



# Flutter analysis of a hybrid aero-servo-viscoelastic wing in the time domain

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*Abstract: This work focuses on the construction of a hybrid aeroelastic model in which it is possible to perform dynamical stability analysis in the time domain. The hybrid model is also called aero-servo-viscoelastic and it takes into account the presence of a viscoelastic material that rules the passive control as well a proportional derivative (PD) controller which performs the active approach. These techniques are concomitantly applied to a straight and homogeneous wing with a control surface coupled to the trailing edge. The physical model considers two degrees of freedom, while the aerodynamic model is unsteady and has approximations to inhibit the reduced frequency factor dependence. Since the global investigation is performed in time domain, the frequency and temperature dependent viscoelastic behavior was modeled using the fractional derivative approximation. In the hybrid model analysis, it is possible to see that the efforts of the derivative control are considerable in this application and the different types of control, when combined, result in a good improvement of the flutter envelope.*

**Keywords:** Aeroelasticity in time domain, Flutter, PD Control, Viscoelasticity, Fractional derivative model

## INTRODUCTION

The Brazilian aeronautical legislation in line with the American and European aeronautical legislation, brings airworthiness requirements in the area of aeroelasticity in order to promote the safety of aircraft. The airplane should be free of *flutter*, control reversion and divergence at all speeds within and sufficiently beyond the structural design envelope, in any configuration and operating condition, taking into account the critical degrees of freedom, failures of any nature and critical malfunctions. It is further required that the manufacturer determine tolerances for all quantities affecting flutter ANAC [2019]. Therefore, it is of utmost importance not only to study but to develop flutter analysis procedures in order to meet the design requirements, the legislation requirements, and to prevent future accidents from occurring.

Most studies aimed at mitigating aeroelastic phenomena focus on the implementation of control techniques, which can assume a passive or an active approach. The passive control can be reached by means of mass balancing, structural modification, insertion of energy dissipation mechanisms, stiffness increase, etc. In despite of the attractiveness they may impair the performance of the aircraft, add mass to the structure, as well as bring complications in the design phase. Among the strategies used to provide passive control, this work highlights the use of viscoelastic materials, which have the ability to dissipate vibration energy in the form of heat (Jones [2001], Martins et al. [2017]). This type of material has been frequently used in the construction, aeronautics, aerospace, and automobile industries. The mathematical model used to represent the mechanical behavior of viscoelastic material is based in a derivative fractional model, detailed in the paper of Galucio et al. [2004].

The suppression of aeroelastic flutter instability without penalty addition of mass, makes possible the best performance of modern aircraft and its ability to operate safely to the limit of its flight envelope. All active flutter suppression requires sensors to detect the movement of the control surface and to activate it accordingly with a pre-defined control law. In this statement lies the problem of positioning sensors and actuators. In addition, the cost of maintenance, assembly and operation is relatively high, which demands investment in sources of alternative power (Casciati et al. [2006]). When active control is considered in aeroelastic models, the term aeroservoelasticity comes into the picture (Wright and Cooper [2014]), where the forces coming from the control devices also interact with the other loads acting on the wing. The PD controller (proportional and derivative) is highlighted in this job. Despite its simple formulation, this type of controller allows the analysis of aeroelastic phenomena involving control surfaces and other types of aeroelastic interactions.

Thus, the motivation of this study lies in the fact that, after understanding the advantages and disadvantages of each of the control techniques mentioned, it becomes important to investigate their combined action, in order to prevent or postpone the onset of the *flutter* phenomenon. Specifically, this work deals with the hybrid aeroelastic problem built in the time domain, an approach not very common, since aerodynamic models are described as a function of reduced frequency. In this approach it is possible to visualize the displacements of the model, as well the controller efforts and, with this information, make assumptions and analyses that, in the frequency domain, can not be done on a direct way.

## AERO-SERVO-VISCOELASTIC MODELLING IN TIME DOMAIN

The temporal analysis of dynamic instability has, like any method, its advantages and disadvantages. The analysis in the time domain allows the visualization of the response in displacement, velocity and acceleration of the generalized coordinates involved, allows the study of single or multiple arbitrary inputs, varying or not with time, and can also be appropriate for the association of control laws, given the representation in state space. However, if necessary, the expansion in state space doubles the order of the system, making the calculations more expensive from a computational point of view.

The model used in this paper is based on the one presented in the work of Martins et al. [2022], and it is showed in Eq. 1:

$$[A]\{\ddot{q}(t)\} + \left(\frac{1}{2}\rho V[B] + [D] - [F]\right)\{\dot{q}(t)\} + \left(\frac{1}{2}\rho V^2[C] + [E] + p_b[E_v(\omega, T)] - [G]\right)\{q(t)\} = \{0\}, \quad (1)$$

where  $[A]$ ,  $[D]$  and  $[E]$  are respectively, the inertia, damping and stiffness matrices of the wing model. The  $q(t)$  represents the generalized coordinates of the problem.

The matrices  $[B]$  and  $[C]$  are related to aerodynamic model and  $\rho$  represents the air density while  $V$  is the airflow velocity. For simplification purposes, the Hancock Aerodynamic Model is adopted ([Hancock et al., 1985]), in which the aerodynamics are not time-dependent. Hence, it is not necessary to apply any procedure on the aerodynamic matrix to use in time domain analysis. The matrices  $[F]$  and  $[G]$  carries out the information about the proportional and derivative control, including the gains  $k_v$  and  $k_d$ , respectively.

The Matrix  $[E_v(T, \omega)]$  represents the viscoelastic material, which have temperature and frequency dependency. The viscoelastic geometric factor  $p_b$  represents the form factor of the viscoelastic device, which can be constructed in several different ways.

All the constructive details of the above described matrices can be better understood in Martins et al. [2022].

### Fractional derivative viscoelastic model

The approach suggested by Galucio et al. [2004] was chosen to represent the viscoelastic model. It suggests that the stress  $\sigma(t)$  in viscoelastic material could be expressed as Eq. 2:

$$\sigma(t) = \left[1 + v \frac{E_\infty - E_0}{E_\infty}\right] E_0 \varepsilon(t) + v E_\infty \sum_{j=1}^{n_p} A_{j+1}(\xi) \bar{\varepsilon}(t - j\Delta t). \quad (2)$$

The non-dimensional coefficient is  $v = \tau_r^\xi / (\tau_r^\xi - \Delta t^\xi)$ , where the non-dimensional time is  $\tau_r$  and the low and high material frequency modules are indicated by  $E_0$  and  $E_\infty$ , respectively. The strain is  $\varepsilon(t)$ . The variable  $\xi$  is the order of fractional derivative. The  $n_p$  is the number of points chosen to the summation. The Grünwald coefficients could be calculated as Eq. 3.

$$A_{j+1}(\xi) = \frac{\Gamma(j - \xi)}{\Gamma(-\xi)\Gamma(j+1)} = \frac{j - \xi - 1}{j} A_j; \quad (3)$$

where  $A_1(\xi) = 1$ .

Analyzing Eq.2, it can be said that the deformation of the viscoelastic material, fixed at any instant of time, has more to do with the recent history of its behavior than with occurrences that took place earlier. Such a deformation can be incorporated into traditional equations of motion where the deformations are associated with stiffness, and the portions calculated in the previous time (Grünwald terms) can be associated with external forces, since their value from the previous iteration is already known, as shown in the equation below.

$$[A]\{\ddot{x}(t)\} + [D]\{\dot{x}(t)\} + \left[E + \left(1 + v \frac{E_\infty - E_0}{E_\infty}\right) p_b [E_{v,0}]\right]\{x(t)\} = f(t) - v \frac{E_\infty}{E_0} \sum_{j=1}^{n_p} A_{j+1} p_b [E_{v,0}] \bar{x}(t - j\Delta t) \quad (4)$$

where,  $[E_{v,0}]$  is the stiffness of the viscoelastic material, equivalent to a zero frequency condition.

## NUMERICAL SIMULATION IN TIME DOMAIN

The hybrid model is associated with a a straight and homogeneous wing with a control surface coupled to the trailing edge, which is characterized in Fig. 1. The related properties of the wing are listed in Tab. 1.

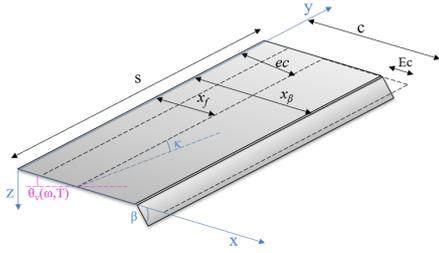


Figure 1: Representation of three-dimensional simplified wing model.

Parameter	Value [Unit]
Span ( $s$ )	7,5m
Chord ( $c$ )	2m
Surface control length ( $E$ )	0,10c
Elastic axis ( $x_f$ )	0,48c
Aero and elastic axis difference ( $ec$ )	0,5c
Mass distribution	200kg/m <sup>2</sup>
Flexural stiffness ( $EI$ )	2.10 <sup>7</sup> Nm
Torsional Stiffness ( $GJ$ )	2.10 <sup>6</sup> Nm
Curve Slope ( $a_w$ )	2 $\pi$

Table 1: Geometrical and physical parameters of the rectangular wing used in the numerical simulation.

The temporal solution is given through the integration of the equation of motion in state space ( Eq.4). To this end, the 4th order Runge-Kutta method associated with an initial condition of vertical displacement of 10cm and other null initial conditions was used. With this methodology, it is possible to obtain the displacements and velocities, as a function of time. This behavior can be characterized for each speed of interest in the flight envelope.

Four simulated cases are defined, in order to compare the proposed control types acting separately and in combination:

- Case 1 = Aeroelastic: Aeroelastic wing without control of any nature;
- Case 2 = Aeroservo: Aeroelastic wing with active control acting on the gains  $k_v = -0.04$  and  $k_d = 0.4$ ;
- Case 3 = Aeroviscoelastic: Aeroelastic wing with passive control with  $p_b = 0.1m$  and  $T = 300K$ ;
- Case 4 = Hybrid: Cases 2 and 3 combined.

As can be seen in Figure 2, the oscillations are shown to be decreasing for all observed control types, in the vicinity of the critical speed, which is equivalent to 82,2m/s.

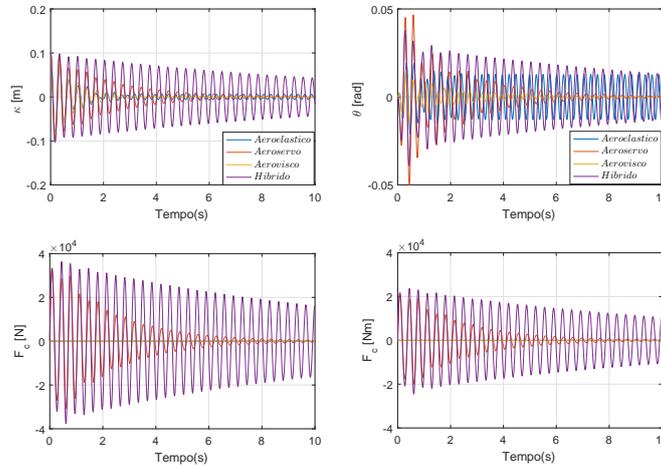


Figure 2: Comparison of cases at the aeroelastic critical speed equivalent to 82,2m/s.

It can be indicated that the critical condition of the hybrid model is approximately 106m/s, since in this situation the amplitudes do not show significant increase in the time period analysed. Similarly, the aeroservoelastic model with the gains shown in case 2 equates to a critical speed of approximately 95.3m/s. The investigated aeroservoelastic model showed a velocity around 98.5m/s when the same process was performed. It is also valid to note that the amplitudes for the aeroservoelastic model are larger than those observed for the aeroviscoelastic model. However, for the hybrid case, the amplitudes are larger still, despite the stabilization tendency of the model. With respect to controller effort, it is again observed that the hybrid control values are above the active control and that both present high values in modulus. In this way, the observation regarding the proportional active control, which, as seen in previous simulations, is the one that carries a large portion of the controller's effort, is again valid. In this way, a hybrid model with only the derivative active control is proposed, and the simulations are repeated and illustrated in Figure 3.

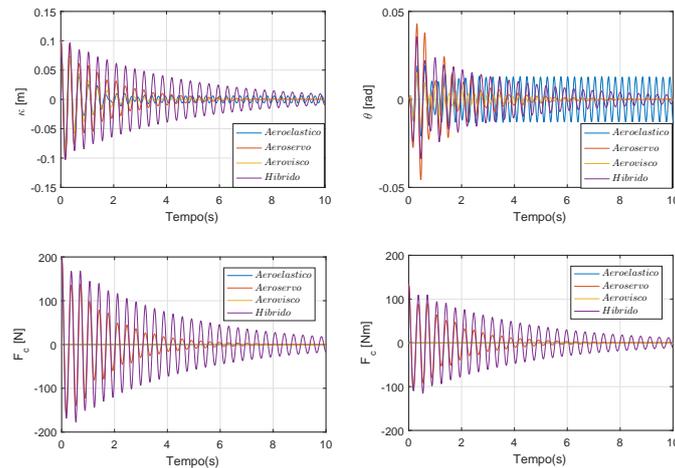


Figure 3: Comparison of the cases at the aeroelastic critical velocity equivalent to 82.2m/s without proportional active control.

It is important to highlight that the presented results are associated with a single aeroelastic physical model, i.e. one cannot generalize or extrapolate such a statement to all models that can be simulated. Due emphasis should be given to the aero-viscoelastic analysis model built in the time domain, which makes it possible to evaluate various configurations of hybrid control, and given the specific characteristics of each model, to infer specific conclusions.

## CONCLUSION

The hybrid model was built and implemented taking into account the peculiarities of each type of control applied, as well as the representation of aerodynamics in the time domain. The time domain approach is complementary to the classic aeroelastic model in the frequency domain, since only in the time domain it is possible to visualize oscillation amplitudes and controller efforts directly. In contrast, the analyses are done for one velocity value at time. The proposed hybrid model proved to be efficient when compared with other types of disassociated control strategies. Even presenting larger oscillation amplitudes, the system is stabilized in conditions in which the passive and active systems could not achieve this goal when applied separately.

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