19th Brazilian Congress of Thermal Sciences and Engineering
November 6th–10th, 2022, Bento Gonçalves - RS - Brazil

ENC-2022-0426
NUMERICAL ANALYSIS OF THE RHEOLOGICAL PROPERTIES EFFECTS OVER THE RE-START FLOW

Luiz Paulo Borges Miranda
Daniel Dall'Onder dos Santos
Federal University of Uberlândia, João Naves de Ávila Av., 2121, Uberlândia, MG, 38.408-100, Brazil
luiz.miranda@ufu.br, dallonder@ufu.br

Abstract. *The re-start problem is a relevant issue in several applications, specially when the non-Newtonian behaviors are considered. For example: during the oil transport, if the system shuts down and the fluid temperature falls below a certain point, the oil can present a rheological behavior that can completely block the pipeline. If at a Newtonian isothermal internal flow the Reynolds number is the main dimensionless quantity necessary to define the flow, when a non-Newtonian behavior is considered, at least one more dimensionless quantity is needed to describe the flow. Therefore, the present paper search to discuss the effects of some dimensionless quantities related to these non-Newtonian behaviors over a transient flow inside a planar channel. These dimensionless quantities describe the rheological behavior of the fluid, considering, for example, the local shear rate. The flow was solved using the open-source CFD software OpenFOAM, adopting the isothermal and the Poiseuille flow conditions, where a pressure gradient is imposed over a static flow. The present results show how these dimensionless quantities affect the time to reach the developed flow condition. Increasing the flow behavior index decreases the time to reach the developed flow condition while increasing the plastic number increases this time. These dimensionless quantities change the viscous forces profile, and therefore also change the acceleration of the flow.*

Keywords: *OpenFOAM, Re-start flow, plastic number, Poiseuille flow*

1. INTRODUCTION

The offshore oil production, and especially deepwater oil production, has shown consistent and important growth in the last decades. These operations have several critical points, and one of them is the re-start problem. When the production has to shut down, because of a failure or because of the maintenance schedule, for example, the oil within the pipes can change its rheological behavior, increasing the difficulty to re-start the flow. Several works in the literature seek to investigate this kind of flow aiming to increase the knowledge about it.

Chala *et al.* (2018) presented a review to support the process of predicting the re-start pressure as accurately as possible. During cooling process, the waxes in the crude oil can precipitate and make the restart-ups processes challenging and sometimes even impossible. The gelled crude oil exhibits non-Newtonian behavior with complex rheological properties below pour point temperature, and its rheological properties can be dependent on temperature, cooling rates and time. Due this complexity, the authors reported that the theoretical re-start pressure was usually much higher than the re-start pressure developed by the oil industry.

Abedi *et al.* (2019) carried out an experimental study with viscoplastic fluids with and without thixotropic behavior, seeking to identify the relationship between the rheological properties and the minimum pressure gradient needed to re-start the flow in a tube. The authors came to the conclusion that a simple balance of forces can provide a good estimate of the pressure gradient, a result that contradicts part of the literature on the subject, where the minimum gradient is typically smaller than that obtained in the balance of forces. As this study takes into account materials with and without thixotropic behavior, its results suggest that the difference between the force balance value and the value obtained in practice is not due to the thixotropic behavior.

Dalla *et al.* (2019) performed experiments to obtain the critical stress required for the re-start of the flow of crude oil and a Carbopol solution in a tube. The results obtained were compared with the values of the yield stress obtained in a rheometer, taking into account the four main reasons that could lead to discrepancies between these two values: the thermal and stress history, the radial temperature distribution, the contraction of the fluid during the cooling process and the different ways of imposing a flow of crude oil. Taking these parameters into account, the difference between the values obtained in the experiment and in the rheometer was considered small. The differences observed in the experiments with the Carbopol solution can be related to the slipping condition on the walls.

Sargentini (2013) used the software OpenFOAM to develop numerical simulations of a viscoplastic flow, adopting the Bingham function to model the viscoplastic behavior and considering the effect of temperature on several rheological parameters. The author concludes that the conditions for the resumption of flow do not depend only on the pressure difference, but also on the local temperature. Furthermore, the pressure difference required for the re-start operation is much greater than the pressure difference to maintain the flow once it reaches steady state.

To discuss the re-start flow, it is important to establish a comparison between it and the entrance region flow. The re-start flow has an imposed pressure gradient, and the flow rate is calculated over time. On the other hand, the entrance region flow has an imposed flow rate, and the pressure gradient needed to obtain such a flow rate is posteriorly determined. Both problems model how the flow achieve the developed condition, but the first one can be independent of the axial position within the pipe, while the latter can be independent of the time. White (2002) shows through dimensional analysis that the entrance length of a Newtonian entrance region flow can be written as a function of the Reynolds number, and usually, for laminar flows, the entrance length is a linear function of the Reynolds number. The same strategy can be applied to the Newtonian re-start flow, leading to a similar conclusion: the development length (and the development time) can also be expressed as a function of the Reynolds number. Due to re-start flow characteristics, the development time usually is more relevant than the development length for this problem, thus the present work has adopted the development time over the development length. When non-Newtonian behaviors are considered, using only the Reynolds number is not enough to accurately estimate the development time or length, as the rheological properties also influence the flow. Thus, the present work seeks to propose a correlation between the effects of the Reynolds number and the rheological dimensionless parameters over the development time of the re-start flow problem. A viscoplastic model proposed by de Souza Mendes and Dutra (2004) was chosen, and the influence of the plastic number, the power-law index, and the Reynolds number was evaluated. As already mentioned, the rheological behavior of the re-start flow can be really complex, with time and temperature depending properties, but in the present work only the viscoplastic behavior is considered.

2. METHODOLOGY

2.1 Mechanical model and geometry

The mathematical model for the incompressible flow of a generalized Newtonian fluid through a two-dimensional planar channel consists of the balance equations of mass, and momentum for a control volume:

$$\nabla \cdot \mathbf{u} = 0$$
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\nabla \mathbf{u})\mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau} \tag{1}$$

where $\mathbf{u}$ is the velocity vector, t is the time, ρ is the specific mass of the fluid, p is the hydrostatic pressure, and $\boldsymbol{\tau}$ is the extra-stress tensor defined by the following constitutive equation:

$$\boldsymbol{\tau} = 2\eta(\dot{\gamma})\mathbf{D}(\mathbf{u}) \tag{2}$$

where $\mathbf{D}(\mathbf{u})$, $\eta(\dot{\gamma})$ and $\dot{\gamma}$ are defined as:

$$\mathbf{D}(\mathbf{u}) = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T) \tag{3}$$
$$\eta(\dot{\gamma}) = \left[1 - exp\left(\frac{-\eta_0}{\tau_0}\dot{\gamma}\right)\right]\left(\frac{\tau_0}{\dot{\gamma}} + K\dot{\gamma}^{n-1}\right) \tag{4}$$
$$\dot{\gamma} = \sqrt{2tr\mathbf{D}(\mathbf{u})^2} \tag{5}$$

where η_0 is the viscosity for very low values of the shear rate $\dot{\gamma}$, τ_0 is the yield stress limit of the material, K is the consistency index and n is the power-law index. Equation (4) represents the SMD model, which models the apparent viscosity η as a function of the shear rate, not considering any time or temperature effects.

The schematic representation of the geometry is shown in Fig. 1. As the planar channel has a symmetry line, only half of it was simulated. The channel is considered infinite, and a constant imposed pressure gradient is applied. On the walls, the no-slip condition is adopted.

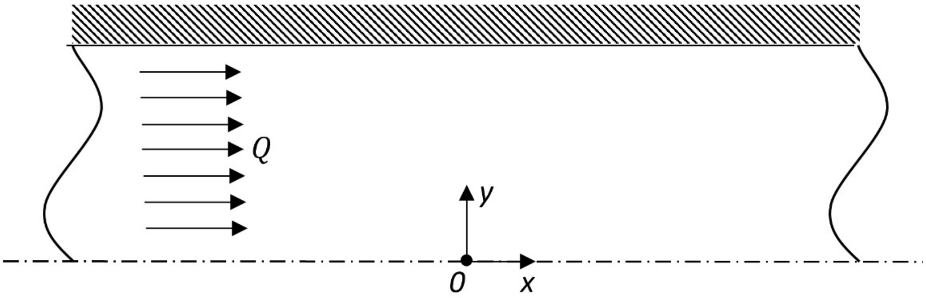

Figure 1: Schematic representation of the adopted geometry.

2.2 Dimensionless groups of interest

In this work, the dimensionless groups of interest are: the Reynolds number, the jump number, the power-law index, and the plastic number. The adopted expressions for the Reynolds number (Re), and plastic number (Pl) were proposed by Thompson and Soares (2016), as follows:

$$Re = \frac{\rho {V_c}^2}{\tau_0 + K(\frac{V_c}{L_c})^n + \eta_\infty(\frac{V_c}{L_c})} \tag{6}$$

$$Pl = \frac{\tau_0}{\tau_0 + K(\frac{V_c}{L_c})^n + \eta_\infty(\frac{V_c}{L_c})} \tag{7}$$

where L_c and V_c are the characteristic length and the characteristic velocity, respectively taken as the the half of the channel height and mean value of the velocity when the flow is fully developed.

The jump number (J) is an dimensionless group proposed by de Souza Mendes *et al.* (2007) that gives a relative measure of the shear rate jump that occurs at $\tau = \tau_0$ and its expression is:

$$J = \frac{\eta_0 \dot{\gamma}_1}{\tau_0} - 1 \tag{8}$$

where $\dot{\gamma}_1 = \left(\frac{\tau_0}{K}\right)^{\frac{1}{n}}$.

The dimensionless quantities of the problem are defined according to the following expressions:

$$\mathbf{x}^* = \frac{\mathbf{x}}{L_c} \qquad t^* = \frac{tV_c}{L_c} \qquad \mathbf{u}^* = \frac{\mathbf{u}}{V_c} \qquad \dot{\gamma}^* = \frac{\dot{\gamma}}{\dot{\gamma}_1} \qquad \tau^* = \frac{\tau}{\tau_0} \qquad \eta^* = \frac{\eta \dot{\gamma}_1}{\tau_0} \tag{9}$$

In the present work, the influence of the Reynolds number, the power-law index and the plastic number is evaluated by adopting a constant $J = 1000$. The Reynolds number ranges from $Re = 0.1$ to 100, the plastic number ranges from $Pl = 0$ to 0.9 and the power-law index ranges from $n = 0.4$ to 1.3. To fit the correlation between these parameters and the developing time, ten values of the Reynolds number and the plastic number were chosen, and four values of the power-law index. The values of the Reynolds number are logarithmically spaced within its range. This way, four hundred simulations were used to obtain the coefficients of the proposed correlation.

The developing time was measured by searching the time step where the simulated flow rate reaches less than two percent of error concerning the developed flow rate. The developed flow rate can be obtained by solving Eq. (1) imposing the developed flow conditions, as the null time derivatives.

To evaluate the viability of the proposed correlation, the coefficient of determination (r^2) was used.

$$SSE = \sum_{i=1}^{s} (\phi_i - \hat{\phi}_i)^2 \tag{10}$$

$$r^2 = 1 - \frac{SSE}{\sum_{i=1}^{s} (\phi_i - \bar{\phi})^2} \tag{11}$$

$$\tag{12}$$

where s is the number of observations. ϕ_i is the reference value at the observation point i, $\hat{\phi}_i$ is the value obtained with the proposed correlation at the observation point i and $\bar{\phi}$ is the mean value of the reference values. In the present work, the reference values are the values obtained in the numerical simulation.

3. RESULTS

In this section, the obtained development times are shown. Initially, the results are presented by discussing the individual effect of each one of the three chosen parameters: the Reynolds number, the power-law index, and the plastic number. Then, a correlation is presented, and its viability is discussed.

3.1 Reynolds number

To understand the influence of the Reynolds number over the development time, a range from $Re = 0.1$ to 100 was chosen. All these values lies inside the laminar flow range, and, as already mentioned, when the hypothesis of Newtonian flow is considered, the development time can be written as a linear function of the Reynolds number. Figures 2 and 3 indicate that this linear relationship is kept even when a viscoplastic behavior is present. Even when the power-law index is not unitary and the plastic number is non-null, the development time increases linearly with the Reynolds number adopted in the present work. It is possible to find several other expressions for the Reynolds number for non Newtonian flows in the literature, and this conclusion is restricted to the expression proposed by Thompson and Soares (2016).

The Reynolds number means the ratio between the inertial forces over the viscous forces, so a higher Reynolds number means that the inertial forces are more relevant concerning the viscous forces when compared with a lower Reynolds number flow. The viscous forces are represented by the diffusive term of the balance equation of momentum, this way, when the viscous forces are less relevant, the momentum diffusion is also lower, and therefore the flow takes longer to achieve the developed state.

3.2 Power-law index

The power-law index is one of the rheological parameters that rule the region of the flow where $|\boldsymbol{\tau}| > \tau_0$. When $n > 1$, the apparent viscosity increases with the shear rate, increasing the influence of the viscous forces within this region. The opposite is also true: When $n < 1$, the apparent viscosity decreases with the shear rate, decreasing the influence of the viscous forces within this region. Its influence is clearly non-linear: $n < 1$ means a lower apparent viscosity, which tends to increase the shear rate, decreasing the apparent viscosity even more. Another important aspect is that the power-law index rules only over a portion of the flow. This way, the influence of this region is more relevant when it plays as a bottleneck. When the power-law index increases the influence of the viscous forces within a region, another region of the flow can become the critical region to reach the developed state, decreasing the influence of the power-law index over a global parameter, as the development time.

For these reasons, it is clear that the influence of the power-law index is not linear. Increasing the power-law index can decrease the development time, even though the higher the power-law index, the lower its influence. Thus, the correlation proposed to model the effects of the power-law index is a power function, in the form of $t^* = an^{-b}$, where a and b are positive values. In this correlation, a is a function of the other parameters (J, Re, and Pl), while b is constant. The proposed function can model the obtained values with great precision, but its results must be treated with caution, as there are only four different values of n, and there are two fitting coefficients (a and b). This way, the fitting success of this function may be only because of the relatively high number of fitting coefficients, when compared with the number of values of n. An example of the influence of n is shown in Fig. 4.

3.3 Plastic number

The plastic number is a dimensionless parameter associated with the viscoplastic behavior, and it is, by definition, the ratio of the yield stress to the characteristic stress of the problem. This way, the plastic number defines the portion of the domain where $|\boldsymbol{\tau}| > \tau_0$, for example. The higher the plastic number, the more relevant is τ_0 when compared to other rheological parameters, as can be seen in Eq. (7). This way, the higher the plastic number, the higher the development time, as there is always a region near the wall where $|\boldsymbol{\tau}| > \tau_0$ for this range of Reynolds number, even when the majority of the flow has a $|\boldsymbol{\tau}| < \tau_0$. This region where $|\boldsymbol{\tau}| > \tau_0$ can play as a bottleneck, increasing the development time. This way, increasing the plastic number can influence the development time in two ways: one is decreasing the influence of the rheological parameters other than τ_0, and the other is decreasing the region where $|\boldsymbol{\tau}| > \tau_0$, creating a bottleneck.

As can be seen in Fig. 5, the correlation between the development time and the plastic number can be particularly complex, with a region with relatively low Pl where the development time increases almost linearly, and a region with higher Pl that shows a non-linear behavior. Another important fact is that the plastic number can be null, as it is in a Newtonian flow, for example. Thus, the proposed correlation takes the form of $t^* = a(Pl^b + c * Pl + d)$ where a is a function of the other parameters (J, Re, and n), while b ,c, d are constants.

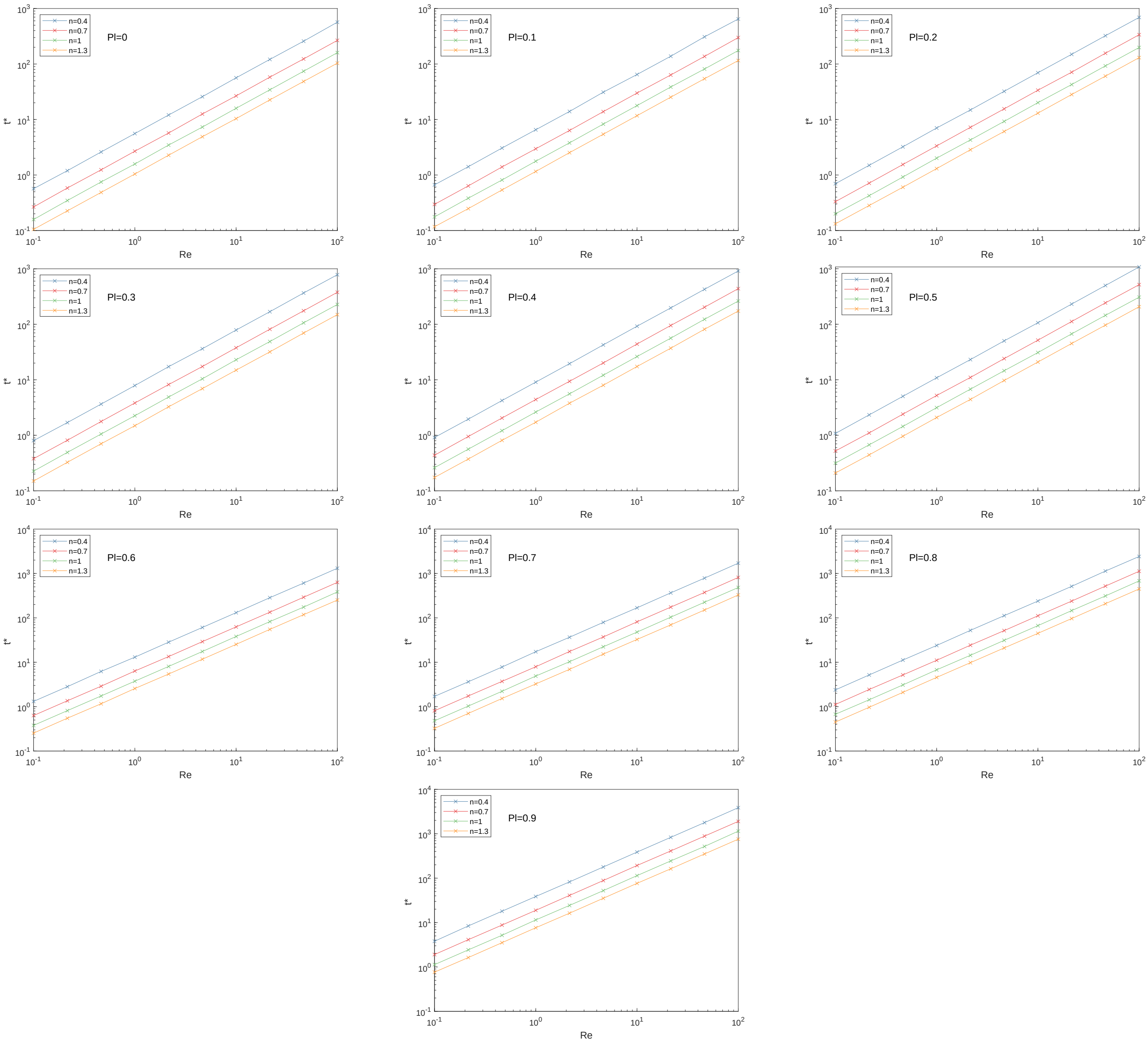

Figure 2: Developing time as a function of the Reynolds number, from $Pl = 0$ to 0.9.

3.4 Fitted curve

By combining all the correlation functions presented in this work, it is possible to obtain the correlation between the development time and the studied parameters. The correlation function is equal to:

$$t^*(Re, Pl, n, J = 1000) = 14.45\, Re\, n^{-1.352}(Pl^{6.678} + 0.1944\, Pl + 0.107) \quad (13)$$

The proposed correlation has an $r^2 = 0.9988$, and Fig. 6 compare graphically the results of the numerical simulation and the proposed correlation. The correlation is more effective for $n \leq 1$, which is the region where all the examples from the work of de Souza Mendes and Dutra (2004) are located. More complex correlations can be built, especially considering how one parameter interacts the others (how the plastic number interacts with the power-law index, for example), but the proposed correlation has a good compromise between simplicity and surface fitting.

4. FINAL REMARKS

The present work analyzed how the rheological parameters influence the development time. As discussed, the Reynolds number increases linearly the development time, even when a viscoplastic behavior is considered. The plastic number also has a positive correlation with the development time, but this relationship is not linear. Finally, the power-law index has a negative effect over the development time, but its effects decrease for higher values.

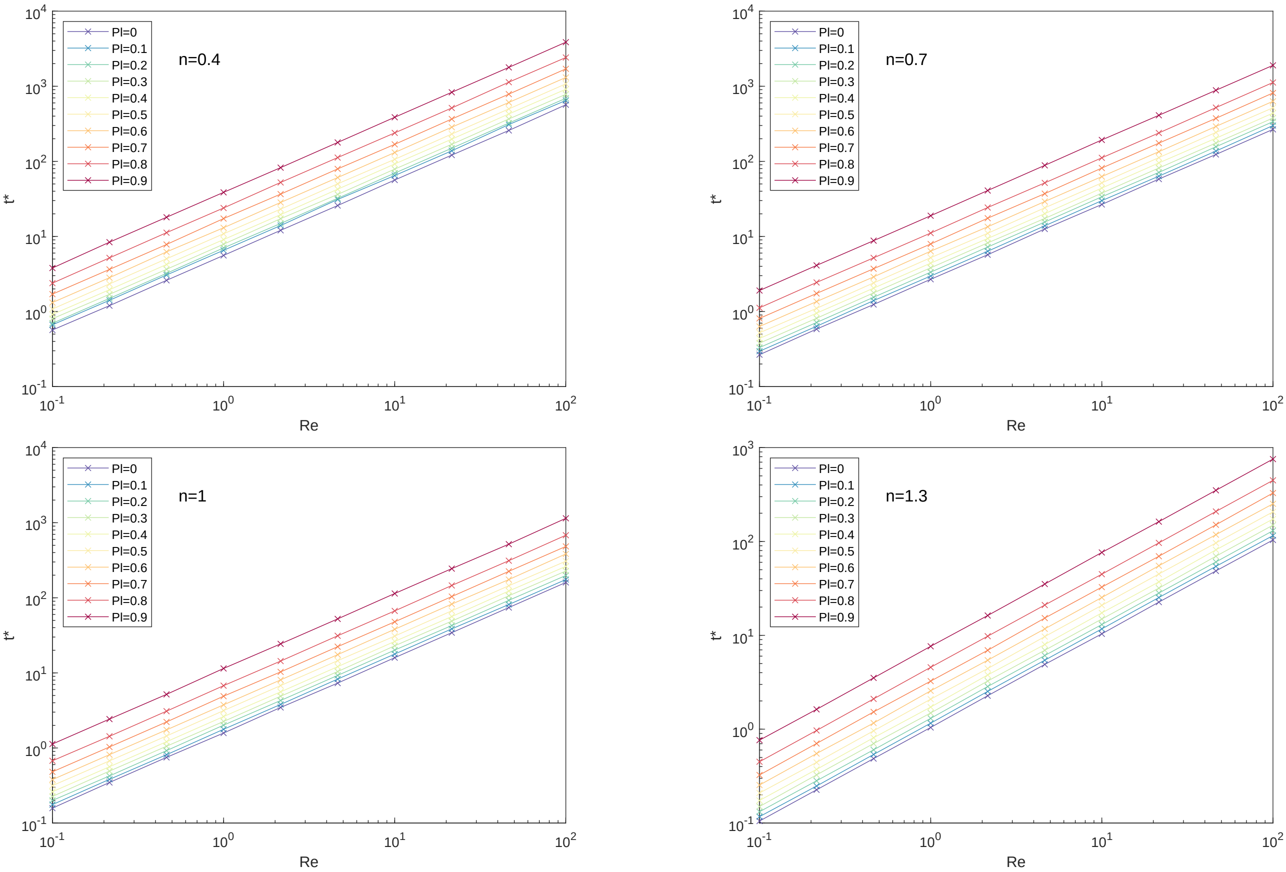

Figure 3: Developing time as a function of the Reynolds number, from $n = 0.4$ to 1.3.

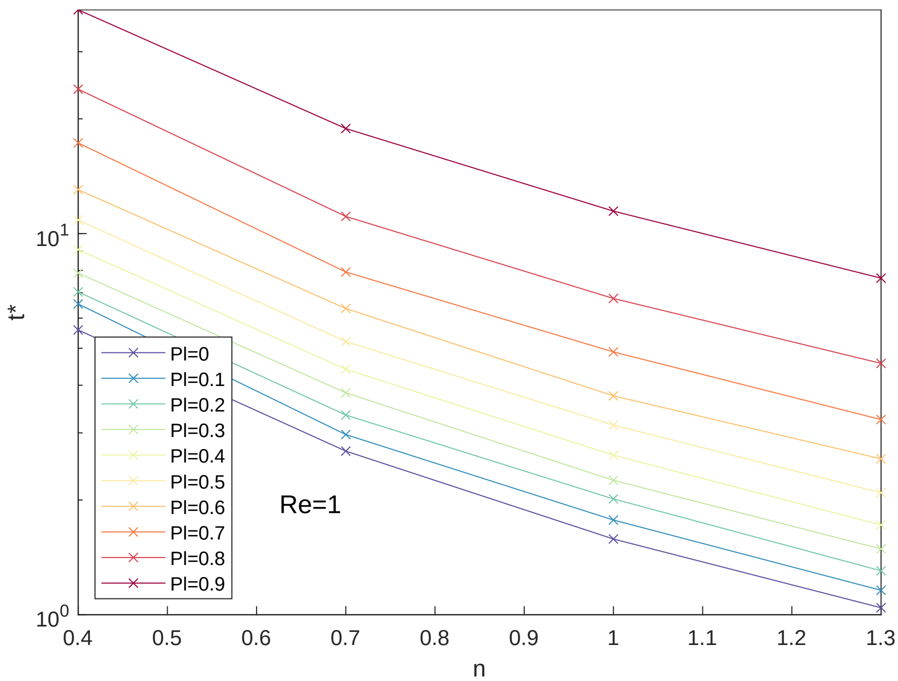

Figure 4: Example of the influence of the power-law index.

To fit the proposed correlation, four hundred simulation results were used. The correlation has an $r^2 = 0.9988$ and it is particularly effective for $n \leq 1$. However, the present work only adopted four different values for the power-law index. A more detailed correlation may be needed to confirm the precision of the proposed correlation.

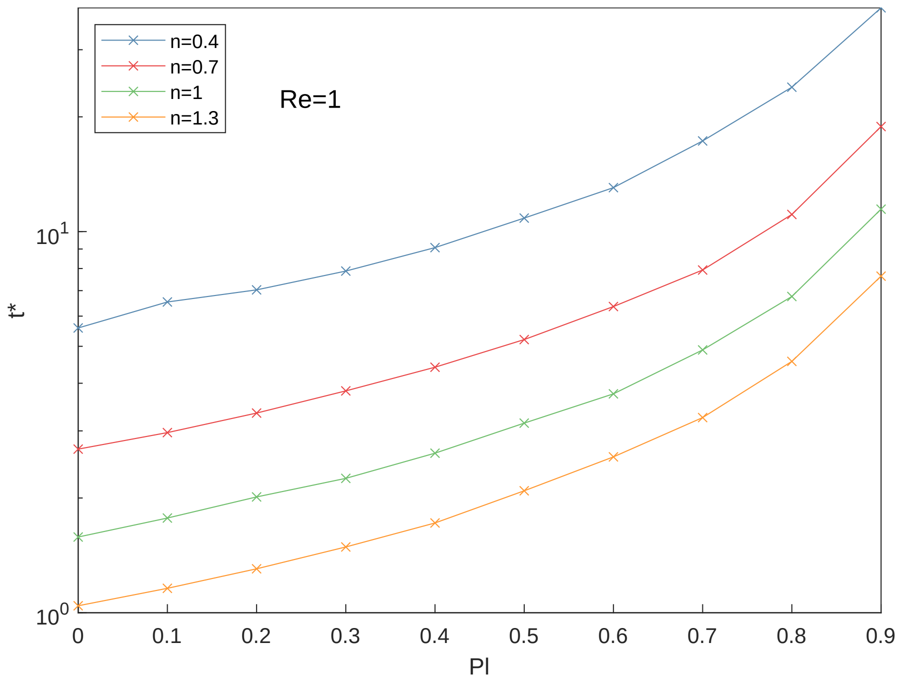

Figure 5: Example of the influence of the plastic number.

5. ACKNOWLEDGMENTS

L.P.B. Miranda is grant holder of CNPq, process number 140729/2020-8. The authors acknowledge CAPES, FAPEMIG, and the Faculty of Mechanical Engineering (FEMEC/UFU) for financial support.

6. REFERENCES

Abedi, B., Mendes, R. and de Souza Mendes, P.R., 2019. "Startup flow of yield-stress non-thixotropic and thixotropic materials in a tube". *Journal of Petroleum Science and Engineering*, Vol. 174, pp. 437–445.

Chala, G.T., Sulaiman, S.A. and Japper-Jaafar, A., 2018. "Flow start-up and transportation of waxy crude oil in pipelines-a review". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 251, pp. 69–87. ISSN 0377-0257. doi:https://doi.org/10.1016/j.jnnfm.2017.11.008. URL `https://www.sciencedirect.com/science/article/pii/S0377025717300903`.

Dalla, L.F., Soares, E.J. and Siqueira, R.N., 2019. "Start-up of waxy crude oils in pipelines". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 263, pp. 61–68.

de Souza Mendes, P.R., Naccache, M.F., Varges, P.R. and Marchesini, F.H., 2007. "Flow of viscoplastic liquids through axisymmetric expansions-contractions". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 142, pp. 207–217.

de Souza Mendes, P.R. and Dutra, E.S.S., 2004. "Viscosity function for yield-stress liquids". *Applied Rheology*, Vol. 14, pp. 296–302.

Sargentini, R., 2013. *Simulação numérica do deslocamento de óleos parafínicos em dutos considerando efeitos térmicos e não newtoninaos*. Master's thesis, Universidade Federal Fluminense.

Thompson, R.L. and Soares, E.J., 2016. "Viscoplastic dimensionless numbers". *Journal of Non-Newtonian Fluid Mechanics*, Vol. 238, pp. 57–64.

White, F.M., 2002. *Mecânica dos fluidos 4ª Edição*. McGraw Hill Brasil.

7. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.

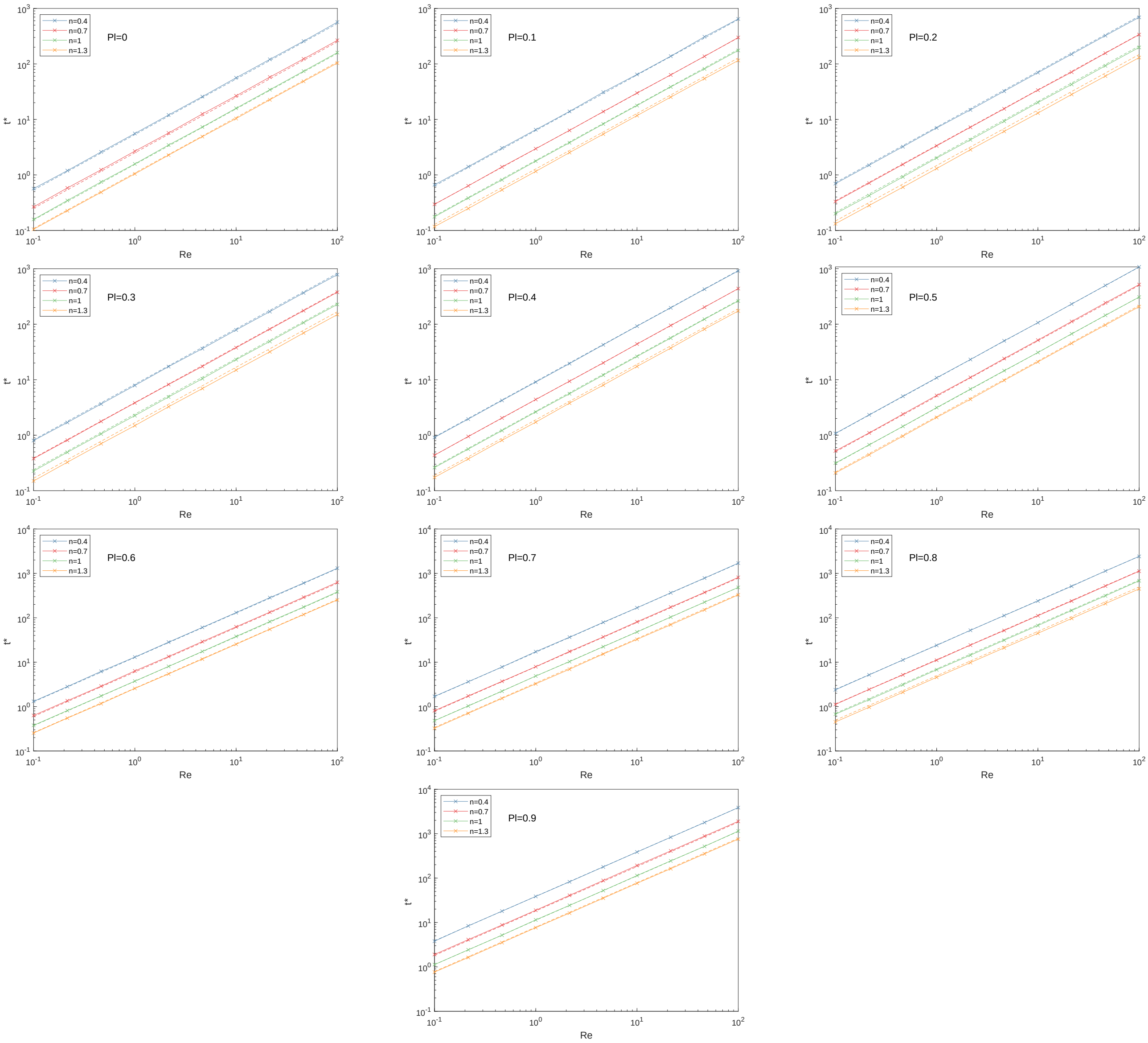

Figure 6: Comparison between the obtained results and the proposed correlation.