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# GEOMETRY EVALUATION OF A LID-DRIVEN CAVITY INSERT AT DIFFERENT POSITIONS

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**Abstract.** *The present work aims to analyze and optimize the geometry of a fin inserted in the lower surface at three different positions in a lid-driven cavity using the Constructal Design method and exhaustive search. The system is two-dimensional and subjected to mixed, laminar, and steady-state convection. The performance indicator is the dimensionless heat transfer rate ( $q^*$ ). The degrees of freedom investigated were the fin position and ratio between fin height and width ( $H_1/L_1$ ). For all simulations, the Reynolds, Richardson, and Prandtl numbers were kept constant, equal to 400, 0.1, and 6.0, respectively. The mass, momentum, and energy conservation equations were solved using the Finite Volume Method. Compared with the case without a fin, the addition of the fin at the best geometric position showed an improvement of 65027%. As a general trend, the insertion of a fin improves the performance of the system. Besides, the fin inserted more to the right of the cavity provides a significant increase in the performance indicator, increasing up to 172.96% compared to the baseline geometry ( $S = 0.25$ ,  $H_1/L_1 = 1.0$ ). Regarding the effect of the  $H_1/L_1$  ratio inserted at three different positions, the geometric evaluation allows a gain of approximately 124%.*

**Keywords:** *Constructal design, Lid-driven flows, Cavity, Rectangular fin, Mixed convection*

## 1. INTRODUCTION

The search for the development of increasingly smaller and more efficient equipment has motivated numerous studies of fluid flow and heat transfer. According to DeWitt et al. (2020), due to the various applications related to convection heat transfer, equipment such as heat exchangers, solar heating systems, component cooling and others, highlight the need to investigate the behavior of flows. Among the studies related to heat transfer by convection, cavity flow has been widely discussed and studied in the area of computational fluid dynamics (CFD). Cavities with the insertion of fins represent several ideal engineering problems, such as cooling in rooms, solar panels, and space between fins in heat exchangers. In this context, studies have been carried out to improve the understanding of the behavior of flows in cavities (Erturk and Gökçöl, 2006; Shankar and Deshpande, 2000).

Several researchers have used the Constructal Design method to analyze problems related to cavities (Dos Santos et al., 2013; Aldrighi et al., 2016), which is a method of geometric evaluation and seek the geometry of least resistance to flow, assuming that the flow keeps the system alive and that the system is finite size and subjected to geometric constraints. Constructal Law states that nature's designs are not random (Bejan, 2000; Bejan and Lorente, 2008; Bejan and Zane, 2012). Constructal Design is a method based on the physical principles of the Constructal Theory. The Constructal Law states that for any finite animate or inanimate flow system to persist in time (survive), its design must evolve in a way that facilitates access for the currents flowing through the system (Bejan, 2000; Bejan and Lorente, 2008; Bejan and Zane, 2012). In this context, Mustafa (2019), using Constructal Design Method, analyzed a multi-scale

diamond-shaped pin fin cooled by mixed convection and maximized under pressure drop. It was shown that the heat transfer density could be maximized twice for all the studied angles of the smaller fins ( $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ ) and all ratios studied ( $Ra/Be = 0.1, 1, \text{ and } 10$ ). Also, the highest value of the second maximized heat transfer density is found at the angle of the smaller fins ( $\lambda_1 = 30^\circ$ ) for all ratios ( $Ra/Be$ ). Hazarika et al. (2019) designed and analyzed a set of fork-shaped wet fins attached to a circular tube under dehumidifying conditions, using the Constructal Design method associated with Particle Swarm Optimization, to investigate the heat transfer rate that leads to the ideal geometry of this system. According to the authors, the optimum fork-shaped fin array with two branches gives a higher heat transfer rate than the corresponding optimum rectangular fin array. More recently, Rodrigues et al. (2020) studied a geometric evaluation using the Constructal Design method and the exhaustive search to maximize the heat transfer rate and the Nusselt number for two fins inserted in a lid-driven with two rectangular intrusions under mixed convection heat transfer.

In this context, the present work intends to analyze which geometry leads to the best performance of a lid-driven cavity subjected to mixed convection under laminar and incompressible flow in a two-dimensional domain. The performance indicator is the dimensionless heat transfer rate ( $q^*$ ). The geometry is a square cavity with a fin inserted into the lower surface, which varies by applying the Constructal Design method associated with an exhaustive search to obtain the geometry that leads to the best performance. For all simulations,  $Re_{A^{1/2}} = 400$ ,  $Pr = 6.0$  and  $Ri (Gr_{A^{1/2}}/Re_{A^{1/2}}^2) = 0.1$  were considered, which represents a flow dominated by forced convection. The effect of the fin position is investigated, as well as the ratio of height to width ( $H_1/L_1$ ).

## 2. MATHEMATICAL MODELING

The present work proposes numerically evaluating the behavior of a system of a lid-driven cavity and a rectangular fin submitted to mixed convection. The geometric domain is considered two-dimensional. The flow is assumed to be steady-state, laminar, and incompressible. The objective is to maximize the heat transfer rate between the fin and the flow. The thermophysical properties of the flow are kept constant throughout the entire domain, except for the density, which considers the use of the Boussinesq approximation (Gray and Giorgini 1976) to account for body forces. For the flow subject to mixed convection in a laminar regime, the equations of conservation of mass, momentum, and energy for incompressible flows are given by Bejan (2013):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_0) \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions,  $\rho_0$  is the density of the fluid at the reference temperature,  $\beta$  is the coefficient of thermal expansion,  $P$  is the pressure,  $\nu$  is the kinematic viscosity of the fluid,  $\alpha$  is the thermal diffusion,  $T$  is the temperature,  $T_0$  is the reference temperature and  $g$  is the gravity vector component in the  $y$  direction.

### 2.1 Problem formulation and geometric evaluation procedure

The system consists of a square cavity filled with a fluid, with one fin subjected to tangential flow to the upper boundary and the effects of gravity, characterizing a case of mixed convection. Figure 1 presents a schematic of the computational domain to be investigated and its boundary conditions and geometric variables to be studied. The cavity has sides  $H$  and  $L$ , both equal to 1.0, where  $H_1$  and  $L_1$  represent the dimensions of the fin and  $\phi_1$  represents the fraction of the area. For the problem, stable stratification was considered, i.e., the upper surface of the cavity is subjected to a temperature higher than that of the fin. It is also considered a stable stratification convective flow, i.e., the prescription of the highest temperature is performed on the upper surface. Regarding the thermal field, the upper surface of the cavity (infinite plate) is at a dimensionless temperature  $T_s^* = 1.0$ , while the fin is at a lower temperature,  $T_f^* = 0$ . The side and bottom surfaces of the cavity are thermally insulated. As for fluid dynamic fields, it is imposed non-slip and impermeability conditions, i.e., dimensionless velocities are prescribed as null ( $v^* = w^* = 0$ ). For all cases, fixed Prandtl and Richardson numbers are kept constant and equal to 6.0 and 0.1, respectively.

The parameter results are presented in a dimensionless way, as shown below:

$$x^*, y^*, H^*, L^*, H_1^*, L_1^* = x, y, H, L, H_1, L_1 / A^{1/2} \quad (5)$$

where  $H$  is the cavity height,  $L$  is the cavity width,  $H_1$  is the fin height and  $L_1$  is the fin width. The geometric evaluation is based on the definition of two constraints, which are the cavity area and the fin area ( $A, A_1$ ), given respectively by:

$$A = HL \text{ and } 1 = H^*L^* \quad (6)$$

$$A_1 = H_1L_1 \quad (7)$$

The areas of the fins can be represented by the ratio between the areas of the fins and the area of the cavity, given by:

$$\phi_1 = \frac{A_1}{A} = 0.1 \quad (8)$$

The problem has two degrees of freedom (DOF): position of fin ( $S$ ) and width) and  $H_1/L_1$  (ratio between fin height and width). The main objective is to identify which fin ratio maximizes the heat transfer between the fin and the surrounding flow, that is, the best value for the dimensionless heat rate transfer ( $q^*$ ), given by:

$$q^* = \frac{q'}{k(T_\infty - T_s)} H \quad (9)$$

where  $q'$  is the heat transfer rate per unit length,  $k$  is the thermal conductivity of the fluid,  $T_s$  is the surface temperature and  $T_\infty$  is the free stream temperature and  $H$  is the cavity height.

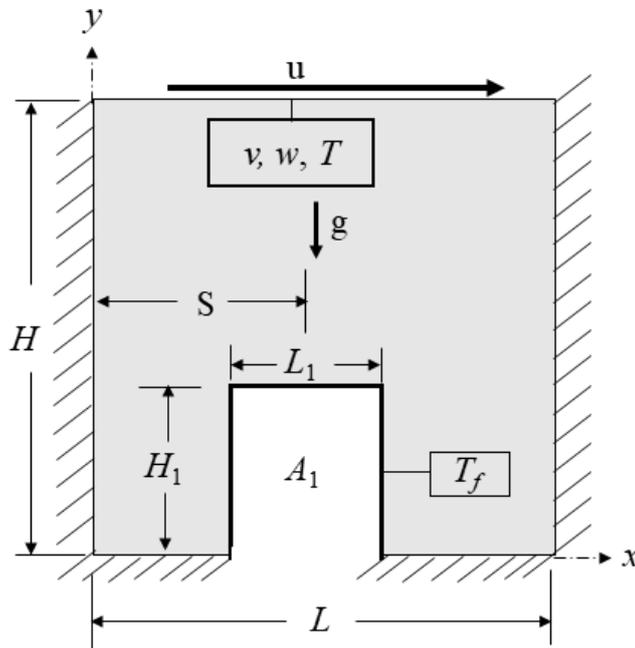


Fig. 1. Schematic representation of studied domain.

Fig. 2 presents the flowchart of the geometric evaluation carried out through the Constructal Design method associated with an exhaustive search. In order to find the geometry that maximizes  $q^*$ , in all simulations, the area fraction is kept constant ( $\phi_r = \phi_1 = 0.1$ ) and both the fin positions ( $S = 0.25, 0.5$  and  $0.75$ ) and  $H_1/L_1$  ratio ( $1.0 \leq H_1/L_1 \leq 9.0$ ) are varied. The evaluation process is conducted as follows: First, the  $S$  position is defined and the simulation is performed for all  $H_1/L_1$  ratios. Later, the process is repeated for the other positions ( $S$ ), and  $H_1/L_1$  optimized,  $(H_1/L_1)_o$ , and heat transfer maximized,  $(q_m^*)$ , are found.

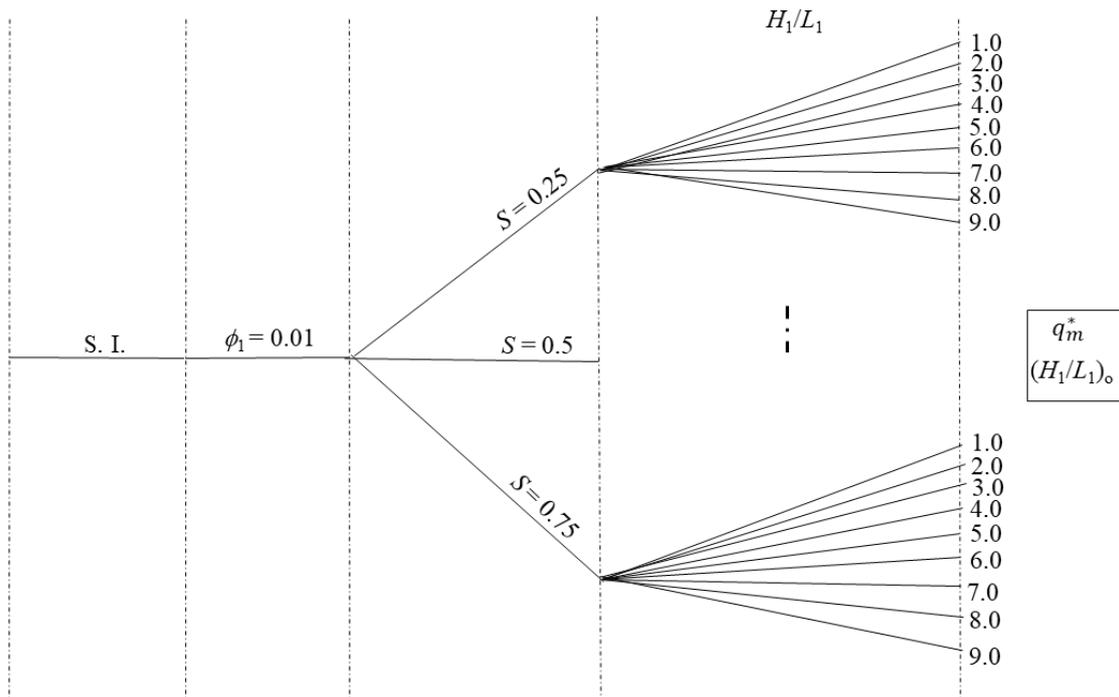


Figure 2: Scheme of the geometric evaluation carried out with the Constructal Design associated with the Exhaustive Search.

### 3. NUMERICAL MODELING

For all simulations, geometries, and meshes, the commercial package CFD FLUENT 16.0 was used. The Finite Volume Method was used to solve Eqs. 1-4 for all investigated geometries. SIMPLEC (Semi-Implicit Method for Pressure-Linked Equations Consistent) algorithm is used for the pressure-velocity coupling. For the spatial discretization of the advective terms, the second-order upwind scheme was used. Calculations are considered convergent when the residuals for conservation of mass, conservation of momentum, and conservation of energy are less than  $10^{-6}$ ,  $10^{-6}$ , and  $10^{-8}$ , respectively.

For the mesh independence test, the Grid Convergence Index (GCI) method proposed by Celik et al. (2008) was used. This method consists of comparing three meshes with different refinements, seeking to find a mesh refined enough that generates satisfactory results and that the computational cost is the minimum necessary.

To calculate the GCI, three meshes were analyzed. The coarsest mesh contains 24,278 elements, the intermediate mesh 36,265 elements, and the most refined mesh (Fig. 3), the mesh adopted, 59,543 elements. The variable of interest analyzed for each mesh was the Nusselt number on the top lid of the cavity. The GCI found was 0.19%. Thus, the result found in the calculation of the GCI was satisfactory, as it is recommended to be less than or equal to 5%. The GCI calculation for the most refined mesh represents the numerical uncertainty for the analyzed variable of interest.

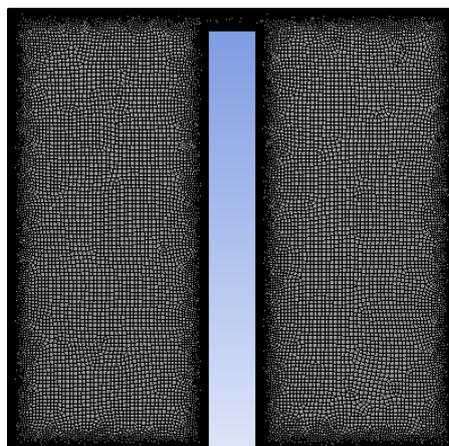


Figure 3: Refined mesh adopted for this study.

To verify the numerical methodology, the studies by Dos Santos et al. (2011) and Ji et al. (2007) were compared. The results obtained were in agreement with the studies cited above (Rodrigues, 2018).

#### 4. RESULTS

Figure 4 shows the temperature fields for the case without fins (Fig. 4 (a)) and for the worst (Fig. 4 (b)) and best (Fig. 4 (c)) cases analyzed with an inserted fin. For the case without a fin, it is observed that the heat transfer is dominated by conduction. This is due to the fact that the displacement of the directed plate is not enough to overcome the buoyance forces that tend to stagnate the fluid. The circulation in the upper corner of the cavity evidences the competitive effect between natural convection and forced convection. The dimensionless heat transfer rate ( $q^*$ ) is equal to 0.03063. With the addition of a fin, even for the worst case, Fig. 4 (b), it is possible to observe an increase of 23759% higher on the value of  $q^*$ . In the case of Fig. 4 (c), the best case, it is observed that the insertion of the fin changes the behavior of the flow, i.e., the fin furthest to the right behaves like a wall. If compared with the case without a fin, the addition of the fin in this position ( $S = 0.75$ ,  $H_1/L_1 = 9.0$ ) shows a great increase over  $q^*$ , revealing an improvement of 65027%. As a general trend, the insertion of a fin improves the performance of the system. Besides, the fin inserted more to the right of the cavity for the highest ratio  $H_1/L_1$  investigated (fig. 4c) provides a significant increase in the performance indicator, providing an increase up to 172.96% when comparing to the worst case (fig. 4b).

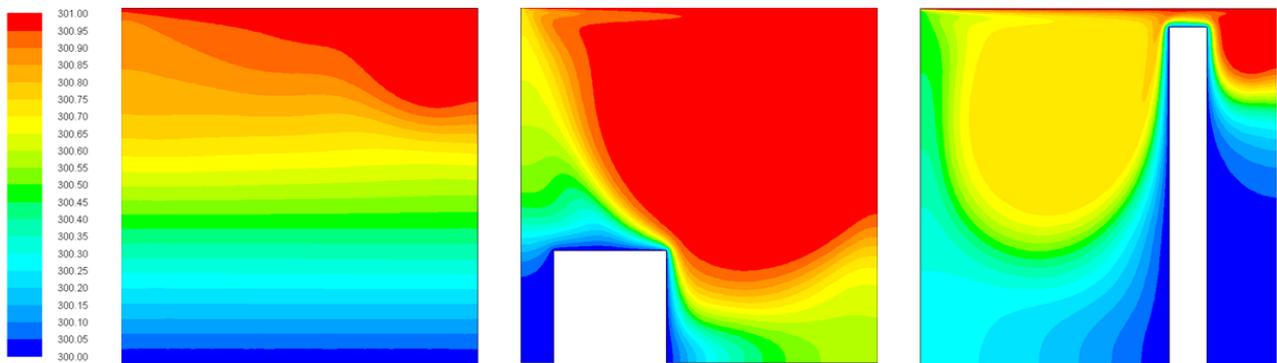


Figure 4. Temperature Fields for  $Re_{A^{1/2}} = 400$ ,  $Ri = 0.1$ ,  $Pr = 6.0$ , (a)  $q^* = 0.03063$ , (b)  $S = 0.25$ ,  $H_1/L_1 = 1.0$ ,  $q^* = 7.3081$  (c)  $S = 0.75$ ,  $(H_1/L_1)_o = 9.0$ ,  $q_m^* = 19.9484$ .

Three different positions ( $S$ ) were investigated for the fin (a)  $S = 0.25$ , (b)  $S = 0.5$  and (c)  $S = 0.75$ , varying the  $H_1/L_1$  ratio from 1.0 to 9.0 for  $\phi_r = 0.1$ . Figure 5 shows the temperature fields for the best results obtained. Comparing the three cases presented, it is noted that the fin has a large insertion in the cavity for the best cases, as it ends up behaving like a wall. However, the position in which it is inserted interferes in the temperature field. In Fig. 5 (a), a stagnation region of the flow is observed with the cold (denser) fluid in the lower left region of the cavity. Still, it is observed that the fluid flows more freely than in the other cases observed in Figs. 5 (b) and (c). This is justified by the fact that the fin is inserted more to the left of the cavity and the flow moves from right to left. The fin height ends up causing recirculation regions in the left region of the three investigated cases. In Fig. 5 (b), it can be seen that the fin generates an obstacle to the flow.

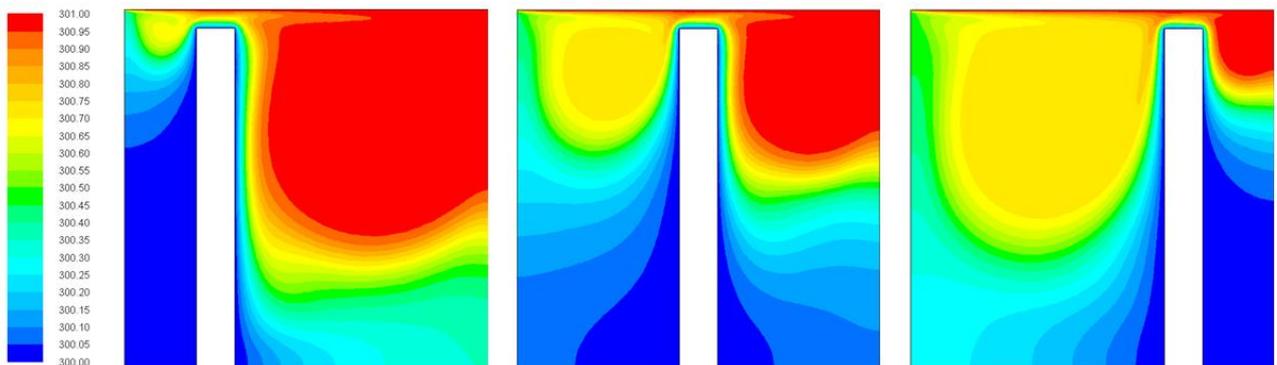


Figure 5. Temperature fields for  $Re_{A^{1/2}} = 400$ ,  $Ri = 0.1$ ,  $Pr = 6.0$ ,  $\phi_r = 0.1$ : (a)  $S = 0.25$ ,  $H_1/L_1 = 9.0$ ,  $q^* = 17.2003$ ; (b)  $S = 0.5$ ,  $H_1/L_1 = 9.0$ ,  $q^* = 19.113$ ; (c)  $S = 0.75$ ,  $H_1/L_1 = 9.0$ ,  $(H_1/L_1)_o = 9.0$ ,  $q_m^* = 19.9484$ .

Fig. 6 shows the effect of the  $H_1/L_1$  ratio on the heat transfer rate ( $q^*$ ) for a fin inserted at three different positions ( $S = 0.25, 0.5$  and  $0.75$ ) for flow with  $Re_{A^{1/2}} = 400$ ,  $\phi_T = 0.1$ ,  $H/L = 1.0$ . The worst case was for  $S = 0.25$ ,  $H_1/L_1 = 1.0$  and  $q^* = 7.3081$  and the best case was for  $S = 0.75$ ,  $(H_1/L_1)_o = 9.0$  and  $q_m^{*'} = 19.9484$ . Comparing the best and worst case, the geometric evaluation allowed a gain of approximately 173% in the value of the heat transfer rate. As seen in Fig. 6, as the  $H_1/L_1$  ratios increase, the heat transfer rate value increases considerably. For  $S = 0.25$ , there is almost a straight line and the worst results when compared to the other investigated positions. However, a significant gain of approximately 135% in the value of  $q^*$  is obtained. For  $S = 0.5$ , there is an initial peak on the value of  $q^*$ , followed by a small decrease and then a continuous growth for  $H_1/L_1$  ratios greater than 5.0. Comparing the best case ( $H_1/L_1 = 9.0$ ) and the worst case ( $H_1/L_1 = 3.0$ ) for  $S = 0.5$ , a gain of approximately 124% in the value of  $q^*$  is obtained. For  $S = 0.75$ , a slight increase in the value of  $q^*$  is observed for  $1.0 \leq H_1/L_1 \leq 6.0$  and for  $H_1/L_1$  ratios  $\geq 7.0$  an abrupt increase in the value of the heat transfer rate is observed.

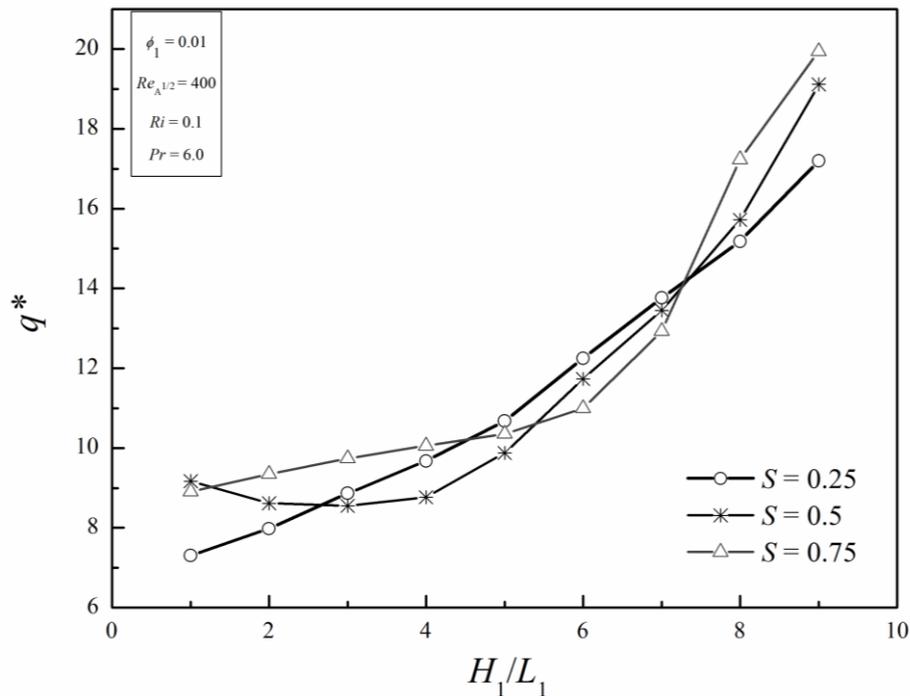


Figure 6. Effect of  $H_1/L_1$  ratio over dimensionless heat transfer rate for  $\phi_T = 0.1$ ,  $Re_{A^{1/2}} = 400$ ,  $Ri = 0.1$  and  $Pr = 6.0$ .

## 5. CONCLUSIONS

The present work performed a geometric analysis and investigation of a lid-driven cavity subjected to mixed convection with a fin inserted into the lower surface of the cavity at different positions ( $S = 0.25, 0.5$ , and  $0.75$ ). The flow was considered a steady-state, laminar, incompressible and two-dimensional. The dimensionless heat transfer rate per length ( $q^*$ ) was used as a performance indicator. The Constructal Design method and the exhaustive search method led the study to the search for the best performance. For all simulations, the investigated area fraction was equal to 0.1,  $Ri = 0.1$ ,  $Re_{A^{1/2}} = 400$  and  $Pr = 6.0$  were kept constant.

The results indicated that inserting a fin in the directed cavity, regardless of geometry, leads to a better performance when compared to the case without a fin (up to 65,027%). It is interesting to note that, for all investigated positions, the fin with the highest insertion, that is, the greater  $H_1/L_1$ , leads to the best performance of the dimensionless rate of heat transfer ( $q^*$ ) because the fin behaves like a wall. Regarding the effect of the  $H_1/L_1$  ratio on the heat transfer rate ( $q^*$ ) for a fin inserted at three different positions ( $S = 0.25, 0.5$  and  $0.75$ ), the geometric evaluation allows a gain of approximately 124% compared the worst ( $S = 0.25$ ,  $H_1/L_1 = 1.0$ ) and the best case ( $S = 0.75$ ,  $(H_1/L_1)_o = 9.0$ ).

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