity in the test dataset.

Additionally, we test the ability of the proposed transformer model to predict the amplitude of vibration as a function of reduced velocity for the cross-flow direction. The transformer model is able to yield extremely accurate predictions, as we can see in Fig. 3. The root-mean-squared error (RMSE) between the amplitude of vibration predicted by the transformer model and computed by the phenomenological model is $RMSE = 3.4 \times 10^{-3}$.

Furthermore, we test the performance of the transformer model for a more challenging scenario in which a non-constant mass ratio is assumed, and its value is given by the following probabilistic model,

$$\mu = \bar{\mu}(1 + \delta\xi_\mu) \tag{9}$$

, where $\bar{\mu} = 8.6$ denotes the mean (expected) value of the mass ratio, $\delta = 0.05$ the percentile variation over the mean and ξ_μ independent and identically distributed uniform random variable taking values into $[-1,1]$. Similar to the first

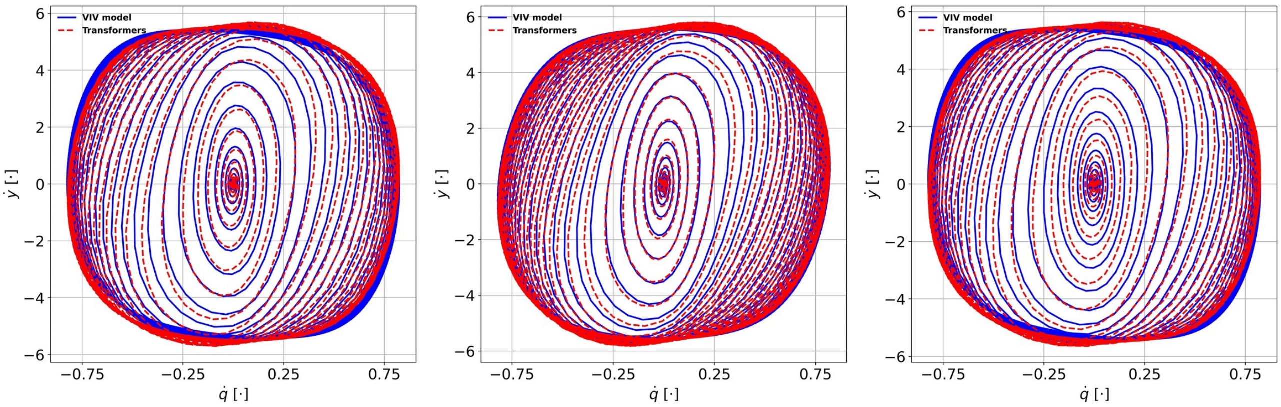


Figure 2: 2D cylinder cross-flow trajectory y in relation to the phenomenological variable q.

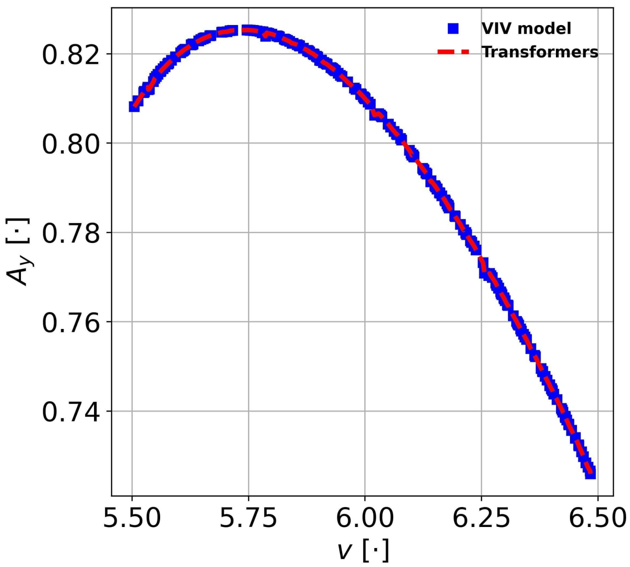


Figure 3: Amplitude dependence on the reduced velocity. $RMSE = 3.4 \times 10^{-3}$

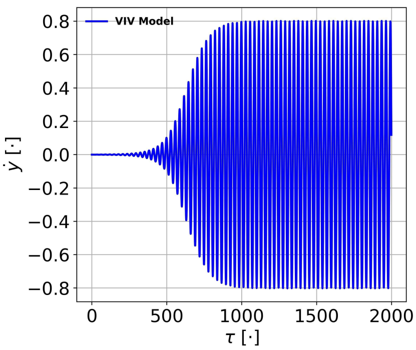
experiment, the training, validation, and testing datasets containing 2048, 64, and 256 time-series of 2000 time-steps containing the solution during unsteady and steady states.

Figure 4 shows one randomly selected test sample for which only the initial state is provided and the parameters of the dynamical system, i.e., reduced velocity and mass ratio. As we can see the machine learning model predicts 2000 time steps forward with high accuracy. Also, the proposed machine learning model predicts the amplitude of vibration for different conditions given by parameters of the dynamical system accurately well, as shown in Fig. 5. Further, the RMSE between the amplitude of vibration predicted by the transformer model and computed by the phenomenological model is $RMSE = 3.6 \times 10^{-3}$.

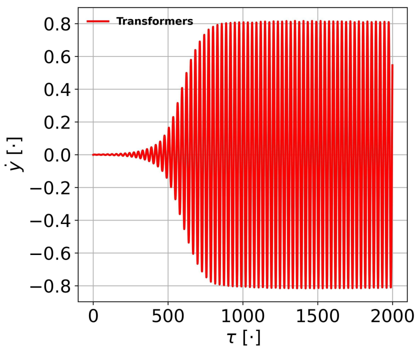
CONCLUSIONS

The present study addresses the construction of reduced-order models of physics-based computational models. Here, the attention goes to the use of the self-attention transformers model for modeling VIV dynamics. The applicability of the proposed approach was tested using data provided by a phenomenological model which captures essential features of the VIV dynamics. More specifically, it was proposed two numerical experiments in which the reduced velocity and mass ratio were assumed following independent and identically distributed probabilistic functions. Such an approach has been revealed to yield extremely accurate predictions for different dynamics conditions given by the parameters of the phenomenological model.

Furthermore, we place our contribution in the emerging area of physics-aware machine learning, where the final model, in many different ways blends two main components: availability of experimental data and/or often expensive computational models, data-driven machine learning techniques. Such a combination allows an understanding of the underlying physics of dynamics systems at a relatively low cost, and also offers a broad spectrum of opportunities to



(a) Phenomenological model



(b) Transformers model

Figure 4: Displacement in the cross-flow y direction predicted by the transformer model and provided by the phenomenological model for a randomly selected reduced velocity and mass ratio in the test dataset.

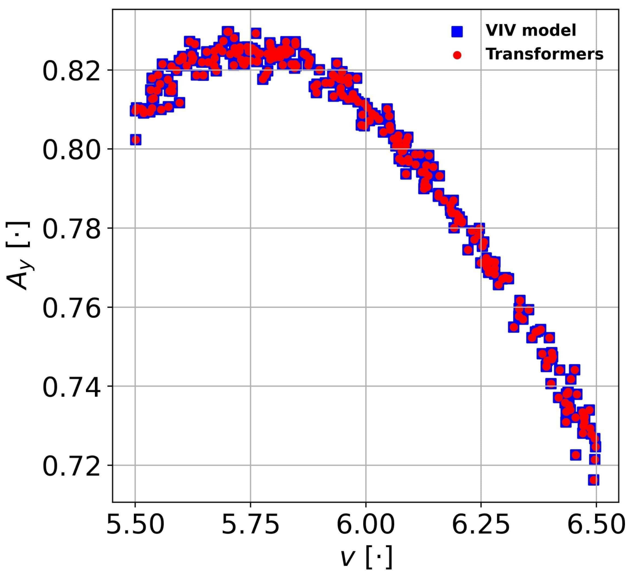


Figure 5: Amplitude dependence on the reduced velocity. $RMSE = 3.6 \times 10^{-3}$

extend the analysis of fluid flows that include a large number of variables and degrees of freedom.

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