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THEORETICAL ANALYSIS OF FREE-SURFACE FLOWS IN RECTANGULAR ANNULAR CROSS-SECTION

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Abstract. *One of the main properties of free-surface flows, as for most industrial flows, is the mean flow velocity. Its estimation can be used to design machines, predict head loss in hydraulic installation, and detect equipment failures in many industrial processes. More specifically, for open-channel flows with rectangular cross-sections, this estimation is straightforward. If one establishes the geometry, the fluid thermophysical properties, and the flow rate, the flow depth and the mean flow velocity can be obtained through analytical and semi-empirical formulae from Fluids Mechanics principles. However, any study reporting the same procedure has not been reported for free-surface flows within channels with annular cross-sections, according to the authors' knowledge. In the present paper, we explore the conservation equations solution for a free-surface flow of a Newtonian fluid in a closed channel with a rectangular-annular cross-section, in steady-state and uniform regimes. The Darcy-Weisbach friction factor, combined with the Churchill equation and hydraulic-diameter approach, are used. Expressions for the flow cross-section area and the wetted perimeter of the partially full annular channel flow, both terms of hydraulic-diameter definition, are derived from trigonometric rules as a function of the flow depth. The Engineering Equation Solver (EES) software is used for calculation. We found that multiple mathematical solutions for flow depth can be found for a fixed set of entry parameters. The influence of inclination angle, flow rate, fluid viscosity, and inner diameter of the annular channel is discussed. As the ultimate result, the variation on the flow rate estimation is presented. The subject brought in the present work poses a problem for engineers and should be taken into account when estimating flow parameters in this scenario.*

Keywords: *free-surface flow, annular-channel flow, hydraulic diameter, friction factor*

1. INTRODUCTION

Flows in partially filled annular-channels are found in several industries, such as Petrochemical, Power generation, Mining, Oil & Gas, and also in rainwater networks and other natural scenarios. Concerning this specific geometry, many approaches can be identified as significant to study and better understand the dynamics of the flow, the friction imposed and possible ramifications of those. In oil production, for example, such scenario is perceived as two-phase flow in circular annular-ducts, where the duct can assume horizontal, vertical or even inclined positions. The positioning of the duct affect how the flow inside the annular duct will behave, sometimes leading to very different flow configuration. In some cases, gas separation can be studied when duct is completely filled with a fluid, characterizing a single-phase approach Vieira *et al.* (2020, 2021b,a).

For some other cases, there might be a clear line of separation between phases inside the duct, thus leading to the necessity of two-phase flow models Hernández Cely *et al.* (2018); Colmanetti *et al.* (2020); Castro and Rodriguez (2021); Barbosa and Rodriguez (2022). Evaluating the free-surface dynamics is paramount for understanding head loss along a pipeline and formation of coherent structures, among others. Several studies in stratified flow-pattern for annular geometry have been reported Hernández Cely *et al.* (2018); Colmanetti *et al.* (2020); Castro and Rodriguez (2021); Barbosa and Rodriguez (2022) which shows that such subject still rises questions and is far from completely understood. More complex scenarios have been studied, considering for example the permeability of porous walls during continuous fluid flows in partially-filled stationary or rotating annulus ducts (e.g. Jha and Yusuf (2022a,b); Poursharif *et al.* (2022)). One particular line of work is interested in partially filled flow in special conditions, such as the recent work from Bochkarev *et al.* (2019) where another degree of generality is inserted considering duct boundaries flexible, thus presenting vibrational modes when interacting with the flow.

More practical questions about the validity of models for annular flows appear for partially filled annulus. Such is the case of the inverted-shroud gas separator in the oil production for down-hole pumped wells (Vieira *et al.*, 2020, 2021b,a). Usually, in such device an important parameter for operation is directly related to the detection and knowledge of the position of the free surface, which should dependent on the balance between gravitational forces (driven forces) and frictional forces (against walls and counter-current air flow).

For different types of annular cross-section, it is possible to obtain the mean flow height and the mean flow velocity based on the liquid flow rate and slope. Classical formulations of friction factor can help deduce such parameters. Usually, they are expressed as function of the hydraulics radius, a parameter dependent on the wetted area and wetted perimeter. The present paper aims to evaluate, theoretically, based on classical friction laws, how the free surface would change as function of the discharge rate for a rectangular annular cross-section geometry.

2. ANALYTICAL FORMULATION

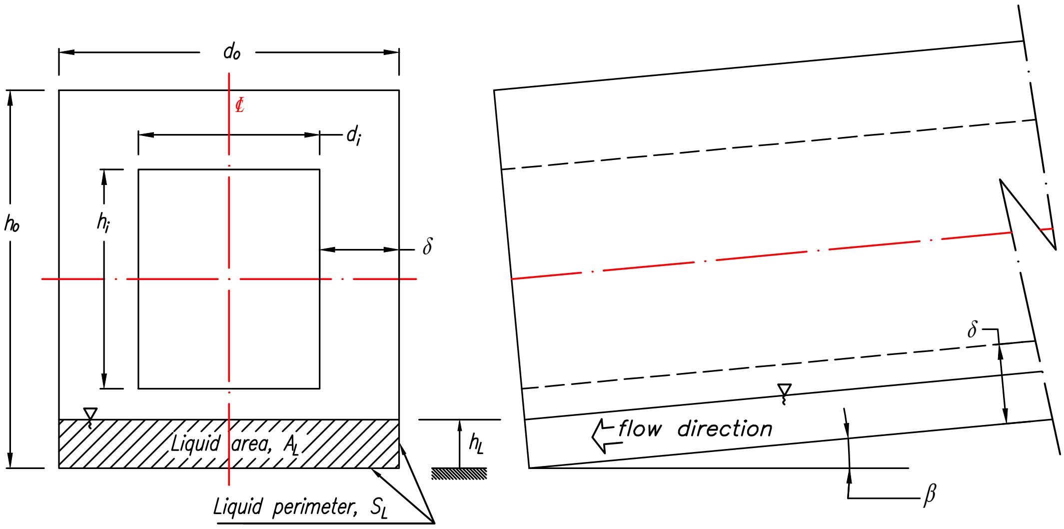

Figure 1. Schematic of a rectangular annular channel. Left: cross-section view. Right: long side view.

Consider a uniform free-surface flow through the rectangular annular channel of Fig. 1 with outer height h_o and width d_o and inner height h_i and width d_i. The geometric flow parameters h_L, A_L, and S_L represent respectively the liquid depth, the cross-section flow area and the wetted perimeter including the solid boundaries of the channel in contact with the liquid. The parameter δ is constant and represents the gap of the annular channel. The channel is inclined β degree. The system can be represented by following equations, Eqs. (1)-(5) (e.g. Çengel and Cimbala (2004)),

$$Q = A_L V \tag{1}$$

$$V = \sqrt{\frac{2gD_h \tan\beta}{f}} \tag{2}$$

$$f = 8\left(\left(\frac{8}{\mathrm{Re}}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right)^{1/12} \tag{3}$$

with $A = \left(-2.457\ln\left(\left(\frac{7}{\mathrm{Re}}\right)^{0.9} + 0.27\left(\frac{e}{D}\right)\right)\right)^{16}$ and $B = \left(\frac{37530}{\mathrm{Re}}\right)^{16}$

$$\mathrm{Re} = \frac{\rho_L D_h V}{\mu_L} \tag{4}$$

$$D_h = \frac{4A_L}{S_L} \tag{5}$$

where Q and V are the flow rate and the uniform flow velocity, respectively. D_h is the hydraulic diameter, and g is the gravity constant. The Churchill friction factor f is appropriate for all regimes ranging from laminar to turbulent flows (Çengel and Cimbala, 2004). The parameters e and Re are absolute surface roughness and Reynolds numbers, respectively. ρ_L and μ_L represent the density and dynamic viscosity of the fluid, respectively. For the case of rectangular annular cross-section, A_L and S_L can be represented as a function of liquid depth h_L, as follows:

- $h_L < \delta = \dfrac{h_o - h_i}{2}$ (6a)

then

$$A_L = h_L d_o \tag{6b}$$

$$S_L = 2h_L + d_o \tag{6c}$$

- $\delta \leq h_L < h_o/2$ (7a)

then

$$A_L = \delta\,(2h_L + d_i) \tag{7b}$$

$$S_L = 4h_L + 2d_i \tag{7c}$$

Based on the set of equations, for a established flow and channel geometry the system is solved as a function of liquid depth h_L. Thus, flow velocity can be obtained from the latter parameter.

3. RESULTS AND DISCUSSION

The equation system presented in the previous section was implemented in the Engineer Equation Solver (EES) software. The inputs are the geometrical properties of the annular duct and fluid thermodynamic properties. Calculations are performed for parameter $h_{adm} = h_L/h_0$, from 0 to 1 with a 0.01 step. The convergence parameter is velocity V. A relative residual of $< 10^{-6}$ is used. According to our simulations, Fig. 2 shows a typical graph for liquid depth versus flow rate for free-surface flow in a rectangular annular channel. We can observe multiple solutions for a generic Q_i when h_L is close to the bottom and the top of the inner duct. ΔQ_i is the variation where multiple solutions occurs and Δh_L is the correspondent liquid depth variation.

Following we present calculations for liquid depth as a function of flow rate. For simulations, it is considered water at standard conditions flowing in a square annular-channel with d_e = 65 mm. The gravity acceleration is considered to be 9.81 m/s^2 and the pipe roughness e is 1.5 μm. Figure 3 show results for three different sizes of inner duct and constant inclination angle. The values for those sizes correspond to commercial acrylic square pipes. The multiple solutions region shifts right for smaller inner duct sizes. Furthermore, the smaller the inner duct size, the higher flow rates to reach the multiple solutions region. The reason for those results is that while liquid does not reach the inner duct, the flow behaves as an open channel flow where liquid depth increases monotonically with flow rate. From a quantitatively viewpoint, the flow rate variation ΔQ_i approximately to 26 lpm, 22 lpm, and 16 lpm decreases with inner duct sizes decreasing. Finally, the liquid depth estimation can variate up to 10%.

Figure 4 show results for three different inclination angles and constant inner duct size (d_i = 25.4 mm). The multiple solutions region shifts right with the inclination angle increasing. Furthermore, the larger values of inclination angle, the higher flow rates to reach the multiple solutions region. The reason for those results is that inclination angles and liquid depth are inversely proportional, so more flow rate is needed to reach the inner duct. From a quantitatively viewpoint, the flow rate variation ΔQ_i approximately to 18 lpm, 26 lpm, 33 lpm increases with inclination increasing. Finally, the variation of the liquid depth estimation is similar to the three cases and can variate up to 10%.

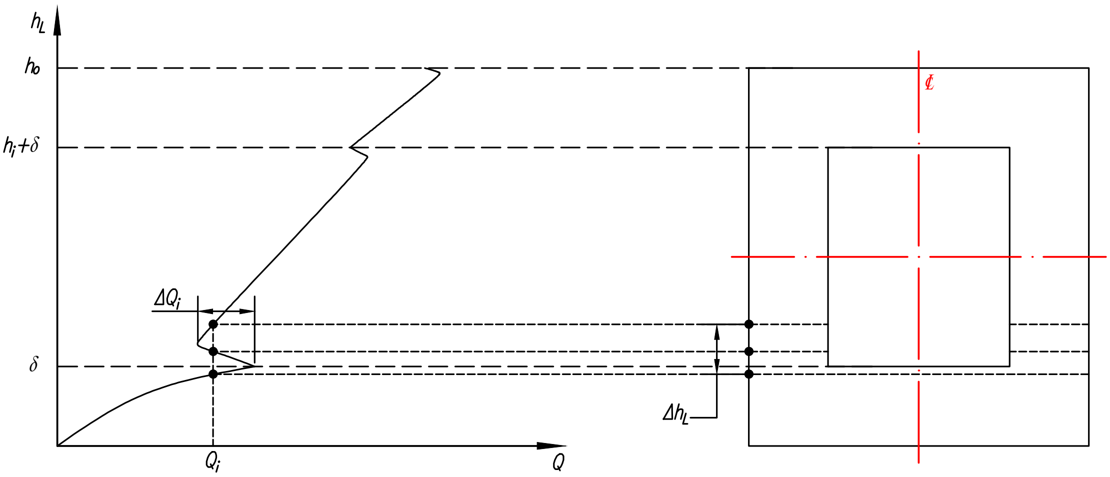

Figure 2. Schematic of multiple solutions for free-surface flow in rectangular annular channel

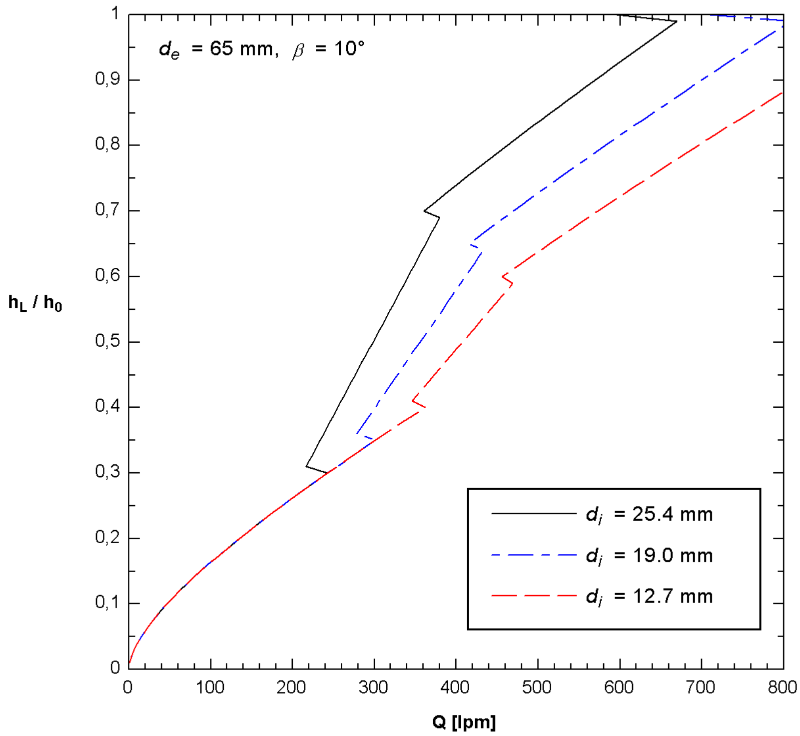

Figure 3. Liquid depth as a function of flow rate for three inner sides (d_i) and constant inclination angle

4. CONCLUSIONS

Based on simple analytical formulation, this paper brought the prediction of liquid depth as a function of flow rate for free-surface flow in square annular cross-section channels. Two main parameters of the flow, namely the inclination angle and the inner duct size, were varied to explore their effect over the flow depth. Multiples solutions (up to three) for the prediction of liquid depth were observed in this kind of geometry located around the y-positions where duct cross-section abruptly change.

In contrast with the flow in open channels with regular cross-section (not annular) where a monotonic relationship be-

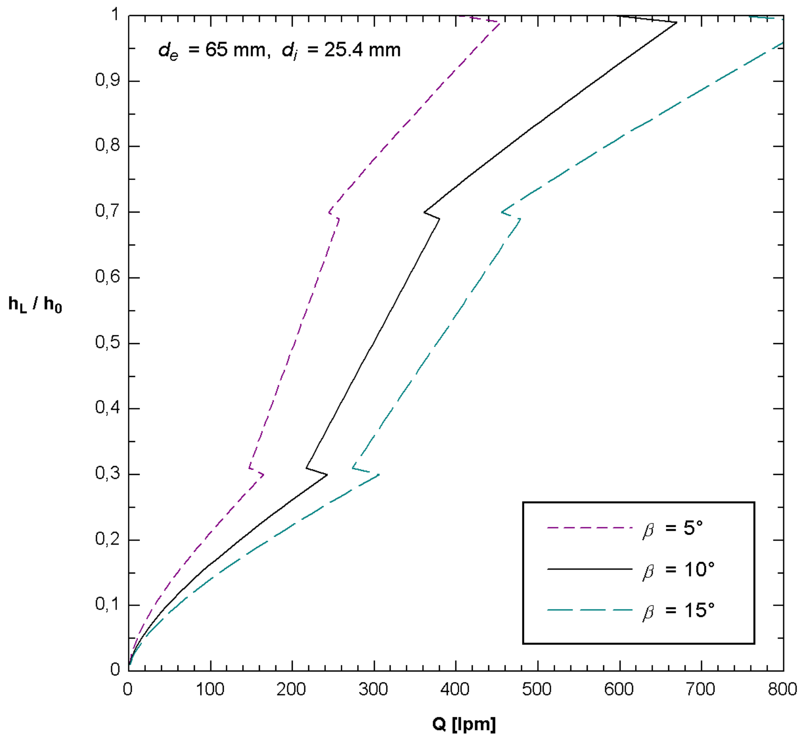

Figure 4. Liquid depth as a function of flow rate for three inclination angles (β) and constant inner side (d_i)

tween liquid depth and flow rate is found, the presence of an inner duct provokes singularities for mathematical solutions. Such singularities lead to multiple solutions of flow depth for a single value of flow discharge. For the tested cases, the liquid depth prediction can variate up to 10%. Discrepancies in flow depth might lead to increase in systematic errors, or even to dubious assessment of flow friction in applications. The authors encourage to development an experimental work to determine which solution of the multiple ones is physically possible.

5. ACKNOWLEDGEMENTS

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