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TURBULENCE MODELING FOR DENSE TWO-PHASE FLOW

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Abstract. *The phenomenon that we study is related to the concept of turbulence modeling and viscosity in a multiphase flow. We want to convey qualitatively and quantitatively what we understand about the concept of modified viscosity when particles are inserted into a fluid. Beforehand, we need to state some preponderant concepts for the topic investigated. It is necessary to differentiate between viscosity modification and modified viscosity. It is important that before we deal with multiphase flows, we need to define something more fundamental—what is the molecular viscosity of a single-phase flow? As the name suggests, molecular viscosity originates at a level below that perceived by continuum mechanics—that is, at the mesoscale. However, its effects are transmitted and detected at the macroscopic level.*
As a premise, below any computational grid, the fluid is a continuous medium, and as such, there are friction effects. These effects come from molecular shocks due to relative motion between the fluid molecules. Theses molecular shocks promote fluctuations in the velocity field. The effects of molecular fluctuations on the macroscopic level are not calculated precisely. For a macroscopic analysis, we model as an average behavior the velocity field and thermodynamic variables of the fluid molecules. Thus, the role of molecular viscosity is to represent effects that originate at the mesoscale but are perceived at the macroscopic level. The molecular viscosity makes up the viscous stresses, which represents a force per unit area that opposes fluid flow. We present a mathematical methodology for turbulent flows that transform point particles (2-way formulation) into point densities (3-way formulation). When comparing the turbulent kinetic energy with the result of physical experiment, we noticed a high agreement with the computational results of the proposed modeling.

Keywords: *Multiphase flow, Multiphase flow modeling, Turbulence Model, bubble induced turbulence*

1. INTRODUCTION

When we make an observation about some phenomenon of nature and raise some questions about the fundamentals of the physics involved, we have as a next step the establishment of hypotheses to explain questions that have not yet been well clarified in the literature. Subsequently, we have one more big question to be answered: are the hypotheses adopted to describe the real phenomenon? The concept of reality is an idealization, and as scientists we must seek it through the scientific method. The phenomenon of nature that we observe and will address is related to the concept of viscosity in a multiphase flow. We want to convey qualitatively and quantitatively what we understand about the concept of modified viscosity when particles are inserted into a fluid.

Beforehand, we need to clarify some preponderant concepts for the topic investigated. It is necessary to differentiate between viscosity modification and modified viscosity. But it is convenient that before we deal with multiphase flows, we need to define something more fundamental - what is the molecular viscosity of a single-phase flow? As the name suggests, molecular viscosity originates at a scale below that perceived by continuum mechanics - that is, at the mesoscale. However, its effects are transmitted and detected at the macroscopic level (continuum mechanics). We know that below the lowest level we are considering for modeling the fluid as continuous (Lifshitz and Pitaevskii, 1995; Reif, 2009), there are friction effects. These effects come from molecular shocks that occur due to internal processes in this particle of size $\mathrm{d}V$. The relative motion between the fluid molecules (Landau and Lifshitz, 2013) generates molecular shocks that promote fluctuations in the velocity field of the molecules. The effect of molecular fluctuations on the macroscopic level are not calculated precisely. For a macroscopic analysis, we model as an average behavior the velocity field and thermodynamic variables of the fluid molecules. Thus, the role of molecular viscosity is to represent effects that originate at the mesoscale but are perceived at the macroscopic level. The molecular viscosity makes up the viscous stresses, which represents a force per unit area that opposes fluid flow.

In two-phase Euler-Lagrange modeled flow, the relative movement of particles induces fluctuations in the Eulerian field (velocity, pressure and temperature) (Lance and Bataille, 1991). Therefore, the above-mentioned fluctuations must be distinguished from the fluctuations induced by turbulence in order to better understand and model the physical behavior

of this kind of complex flow.

In order to bring the influence of the volumetric fraction to the continuous phase of the filtered N-S equations, we need to distinguish between the fluctuations promoted by the relative motion of the particles within the Eulerian field and the fluctuations promoted by the turbulence itself. Consequently, the decomposition of the Eulerian velocity field would have three components. The first one, $(\bar{\mathbf{u}})$, is the velocity field calculated by using the LES (Large Eddy Simulation) methodology. The second part, $\mathbf{u}'$, represents the conventional velocity fluctuation field promoted by turbulent effects. The third part, $(\mathbf{u}'')$, is the fluctuation that occurs when there is a relative velocity between the Lagrangian discrete phase and the Eulerian continuous phase.

We compared the proposed methodology with the 2-way formulation. The results we found were significantly more accurate for turbulence modeling.

2. METHODOLOGY

In this section, we describe the methodology used for the deduction and modeling of the thermo-fluid dynamic equations for two-phase flow in the dense regime, under the concept of Euler-Lagrange modeling.

2.1 Indicator function and filtering properties

In two-phase flows, the indicator function, χ, is defined as a parameter that maps the entire domain and has a value of 1 in phase k and 0 in second phase (Drew, 1983). It is given by

$$\chi^k(\mathbf{x},t) = \begin{cases} 1 & \text{if} \quad \mathbf{x} \in k \text{ phase in time t} \\ 0 & \text{if} \quad \mathbf{x} \in \text{other phases,} \end{cases}$$

where t is the time and vector $\mathbf{x}$ has components x_1, x_2, and x_3.

The filtering of a function corresponds to the convolution integral in the entire domain, between a smooth filtering kernel g (compactly supported) and the function to be filtered. The filtering of the indicator function, which is mathematically represented by brackets symbols, is equal to the volume fraction (ε^k) of the selected phase.

$$\langle\chi^k\rangle(\mathbf{x},t) = \int_{\Omega_\infty} g(\mathbf{x}-\mathbf{y})\chi^k(\mathbf{y},t)d^3\mathbf{y} = \varepsilon^k(\mathbf{x},t), \tag{1}$$

where $\mathbf{y}$ is the vector with components y_1, y_2, y_3; $d^3\mathbf{y}$ the volume element $dy_1dy_2dy_3$ and Ω_∞ being the three-dimensional integration volume.

Before proceeding with the filtering of the equations, it is important to present the necessary filtering properties for all subsequent deductions, as introduced by Drew (1983); Ishii and Hibiki (2010); Sagaut (2006):

$\langle\chi^k a\rangle(\mathbf{x},t) = \int_{\Omega_\infty} g(\mathbf{x}-\mathbf{y})\chi^k(\mathbf{y},t)a(\mathbf{y},t)d^3\mathbf{y}$; $\quad \widetilde{a} = \frac{\langle\chi^k a\rangle}{\varepsilon^k}$; $\quad \bar{a} = \frac{\langle\chi^k f a\rangle}{\varepsilon^k \widetilde{f}^k}$; $\quad \langle a+g\rangle = \langle a\rangle + \langle g\rangle$;

$\langle\langle a\rangle g\rangle = \langle a\rangle\langle g\rangle$; $\quad \left\langle \frac{\partial a}{\partial t} \right\rangle = \frac{\partial\langle a\rangle}{\partial t}$; $\quad \left\langle \frac{\partial a}{\partial x_i} \right\rangle = \frac{\partial\langle a\rangle}{\partial x_i}$.

Where the symbol $\langle a\rangle$ represents the filtered variable a, and $(\widetilde{f}^k)$ is some physical property of phase k, like viscosity or density.

It is worth mentioning that in this work, we use index notation together with Einstein's sum convention. Therefore, x_i symbolizes the components of the position vector in three-dimensional space, with $i = 1, 2$ or 3.

2.2 Filtered continuity equation

The continuity equation for dense two-phase flow is modified by the volumetric fraction of the continuous phase. This procedure is achieved through the identification of the position of the Lagrangian particles within the evaluated domain. Furthermore, it is precisely at this point that we demand to use the concept of indicator function and its total derivative, which is null (Drew, 1983). The modified continuity equation for dense two-phase flow:

$$\begin{aligned} \frac{\partial}{\partial t}(\varepsilon^k\widetilde{\rho}^k) + \frac{\partial}{\partial x_i}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i) &= \left\langle \rho(u_i - v_i)\frac{\partial\chi^k}{\partial x_i} \right\rangle \\ &= -\int_{\partial\Omega^k} g(\mathbf{x}-\mathbf{y})\rho^k(u_i - v_i)^k n_i d^2\mathbf{y} \\ &= -\Gamma, \end{aligned} \tag{2}$$

where

$$\Gamma = \int_{\partial\Omega^k} g(\mathbf{x}-\mathbf{y})\rho^k(u_i - v_i)^k n_i d^2\mathbf{y},$$

and n_i is the components of the normal vector. This surface integral represents the sum over the closed contour ($\partial\Omega^k$) of all particles present in the domain.

2.3 Filtered Navier-Stokes equation

The filtering of the Navier-Stokes equation will be here introduced in its most general form, without any simplification hypothesis. Therefore, in terms of its components, the equation becomes:

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{3}$$

where σ_{ij} is the components of the stress tensor.

$$\sigma_{ij} = -p\delta_{ij} + 2\mu S^d_{ij}, \tag{4}$$

where p is the pressure, δ_{ij} is the components of the Kronecker delta tensor, μ is the molecular dynamic viscosity, and $S^d_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_l}{\partial x_l}\delta_{ij}\right)$ is the components of the deviatoric stress tensor.

To filter the Navier-Stokes equations, we will follow the same procedure applied to the continuity equation to identify the presence of particles in the domain. That is, initially we need to multiply the whole equation by the indicator function. Then, we add a term on the left-hand side that makes it possible to rewrite it in its divergent form:

$$\chi^k \frac{\partial}{\partial t}(\rho u_i) + \chi^k \frac{\partial}{\partial x_j}(\rho u_i u_j) + \rho u_i \left[\frac{\partial \chi^k}{\partial t} + v_j \frac{\partial \chi^k}{\partial x_j}\right] = \chi^k \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{5}$$

that after using the derivative product rule, we have:

$$\frac{\partial}{\partial t}(\chi^k \rho u_i) + \frac{\partial}{\partial x_j}(\chi^k \rho u_i u_j) = \frac{\partial}{\partial x_j}(\chi^k \sigma_{ij}) - \sigma_{ij}\frac{\partial \chi^k}{\partial x_j} + u_i \rho (u_j - v_j)\frac{\partial \chi^k}{\partial x_j}. \tag{6}$$

We can now apply filtering in Eq. (6) and use the filtering properties listed in Sec. 2.1

$$\frac{\partial}{\partial t}(\varepsilon^k \widetilde{\rho}^k \bar{u}_i) + \frac{\partial}{\partial x_j}(\varepsilon^k \widetilde{\rho}^k \overline{u_i u_j}) = \frac{\partial}{\partial x_j}(\varepsilon^k \widetilde{\sigma}_{ij}) - \left\langle \sigma_{ij}\frac{\partial \chi^k}{\partial x_j}\right\rangle + \left\langle u_i \rho (u_j - v_j)\frac{\partial \chi^k}{\partial x_j}\right\rangle. \tag{7}$$

Now we need to discuss the last two terms in Eq. (7). The second term in the RHS of such equation can be written as follows:

$$\begin{aligned}\left\langle \sigma_{ij}\frac{\partial \chi^k}{\partial x_j}\right\rangle &= \left\langle (-p\delta_{ij} + \tau_{ij})\frac{\partial \chi^k}{\partial x_j}\right\rangle \\ &= -p_s \frac{\partial \varepsilon^k}{\partial x_i} - \int_{\partial\Omega^k} g(\mathbf{x}-\mathbf{y})[(p_s - p)\delta_{ij} + \tau_{ij}]^k n_j d^2\mathbf{y}. \\ &= -p_s \frac{\partial \varepsilon^k}{\partial x_i} - M^d_i,\end{aligned} \tag{8}$$

where p_s is the average interfacial pressure (Evrard *et al.*, 2019; Drew, 1983). The last term of Eq. (8) represents the transfer of linear momentum between the Eulerian phase and the Lagrangian phase.

The term $M^d_i = \int_{\partial\Omega^k} g(\mathbf{x}-\mathbf{y})[(p_s - p)\delta_{ij} + \tau_{ij}]^k n_j d^2\mathbf{y}$ is the interfacial force density (Drew, 1983). It is worth mentioning that the surface integral of Eq. (8) represents the sum over the closed contour of all particles present in the domain.

We can also refer to the last term of the RHS of Eq. (7) the same form that we have used for integral of Eq. (8). Where the volumetric integral defined over the whole domain, it is now only calculated in the interface between both phases.

$$\begin{aligned}\left\langle u_i \rho (u_j - v_j)\frac{\partial \chi^k}{\partial x_j}\right\rangle &= \int_{\Omega_\infty} g(\mathbf{x}-\mathbf{y})\left(u_i \rho (u_j - v_j)\frac{\partial \chi^k}{\partial x_j}\right) d^3\mathbf{y} \\ &= -u_{s,i}\int_{\partial\Omega^k} g(\mathbf{x}-\mathbf{y})\rho^k (u_j - v_j)^k n_j d^2\mathbf{y} \\ &= -u_{s,i}\Gamma,\end{aligned} \tag{9}$$

where $u_{s,i}$ is the average interfacial velocity of the k phase.

Finally, by adding Eqs. (8) and (9) to Eq. (7), we give rise to the following expression:

$$\begin{aligned}\frac{\partial}{\partial t}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i)+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\rho}^k\overline{u_iu_j})&=\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\sigma}_{ij})+p_s\frac{\partial\varepsilon^k}{\partial x_i}+M_i^d-u_{s,i}\Gamma\\&=-\varepsilon^k\frac{\partial\widetilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\tau}_{ij})+(p_s-\widetilde{p})\frac{\partial\varepsilon^k}{\partial x_i}+M_i^d-u_{s,i}\Gamma.\end{aligned}\tag{10}$$

Equation (10) can be seen as the filtering of the Navier-Stokes equations for two-phase flows. However, we need to decompose its nonlinear term, which refers to turbulence closure problem in order to turn this equation solvable. Now this is no longer single-phase flow, and we need to contemplate one more feature on the modeling since the presence of the particles promotes a change in the apparent viscosity (Einstein, 1906; Gibilaro *et al.*, 2007; Ishii and Hibiki, 2010; Capecelatro and Desjardins, 2013).

The massive presence of particle, droplets, or bubbles in dense two-phase flows will significantly affect the nonlinear dynamics of information transfer between the turbulent structures that compose a turbulent spectrum. The way to model this information transfer process must be seen carefully when compared with what occurs in diluted or even in single-phase turbulent flows. In the next section, this issue is discussed.

2.3.1 Velocity field decomposition

Several studies have supported the idea proposed and discussed in the present work about the triple decomposition of the velocity and temperature (Sato and Sekoguchi, 1975; Sato *et al.*, 1981a,b; Lance and Bataille, 1991; Balachandar and Eaton, 2010).

With regard to the correlations involving the velocity field, the approach applied in the present work to deal with the closure problem in two-phase flow is analogous to that used for single-phase flow, but brings out some very important complementary aspects. The main difference is that we now want to consider the influence of the Lagrangian phase on the Eulerian field due to relative movement between the particle and the Eulerian field. The proposed idea is that this influence is perceived both in the velocity and in the temperature fields. The velocity field is assumed to be divided into three components. For the velocity field, that is being handled in this section, we have:

$$u_i=\bar{u}_i+u_i'+u_i'',\tag{11}$$

where $\bar{u}_i$: is the filtered Eulerian velocity field, u_i' : corresponds to the fluctuation of the Eulerian velocity field, promoted by turbulence itself, u_i'' : it is the fluctuation of the Eulerian velocity field promoted by the relative movement between the Lagrangian phase and the Eulerian velocity field.

It is noteworthy that (u_i') is different from zero, only if the flow is turbulent and (u_i'') can exist even in laminar or transitional flows. We now turn to discuss the origin of the turbulence closure problem in two-phase flows. Initially, Eq. (11) is introduced in Eq. (10), and after some manipulations, that can be seen in the appendix, we have:

$$\frac{\partial}{\partial t}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i)+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i\bar{u}_j)=-\varepsilon^k\frac{\partial\widetilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\tau}_{ij})-\frac{\partial}{\partial x_j}[\varepsilon^k\widetilde{\rho}^kT_{ij}]+(p_s-\widetilde{p})\frac{\partial\varepsilon^k}{\partial x_i}+M_i^d-u_{s,i}\Gamma,\tag{12}$$

where $T_{ij}=(\overline{u_iu_j}-\bar{u}_i\bar{u}_j)$, which can be conveniently rewritten as the sum of terms:

$$\begin{aligned}T_{ij}&=\overline{u_iu_j}-\bar{u}_i\bar{u}_j\\&=\overline{(\bar{u}_i+u_i'+u_i'')(\bar{u}_j+u_j'+u_j'')}-\bar{u}_i\bar{u}_j\\&=R_{ij}^p+L_{ij}+C_{ij}+R_{ij}^t\\&=R_{ij}^p+\tau_{ij}^t,\end{aligned}\tag{13}$$

where, $R_{ij}^p=\overline{u_i''u_j''}$ represents the filtering of the fluctuations that the particles cause in the Eulerian field, $L_{ij}=\overline{\bar{u}_i\bar{u}_j}-\bar{u}_i\bar{u}_i$ are the components of the Leonard tensor for two-phase flows, $C_{ij}=\overline{\bar{u}_iu_j'}+\overline{u_i'\bar{u}_j}+\overline{u_i''\bar{u}_j}+\overline{\bar{u}_iu_j''}+\overline{u_i'u_j''}+\overline{u_i''u_j'}$ are the components of the cross tensor for two-phase flows. It represents all possible combinations for the interactions between the fluctuations of the Eulerian field, fluctuations promoted by the particle, and the calculated Eulerian field, $R_{ij}^t=\overline{u_i'u_j'}$ are the components of the sub-grid Reynolds stress tensor, and $\tau_{ij}^t=L_{ij}+C_{ij}+R_{ij}^t$ are the components of the sub grid turbulence.

Thus, replacing the terms of Eq. (13) in Eq. (12), results the following open equation:

$$\frac{\partial}{\partial t}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i)+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i\bar{u}_j)=-\varepsilon^k\frac{\partial\widetilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\tau}_{ij})-\frac{\partial}{\partial x_j}[\varepsilon^k\widetilde{\rho}^kR^p_{ij}]-\frac{\partial}{\partial x_j}[\varepsilon^k\widetilde{\rho}^k\tau^t_{ij}]+ \\ +(p_s-\widetilde{p})\frac{\partial\varepsilon^k}{\partial x_i}+M^d_i-u_{s,i}\Gamma. \tag{14}$$

The modeling of the components of tensors R^p_{ij} and τ^t_{ij} must be different because these tensors play different roles from a physics point of view. As already demonstrated by Einstein (1906), the presence of particles somehow affects the flow viscosity. For this case, an useful mathematical hypothesis is that the viscosity is proportional to the viscous shear stress, as shown in the following equation:

$$-\varepsilon^k\widetilde{\rho}^kR^p_{ij}=2\varepsilon^k\mu'\bar{S}^d_{ij}. \tag{15}$$

Modeling the modified viscosity (μ') by the presence of a discrete phase proportional to the viscous stress tensor implies the approximation that the spatial distribution of the Lagrangean phase is homogeneous and isotropic (Batchelor, 2000). Besides, this viscosity also varies according to the particles present in the two-phase flow, as shown by Ishii and Hibiki (2010) and Ishii and Zuber (1979). Thus, we propose a method to model the modified viscosity for two-phase flows, given by:

$$\mu'=\widetilde{\mu}^k[(\varepsilon^k)^{-n-1}-1], \tag{16}$$

where μ^k is the molecular viscosity of Eulerian phase and

$$n=\begin{cases}1 & \text{for bubbly flow}\\ 1.75 & \text{for droplet in liquid}\\ 2.5 & \text{for droplets in gas and particulate flow.}\end{cases}$$

Finally, it is necessary to model another essential term, the sub grid turbulence (τ^t_{ij}). Initially, this tensor will be modeled through using the classic Smagorinsky closure model (Smagorinsky, 1963):

$$-\varepsilon^k\widetilde{\rho}^k\tau^t_{ij}=2\varepsilon^k\mu_t\bar{S}^d_{ij}, \tag{17}$$

where μ_t is the turbulent viscosity.

The modeling we propose for the components R^p_{ij} and τ^t_{ij} only differs from the components of viscosity stress tensor $\widetilde{\tau}_{ij}$ in its coefficient. This characteristic allows one to have the equation for turbulent two-phase flow in the dense regime while substituting Eq. (15) and (17) in Eq. (14).

$$\frac{\partial}{\partial t}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i)+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i\bar{u}_j)=-\varepsilon^k\frac{\partial\widetilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left[2\varepsilon^k(\widetilde{\mu}^k+\mu'+\mu_t)\bar{S}^d_{ij}\right]+(p_s-\widetilde{p})\frac{\partial\varepsilon^k}{\partial x_i}+ \\ +M^d_i-u_{s,i}\Gamma. \tag{18}$$

In summary, the modeling that we propose for turbulent two-phase flows in the dense regime will have an effective viscosity (μ_{ef}), which can be interpreted as the sum of the molecular viscosity of the k phase $(\widetilde{\mu}^k)$, the modified viscosity due to the presence of the discrete phase (μ'), and the turbulent viscosity (μ_t). To put it another way: $\mu_{ef}=\varepsilon^k(\widetilde{\mu}^k+\mu'+\mu_t)$, thus:

$$\frac{\partial}{\partial t}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i)+\frac{\partial}{\partial x_j}(\varepsilon^k\widetilde{\rho}^k\bar{u}_i\bar{u}_j)=-\varepsilon^k\frac{\partial\widetilde{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left[2\mu_{ef}\bar{S}^d_{ij}\right]+(p_s-\widetilde{p})\frac{\partial\varepsilon^k}{\partial x_i}+M^d_i-u_{s,i}\Gamma. \tag{19}$$

The development of this new equation succeeds in representing new features which have not been identified in the already proven models. Some examples we can give are:

- Even for the laminar case, where the turbulent viscosity is zero, it is still necessary to model the fluctuation promoted by the Lagrangian phase, and this is done with the proposed model;
- If the flow approaches the diluted regime, whose $\varepsilon^k\to 1$, the importance of viscosity modified by the presence of the particles decreases, $\mu'\to 0$, as expected;
- In case the flow is turbulent and single-phase, we have $\varepsilon^k=1$ and $\mu'=0$. Therefore, falling into the traditional filtered equation for LES methodology.

2.4 Lagrangian Modeling

In the previous sections, we presented the mathematical formulation of the continuous phase. Now, to conclude, it is necessary to establish the equations of motion for the Lagrangian phase.

2.4.1 Equation of motion of Lagrangian particles

The particle motion equation follows Newton's second law. For dense regimes, we can neglect the pressure gradient forces, added-mass force, and Basset force (Ishii and Zuber, 1979; Evrard *et al.*, 2019). The calculation of the drag force in the two-phase flow that will be proposing takes into account spherical particles and a Viscous Regime (Undistorted-Particle Regime), where the shape of the particles don't change due to interfacial instabilities (Ishii and Hibiki, 2010). Starting from the well-known drag force and the definitions developed by Ishii and Zuber (1979), we present a new form for the components of drag force ($F_{D,i}$) in two-phase flows:

$$\begin{aligned} F_{D,i} &= \frac{1}{2} C_D A \rho^k |u_i - v_i|(u_i - v_i) \\ &= \frac{1}{8} C_D \pi d_p^2 (u_i - v_i)\rho^k |u_i - v_i| \\ &= \frac{1}{8} C_D \pi d_p \mu_m (u_i - v_i) \frac{\rho^k |u_i - v_i| d_p}{\mu_m} \\ &= 3\pi d_p \mu_m (u_i - v_i) \frac{R_{ep} C_D}{24}, \end{aligned} \tag{20}$$

where C_D, R_{ep}, A, d_p, ρ^k, u_i, v_i and μ_m are the drag coefficient, particle's Reynolds number, the projected area of a particle, particle's diameter, density of k phase, Eulerian field velocity, particle's velocity and mixture viscosity, respectively. The definitions of $C_D = 24/R_{ep}\left[1 + 0.1 R_{ep}^{0.75}\right]$, $R_{ep} = \frac{\rho^k |u_i - v_i| d_p}{\mu_m}$ and $\mu_m = \widetilde{\mu}^k (\varepsilon^k)^{-n}$ are found in Ishii and Zuber (1979); Drew and Passman (2006).

Thus, Newton's second law for the particle is:

$$m_p \frac{dv_i}{dt} = F_{D,i} + (\rho_p - \rho^k) V_p g_i, \tag{21}$$

where ρ_p and V_p, are the density and volume of the particle, respectively. The last term of the right-hand side is the force due to the gravity effects (Elghobashi and Truesdell, 1992).

An important way of writing Eq. (20), to incorporate the particle's characteristic time (τ_p), can be seen as:

$$\frac{F_{D,i}}{m_p} = \frac{(u_i - v_i)}{\tau_p} \frac{R_{ep} C_D}{24}, \tag{22}$$

where $\tau_p = \frac{\rho_p d_p^2}{18 \mu_m}$.

Note that equations Eq. (20) and Eq. (22) are valid for two-phase flow that is in viscous regime with spherical particles. Also, these equations are applied to any Reynolds number and are valid for the dense and diluted regime. It is good to remember that the value of n, present in mixture viscosity, always depends on the properties of the Lagrangian phase.

Consequently, we can establish a significant relationship between the mixture viscosity (μ_m) and the modified viscosity (μ') that we propose in this study. Such a relation is given by:

$$\mu_m = \varepsilon^k (\widetilde{\mu}^k + \mu'). \tag{23}$$

3. PHYSICAL MODEL

We will analyze a gas-liquid flow experiment in a square cross-section bubble column. The bubble column has dimensions of $0.15 \times 0.15 \times 0.6\ m^3$ (Deen *et al.*, 2001). We inject the bubbles into the center of the $0.15\ m$ square section bottom. The column has a height of $0.6\ m$. The first $0.45\ m$ of the column is filled with water and the last $0.15\ m$ is filled with air. Deen *et al.* (2001) mixed 4 g of salt per liter of water, to avoid coalescence effects. The gas particles are injected with constant velocity and the diameter of each particle is around 4 mm.

4. COMPUTATIONAL MODEL

The computational model we developed was through the MFSim software, developed at the Fluid Mechanics Laboratory - MFLab, of the Federal University of Uberlândia, with technical and financial support of Petrobras. The data for the

case setup was based on Deen *et al.* (2001). For the Eulerian field, we used a uniform mesh of $32/times32/times128$ in the three Cartesian directions. We used the finite volume method for the discretization and integration of the fluid dynamics equations. We adopted a CFL = 0.5, the SBDF method (Damasceno *et al.*, 2018) for the discretization of the temporal scheme and the Barton method for the discretization of the advective term. For pressure-velocity coupling, we use the fractional step method. The complete numerical Eulerian framework used in the present work can be seen elsewhere in Barbi *et al.* (2018); Vedovoto *et al.* (2015); Castro *et al.* (2021) and Pivello *et al.* (2014).

For the boundary conditions, we adopted non-slip conditions at the bottom of the channel and Neummann conditions for velocities at a height of 0.45 m. For the Lagrangian field, we consider a particle injection (d = 4 mm) that guarantees a flow rate of $1.35\ 10^{-4}\ m^3/s$. The boundary condition between the particles will be free-slip between them and deletion, when it reaches a height of 0.42 m. We also use the 2-way and 3-way formulation proposed by Evrard *et al.* (2019), added to our turbulence closure modeling for dense flows.

In figure 4, we present the pictorial form of our studied problem. With the VOF (Volume of fluid) methodology, we identify the interfaces between water and gas. Where the volumetric fraction (volfrac) is equals to one, it means a liquid phase, and the volumetric fraction is zero, the gas phase.

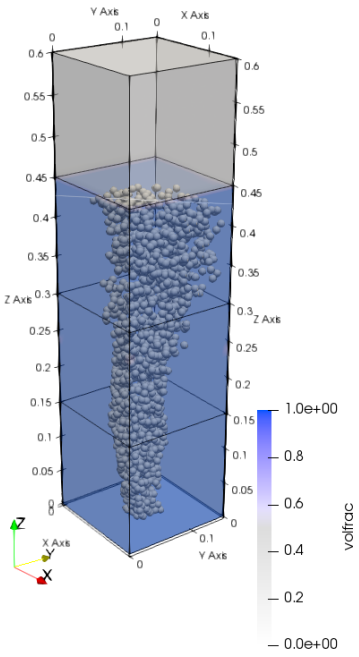


Figure 1: Vertical channel with bubbles injected at the bottom. The volumetric fraction indicates 1 when the phase is Eulerian and 0 when the phase is Lagrangian.

5. RESULTS

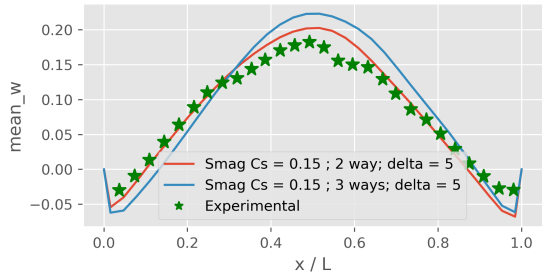
We will present the qualitative results of the impact of the proposed modeling, for dense flows in turbulent regime. The computational results are compared with the results of the material experiment presented by Deen *et al.* (2001). In Figure (2a), we have the values for the average velocity and the standard deviation – Figure (2b) – of the w component of the velocity. The red lines are the results with the 2-way formulation and the blue lines are the results for the 3-way formulation.

In figure (3a), we compare the standard deviation of the u component of velocity. In Figure (3b), we compare the computational and experimental kinetic energy. Again, we compared the solutions between the 2-way and 3-way formulation.

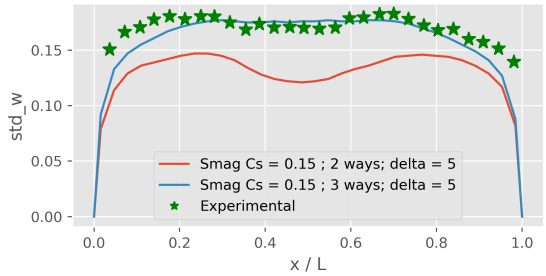
Another important representation for evaluating turbulent kinetic energy is in Fourier space. After performing a Fourier transform of the turbulent kinetic energy, we plot the graph (in log on base 10) of the spectral density of turbulent kinetic energy as a function of frequency. With this we compare the slope of the line in the inertial region.

We can see that the inertial region tends to a slope that is close to -5/3. The inertial region of the turbulent kinetic energy density spectrum coincided with the slope predicted by Kolmogorov's theory. This result is not a consensus in the literature, there are studies that estimate a slope of -25/3 in the evaluated region Liu and Li (2018).

When we seek the answer to the question about the influence of dense modeling on two-phase turbulent flows, we need to evaluate the sub-grid tensor. We know that the tensor components can be decomposed into isotropic and deviatoric (or



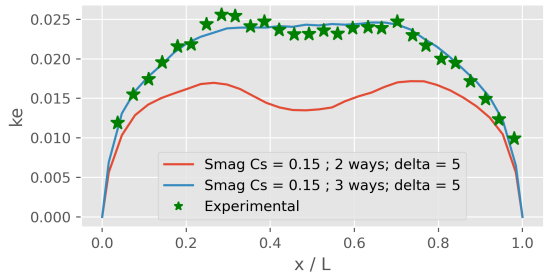
(a) mean velocity of w.



(b) standard deviation of w.

Figure 2: Comparison between experimental and computational results for the component of the velocity w.

(a) Standard deviation of u.



(b) Knetic energy.

Figure 3: Comparison between experimental and computational results for the velocity u and kinetic energy (ke).

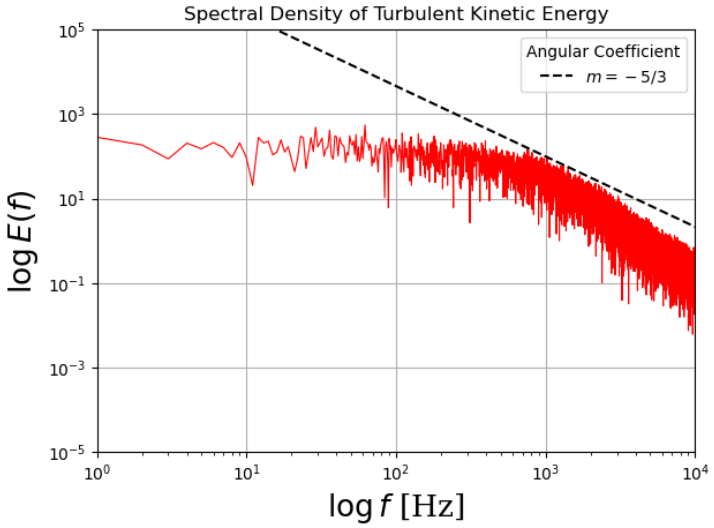


Figure 4: Spectral density of turbulent kinetic energy of the Eulerian field and the line with slope of -5/3.

anisotropic) parts.

$$T_{ij} = T_{ij}^d + \frac{1}{3}T_{pp}\delta_{ij} \tag{24}$$

$$= T_{ij}^d + \frac{2}{3}k\delta_{ij}, \tag{25}$$

where $k = T_{pp} = \frac{1}{2}\overline{u_p' u_p'}$ is the turbulent kinetic energy.

For statistically independent regimes $T_{ij}^d \ll k$ and $T_{ij}^d \to 0$ Sagaut (2006). This is an important factor in assessing whether the flow tends towards homogeneous and isotropic turbulence.

Assuming the experimental result as the ideal, through a posteriori analysis, we can extract the information of the ideal turbulent kinetic energy for the case studied. We can see from the figure 3b that the turbulent kinetic energy was close to the real value with the modeling of the dense. At the same time, it denounces how the modeling of the dense improves the approximation of the real information obtained in the material physical experiment.

As a last quantitative analysis, we can plot the sub-mesh tensor that was modeled ($\mathbf{T}$), and check if the isotropic flow hypothesis is well established. For the 3-way case 1, we have that the $\mathbf{T}$ tensor dimensionless by the largest value is

$$\mathbf{T} = \begin{bmatrix} 2,50 \times 10^{-1} & 5,86 \times 10^{-4} & 3,79 \times 10^{-3} \\ 5,86 \times 10^{-4} & 2,98 \times 10^{-1} & 4,51 \times 10^{-3} \\ 3,79 \times 10^{-3} & 4,51 \times 10^{-3} & 1,00 \end{bmatrix}.$$

In addition to the evident symmetry of the tensor, we can observe that the terms off the main diagonal, that is, the components of the anisotropic part of the tensor, are 3 and 4 orders of magnitude smaller than the tensor trace, and tend to zero. Which is a strong indication of the good modeling of the fluid dynamics of the problem.

6. CONCLUSIONS

With this study, we had an indication that the modeling we propose for two-phase flows in a dense regime is important to capture in more detail the fluid dynamics present in the problem. The methodology we used to propose closure models for the modification of viscosity, and for turbulence, proved to be satisfactory.

When we analyze the average velocity of the Eulerian field, the velocity in the axial direction has little difference between the 2-way and 3-way formulations. However, when it comes to standard deviations, that is, when we want to analyze the influence of the turbulence closure modeling, the 3-way formulation proved to be qualitatively superior to the 2-way modeling.

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