DINAME 2023 - Proceedings of the XIX International Symposium on Dynamic Problems of Mechanics,
A.S. De Paula, A.T. Fabro and M.R. Machado (Editors), ABCM.
Pirenópolis, Go, Brazil, February 26th to March 3rd, 2023

Characterizing the vibration attenuation in Herschel-Quincke beams by the transmitted vibration and transmission loss

Gabriel Angelini [1], José Roberto F. Arruda [1]

[1] UNICAMP – Universidade Estadual de Campinas – Cidade Universitária Zeferino Vaz – Barão Geraldo, Campinas, SP, Brazil.

Abstract: This paper aims at using the well-known Herschel-Quincke (H-Q) acoustic tube principle to design beams that exhibit cancellation of flexural waves at certain frequencies. Unlike H-Q acoustic tubes, where the tube shape does not change the behavior of acoustic waves under the plane-wave assumption, a curved beam couples bending and longitudinal behavior, which makes the problem more complex. We investigate curved shapes used to fit a longer beam between two points, and their effect on the predicted cancellation frequencies. We first simulate an ideal problem, where two straight beams of different lengths are fitted (unrealistically) between two end beams, where cancellation occurs when the difference in the lengths of the two beams is equal to an odd integer number of half wavelengths, so that bending waves reach the receiver beam with opposite phase. Then, a realistic case, where the longer beam is curved, is addressed, and the differences with respect to the ideal case are discussed. The wave attenuation is also characterized using the transmission loss (TL) of the bending waves propagating along the beams. Finally, the H-Q beam TL is compared with the TL of a beam with periodically attached resonators.

Keywords: Herschel-Quincke, beams, vibration, transmission loss, metastructure.

INTRODUCTION

In this paper, we compare two ways of attenuating transmitted vibrations in frame-type structures subjected to harmonic loading. The attenuations are produced by two approaches: by adding resonators periodically along a given section of the beam (metamaterial) and by using structures with a cancellation principle similar to that of the Herschel-Quincke (H-Q) acoustic tube. Both approaches attenuate the vibrations transmitted between a section of the beam where it is excited (with free termination) and a section that is anechoically terminated (semi-infinite). Numerical simulations are performed using the Spectral Element Method (SEM) implemented in MATLAB©. One of the motivations for this study was to start from the concept of the Herschel-Quincke acoustic tube, which basically consists of a connection through two tubes of different lengths so that, at the frequencies where noise cancellation is desired, the acoustic waves arriving through the two ducts have equal pressure amplitudes and opposite phase, so that they cancel each other. The concept was introduced in the 19th century by Herschel and Quincke (Selamet and Dickey, 1994). Several research works have been developed since then using this principle, but all of them using acoustic tubes, where it is desired to eliminate sound waves that propagate in ducts and turbines (Halle and Burdisso, 2001) at certain frequencies. In this paper, this concept will be explored to cancel bending waves in beams.

Since, unlike the case of the acoustic duct, it is not possible to connect two points by straight beams of different lengths, frames with gentle curvature are used in order to increase the propagation length of the bending waves, while trying to keep equal amplitudes. The curvature, even smooth, results in unwanted coupling of the bending waves with longitudinal waves. The conditions under which it is possible to cause the phenomenon of wave cancellation at certain specific frequencies are investigated. The attenuation is calculated by the vibration amplitudes before and after the H-Q beam and by calculating the transmission loss (TL) using, for this purpose, a semi-infinite frame spectral element. The periodic addition of resonators is used for comparison with the attenuation by the proposed H-Q beams. The advantages and disadvantages of the two solutions in attenuating transmitted vibration are discussed, and possible engineering applications are suggested.

SPECTRAL ELEMENT MODEL

SEM can be defined as a finite element method in the frequency domain, in which the elements represent exact analytical solutions within the scope of the theory used (Doyle, 1997). Spectral elements exist for various types of structures, for example rods, shafts, beams, frames and Levy-type plates (Lee, 2009). The dynamic stiffness matrices of the spectral elements are combined to form the global stiffness matrix, exactly as in the traditional finite element method, but the global matrix is expressed in the frequency domain.

The dynamic stiffness matrix of an Euler-Bernoulli beam with length L made of a material with Young's modulus E, mass density ρ, cross section with area A and inertia I using the spectral element method can be written as (Lee, 2009):

$$\begin{Bmatrix} \hat{V}(0) \\ \hat{M}(0) \\ \hat{V}(L) \\ \hat{M}(L) \end{Bmatrix} = EI\beta^2 \begin{bmatrix} i\beta & -\beta & -i\beta e^{-i\beta L} & \beta e^{-\beta L} \\ 1 & -1 & e^{-i\beta L} & -e^{-\beta L} \\ -i\beta e^{-i\beta L} & \beta e^{-\beta L} & i\beta & -\beta \\ -e^{-i\beta L} & e^{-\beta L} & -1 & 1 \end{bmatrix} [G]^{-1} \begin{Bmatrix} \hat{v}(0) \\ \hat{\phi}(0) \\ \hat{v}(L) \\ \hat{\phi}(L) \end{Bmatrix} \tag{1}$$

where $\hat{V}$ are shear forces, $\hat{M}$ are bending moments, $\hat{v}$ are transverse displacements, $\hat{\phi}$ are cross section rotation angles, $\beta = \sqrt{\omega\sqrt{\frac{\rho A}{EI}}}$, and matrix [G] is given by:

$$[G] = \begin{bmatrix} 1 & 1 & e^{-i\beta L} & e^{-\beta L} \\ -i\beta & -\beta & i\beta e^{-i\beta L} & \beta e^{-\beta L} \\ e^{-i\beta L} & e^{-\beta L} & 1 & 1 \\ -i\beta e^{-i\beta L} & -\beta e^{-\beta L} & i\beta & \beta \end{bmatrix} \tag{2}$$

The elementary rod spectral element, on the other hand, is given by:

$$\begin{Bmatrix} \hat{F}(0) \\ \hat{F}(L) \end{Bmatrix} = \frac{EA(ik)}{1 - e^{-i2kL}} \begin{bmatrix} 1 + e^{-i2kL} & -2e^{-ikL} \\ -2e^{-ikL} & 1 + e^{-i2kL} \end{bmatrix} \begin{Bmatrix} \hat{u}(0) \\ \hat{u}(L) \end{Bmatrix} \tag{3}$$

Where $\hat{F}$ are longitudinal forces and $\hat{u}$ are longitudinal displacements. The frame element combines these two matrices as a 6x6 dynamic stiffness matrix. For the curved sections, a piecewise approximation with a number of straight frame elements was used. This number was increased until convergence of the results.

H-Q BEAM

As mentioned earlier, cancellation occurs when two waves of the same amplitude with opposite phases are added. Figure 1 illustrates the H-Q beam shape implemented using curved circular segments. Each quarter-circle curved segment was modeled with 25 straight frame elements. The beam with free end has a length of 0.25m, a width of 0.05 m and is 1 mm thick. The H-Q segments have half the width, the straight segment has a length of 0.6m, the curved segment radius of curvature is 0.1m, and the two segments are attached to a 0.2 m straight upper segment.

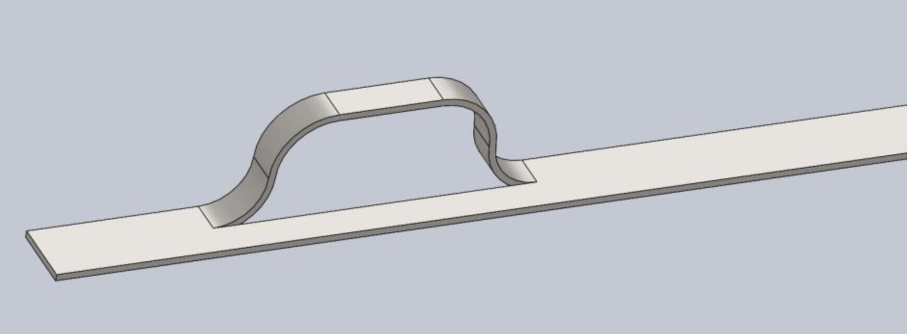

Figure 1 - Proposed H-Q beam structure.

The structure was excited at the free end. The other end is infinite. The frequency response function (FRF) was computed at the junction after the H-Q segment. The TL was computed using the semi-infinite spectral element (anechoic termination). In the excited end, the amplitude of the flexural wave incident at the first junction is computed using the spectral element formulation. At the infinite side, the amplitude of the flexural vibration at a node past the junction is equal to the transmitted wave amplitude, as there is no reflected wave. The TL is the logarithmic ratio of the incident and transmitted waves (before and after the H-Q part) in decibels.

RESULTS

For the ideal case (not physically possible), where two beams of different lengths (0.6 m and 0.8 m, with difference ΔL=0.2 m) are straight and made of the same material with the same cross section, the bending waves propagate in identical ways and with the same wavelength λ. Wave cancellation occurs every time the constant ΔL intersects the curves of odd multiples of $\lambda/2$ (Fig. 2, top). For this ideal case, the FRF and the TL are shown in Fig. 2.

In Fig.3, the results for the geometry shown in Fig. 1 are displayed.

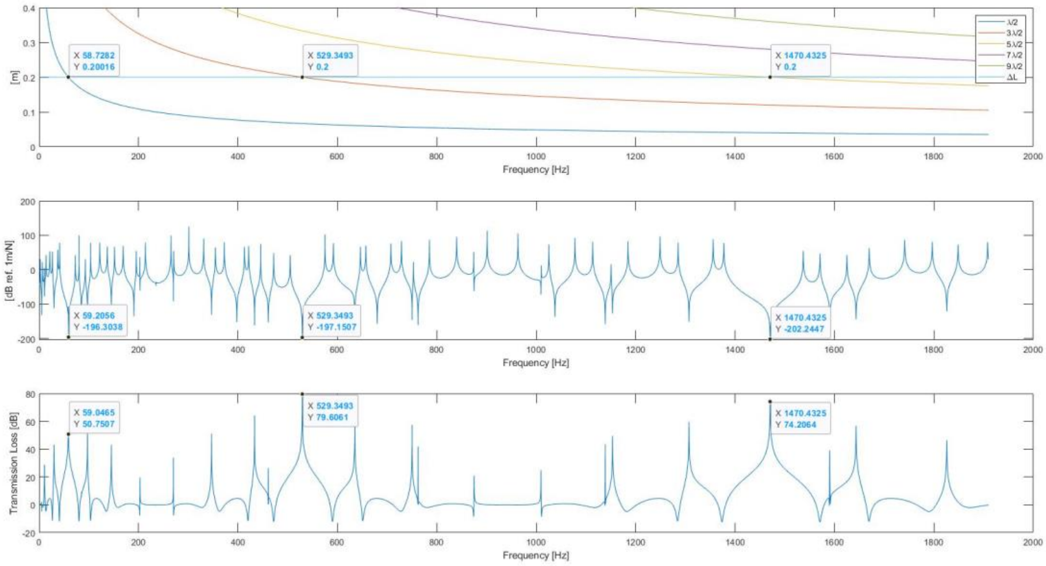

Figure 2 – Results for the ideal case of two straight beams: length difference and odd multiples of λ/2 indicating the cancellation frequencies (top), FRF (middle) and TL (bottom).

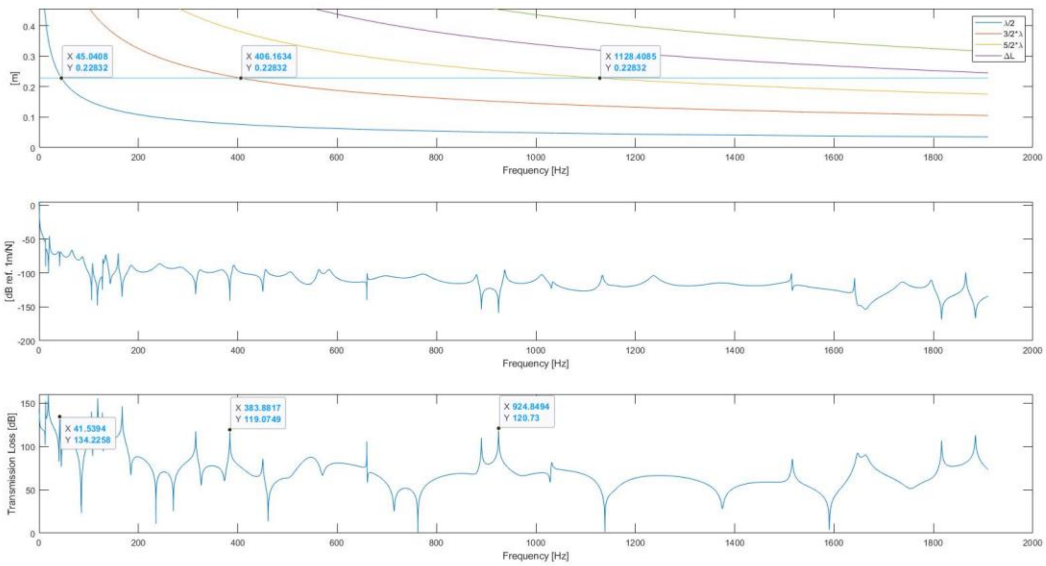

FIGURE 3 – Results for the curved H-Q beam: length difference and odd multiples of λ/2 indicating the cancellation frequencies (top), FRF (middle) and TL (bottom).

The ratio of displacements between the points immediately before and after the H-Q section for the ideal case is around 200dB for a structural damping loss factor of 0.01%. In practical terms, this displacement can be considered to be zero. If zero damping were adopted, we would have a complete cancellation, with the valleys of the logarithmic curves going to negative infinity.

In Fig. 3, where, in fact, we have a physically realizable structure, the model used to predict the cancellation frequencies is only a rough approximation, which can be explained by the fact that the vibration amplitudes are different in the two segments, and there is coupling of the bending and longitudinal waves. Increasing the radius of curvature decreases the coupling of flexural and longitudinal waves, improving the prediction of the cancellation frequencies. Although, in Fig. 3, the cancellation frequencies are not predicted with accuracy, good levels of attenuation are achieved. We have used the same level of damping as in results shown in Fig. 2.

Finally, in Fig. 4, results obtained with 12 resonators of 10g each, half of them tuned to each of the first crossing frequencies (59Hz and 529Hz) of the ideal H-Q beam case are shown. We can see that, using the metamaterial beam, a considerably higher level of attenuation was achieved at frequencies very close to the predicted ones.

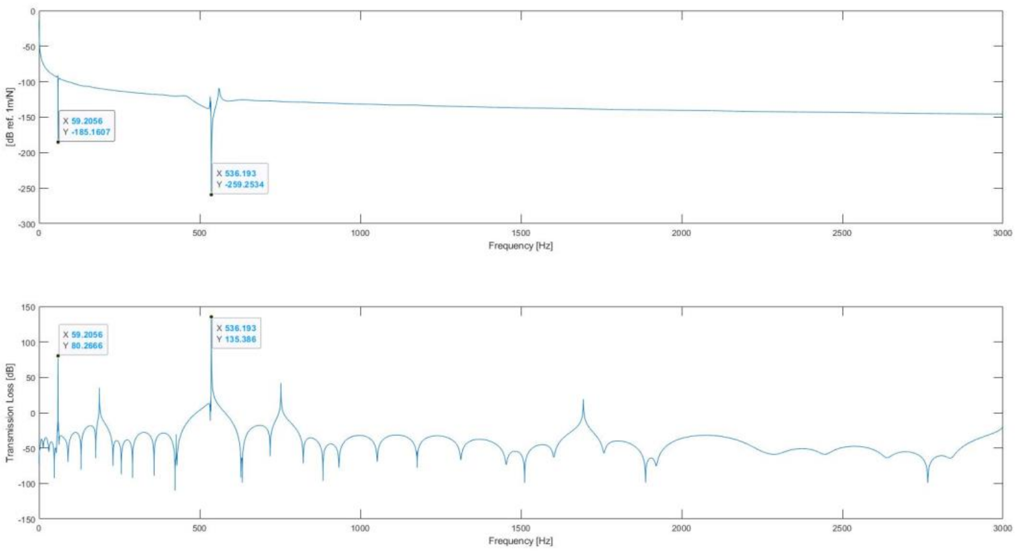

FIGURE 4 – FRF (upper) or TL (lower) for the metastructure case consisting of a straight beam coupled with 12 resonators, half tuned to each frequency of interest (59Hz and 529 Hz).

In order com compare the H-Q solution with the metamaterial solution in a "fairer" way, each of the twelve resonators was attributed the mass of 1/12th of the extra mass introduced by the H-Q solution. In the results shown in Fig. 3, ΔL=0.2283m, so the mass of each resonator will be:

$$M_r = \frac{\rho A L}{12} = 37{,}1g \tag{4}$$

Results obtained for the metamaterial beam with this resonator mass are shown in Fig. 5. Once more, half of them is tuned to each desired attenuation frequency.

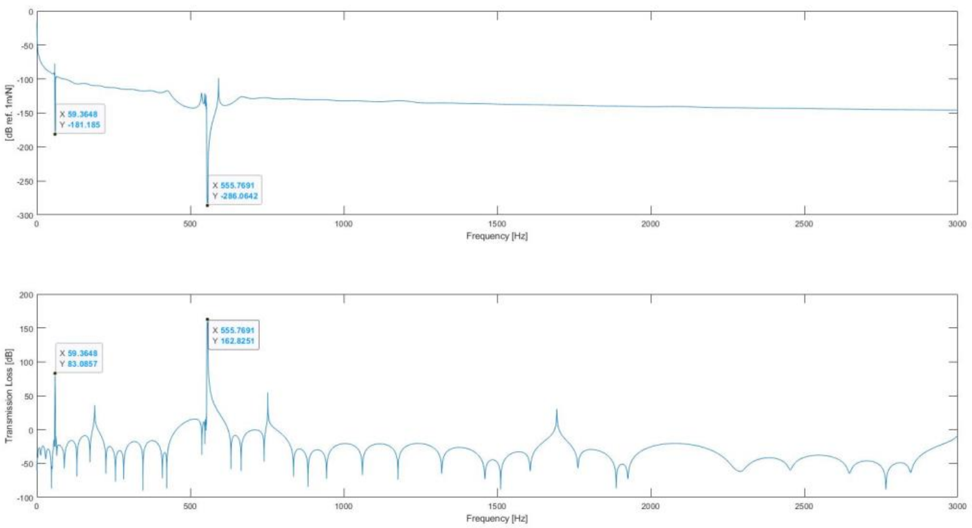

FIGURE 5 – metamaterial composed of 12 equally spaced resonators (59Hz and 529 Hz) with the same extra mass as the H-Q beam.

In Fig. 5 we can see a transmission loss around 160dB at the second cancellation frequency, while in Fig. 4, the same frequency reaches an attenuation of 140dB. The difference found is quite significant, since the power doubles every 3dB.

In both cases we see that the first tuned frequency (59Hz) is attenuated exactly in its value, while the second frequency, which in theory would be 529Hz, is approximately 555 Hz.

We have analyzed two other cases, a thicker structure, and a structure with a larger radius of curvature of the curved sections. Figures 6 and 7 illustrate the result for the H-Q structure (represented in Fig. 1) for the cases where the thickness of the beams is 5mm instead of 1mm, and for the radius of curvature of 0.145m instead of 0.1m, respectively.

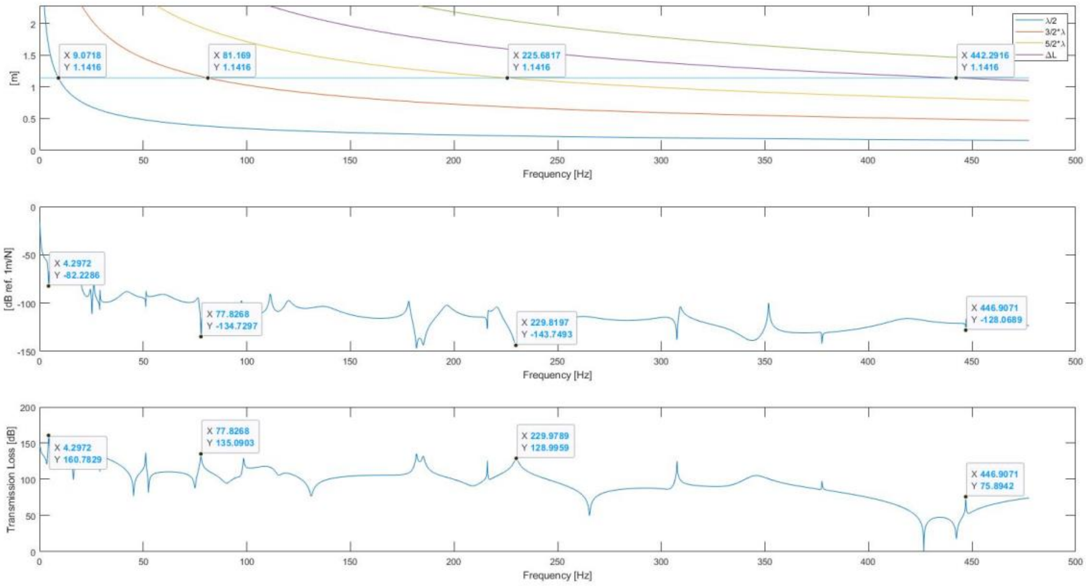

FIGURE 6 – H-Q structure for a thickness of 5mm (five times thicker than the original one).

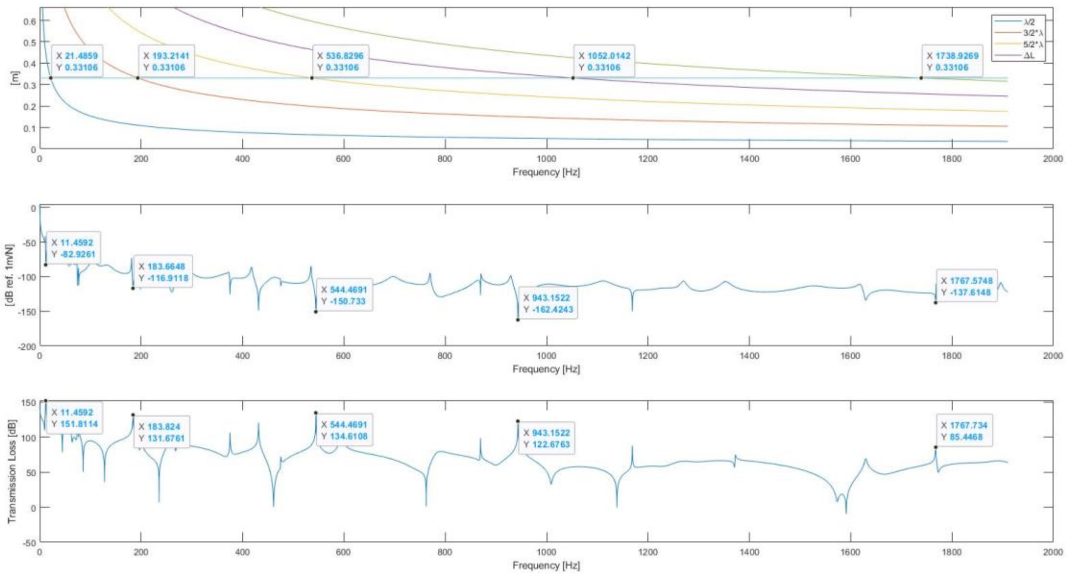

FIGURE 7 - H-Q structure for a bend radius of 0.145m (1.45 times larger than the original radius).

Comparing Figs. 6 and 7, we can state that the larger the radius of curvature, the larger the length of the curved branch and therefore the greater the number of crossings for the same frequency range. Consequently, there is a greater number of attenuated frequencies. To take a basis of the order of magnitude, the original case shown in Fig. 3 had a difference in length of 0.2283m, while in the larger radius case this difference increases to 0.3311m, which generate two more attenuated frequencies in the same band.

CONCLUSIONS

The principle of the H-Q acoustic tube constitutes a simple and efficient passive solution to cancel sound waves in ducts at discrete frequencies. In this work, we have explored the possibility of using the same principle to attenuate vibrations in beams. The difficulty comes from the fact that the propagation of bending waves in curved beams is coupled with longitudinal vibrations, deteriorating the cancellation obtained and making it difficult to predict the cancellation frequencies. However, the simulations presented show that significant attenuation can be achieved if the curved beam is slender and has a large radius of curvature, minimizing the coupling with longitudinal waves. An experimental implementation is currently being carried out.

REFERENCES

Blevins, R. D., Formulas for Natural Frequency and Mode Shape, Van Nostrand Reinhold Company, New York, 1979.

Doyle, J. F., “Wave Propagation in Structures”, Springer, 1997.

Halle, R.F; Burdisso, R.A, 2001, “The Herschel-Quincke Tube Concept as a Noise Control Device for Tubofan Engines”, 39th AAIA Conference.

Lee, U., “Spectral Element Method in Structural Dynamics”, Wiley Online Library, 2009.

Selamet, A., Dickey, N. S., 1994, “The Herschel–Quincke tube: A theoretical, computational, and experimental investigation”, The Journal of the Acoustical Society of America 96, 3177.

RESPONSIBILITY NOTICE

The author are the only responsible for the printed material included in this paper.

APPENDIX A - VALIDATION OF THE CURVED FRAME SEM MODEL USING THE NATURAL FREQUENCIES OF A CIRCULAR RING

In order to validate the spectral element model using straight spectral elements to model a curved beam, a closed circular ring was modeled. The ring has a constant transversal cross section 5cm wide and 5mm thick with a radius of 0.25m. It was subdivided into 100 equal elements, 25 per quadrant. The schematic representation is shown in Fig. 8.

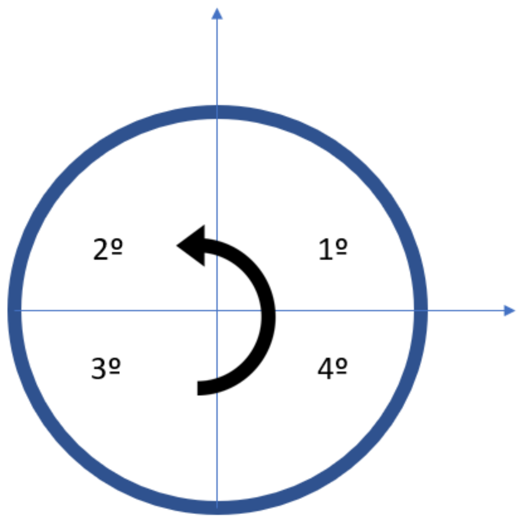

FIGURE 8 – Circular ring with four quadrants.

Each of the quadrants was modeled with straight frame spectral elements, with 6 degrees of freedom, 3 per node: longitudinal displacement, transversal displacement, and rotation, respectively. Next, the global stiffness matrix was assembled by connecting the elements, where the first element contains the GDLs 1 to 6, the second element 4 to 9, and so on, thus totaling 300 degrees of freedom. Since the circular structure is being represented by straight elements, we must keep in mind that each element must have a length equal to the chord of the respective arc section (see Fig. 9), which, for this case is given by:

$$L = 2R\sin(\gamma) \qquad (A1)$$

where $\gamma = \frac{\theta}{2} = \frac{\pi}{4N_e}$ and N_e is the number of elements that make up the circle.

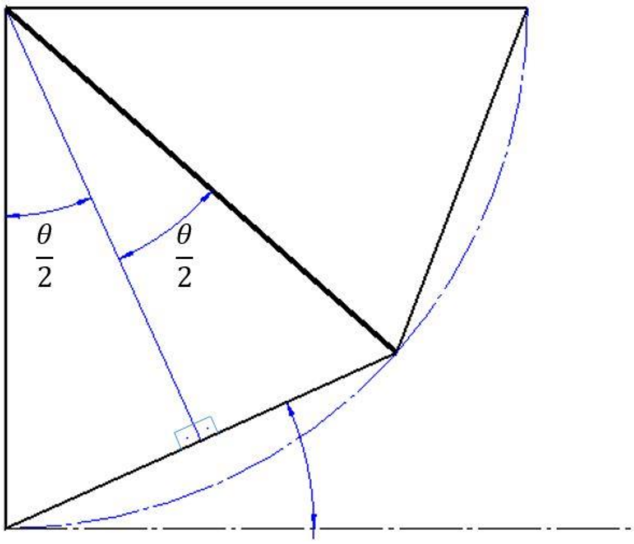

FIGURE 9 - Discretized arc and element length (chord).

Furthermore, it is necessary to rotate each element of its respective inclination. The angle of inclination of each element depends on the number of elements. Equation 6 represents the transformation matrix for a frame element rotated of an angle θ.

$$T = \begin{pmatrix} \cos(\theta) & \mathrm{sen}(\theta) & 0 & 0 & 0 & 0 \\ -sen(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & sen(\theta) & 0 \\ 0 & 0 & 0 & -sen(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (A2)$$

The angle θ that must be considered in the transformation matrix T for the i-th element depends on the position where it is located. Equation A3 represents this slope.

$$\theta_i = \frac{(i-1)2\pi}{N_e} + \frac{\pi}{N_e} \qquad (A3)$$

As is already known, and analogous to the finite element method, there is a grouping to assemble the global matrix, where the terms of the stiffness matrix of a given element are added to the corresponding GDL of the successive element. Two successive elements have one node in common, so the last three GDL of the previous element will be equal to the first three GDL of the subsequent element, and thus we must superimpose the influence of both. Due to the way the global stiffness matrix is constructed, it is a sparse matrix.

The spectral element model was validated by calculating the natural frequencies of a closed circular ring, which have an analytical expression. These natural frequencies for the bending modes are given by (Blevins, 1979):

$$f_n = \frac{2n(n^2-1)}{\pi d^2\sqrt{n^2+1}}\sqrt{\frac{EI_y}{m}} \qquad , \quad n = 1,2,3 \ldots. \qquad (A4)$$

Figure 11 shows the FRF of the spectral element model. The peaks correspond to the natural frequencies.

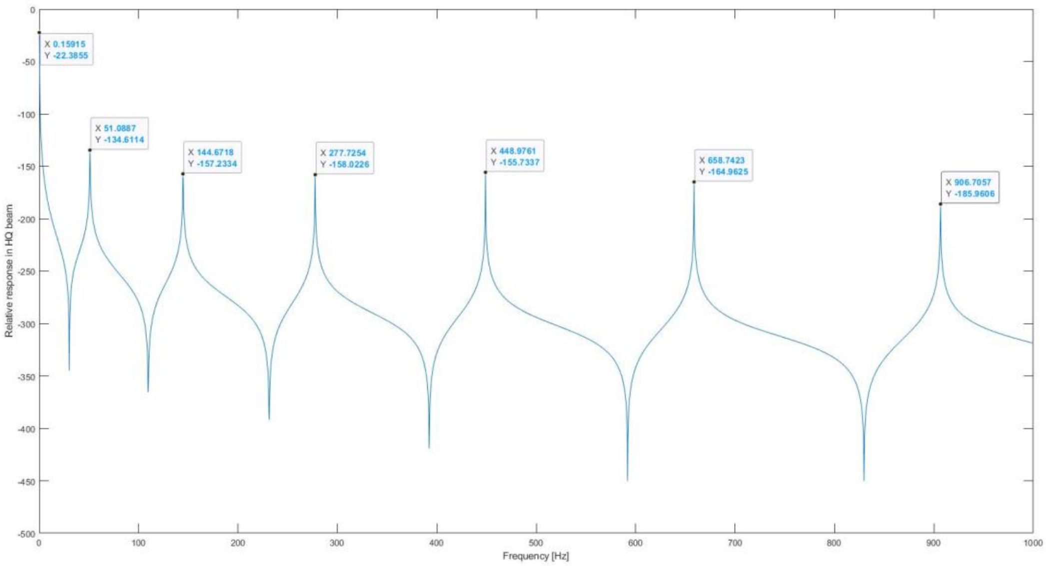

FIGURE 11 - Frequency response function for the modeled ring with straight frames.

Table 1 shows a comparison of the natural frequencies calculated by the two methods. The good agreement validates the spectral element model of the curved beam.

Mode	Analytical Expression (Hz)	SEM – Straight Frames (Hz)
1º Mode	0	0
2º Mode	51,17	51,09
3º Mode	144,74	144,67
4º Mode	277,53	277,73
5º Mode	448,82	448,98
6º Mode	658,42	658,74
7º Mode	906,23	906,71

Table 1 - Comparison of natural frequencies obtained with SEM and the analytical.