ENC-2022-0401
STATE ESTIMATION OF THE HEAT FLUX IN AN 81 MM MORTAR

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Abstract: The study of gun barrel heating allows for the enhancement of the performance of military weaponry and also contributes to the safety of the personnel involved in the operation of the equipment. However, some critical information about relevant parameters is challenging to obtain by direct measurements. Consequently, numerical evaluations of bore temperature distribution or the associated heat flux by inverse analysis procedures become an interesting tool in the design of these devices. Therefore, this work presents an inverse analysis for the estimation of the heat flux that is established at the inner surface of an 81 mm mortar tube during the fire of one round. The associated heat transfer problem is considered in the transient form and applied to a cylindrical geometry. A prescribed heat flux is considered in the inner region while a convective boundary condition is imposed at the outer surface. The direct problem is verified against an analytical solution available in the open literature for one shot of a 155 mm howitzer. The Bayesian Approach is then applied to the inverse analysis where all available information is combined with measurements in order to reduce the uncertainties of the estimation. The Sampling Importance Resampling (SIR) algorithm is applied for the state estimation problem. Synthetic measurements of the inner surface temperature of the weapon are employed in the inverse analysis. Different types of heat flux related to gun heating situations are investigated and the results obtained so far indicate that the estimations performed by the proposed algorithm are within a desirable pattern of accuracy.

Keywords: heat flux, state estimation problem, SIR algorithm, gun heating, inverse analysis.

1. INTRODUCTION

Inverse heat problems are well established techniques for the recovery of variables of interest from available measurements, Beck et al. (1985). In military operations, the firing process in mortars happens when a grenade is ejected from the barrel, which increases the temperature of the weapon barrel. In this context, a large amount of heat is transferred to the inner surface of a gun barrel when the burning of the propellant occurs, which increases the temperature of the weapon barrel. Consequently, a heat flux is generated from the propellant gas and interacts with the gun barrel through thermal conduction. For multiples rounds, the increase of temperature might be a concern in order to avoid the inappropriate ejection of the grenade. For the sake of avoiding this issue, the knowledge about the heat flow rates at the inner surface of a weapon tube is important to prevent elevated bore temperatures and must be known in a direct heat transfers analysis, which is utilized when it is possible to predict a temperature distribution relying on a mathematical model containing well known parameters for boundary and initial conditions.

In this paper, the inverse heat transfer model is employed in order to estimate the heat flux at the inner wall of an 81 mm mortar during the firing of one round. A cylindrical geometry is utilized considering a transient heat transfer problem with a prescribed heat flux in the inner surface while air natural convection occurs at the outer surface as boundaries conditions. Also, the direct problem uses an implicit finite difference scheme and is verified with a previously published analytical solution for a one shot of a 155 mm howitzer. The input-estimation method used in this inverse estimation is a Bayesian approach considering that the variables are state dependent. Therefore, the Sampling Importance Resampling (SIR) algorithm is employed to estimate the heat flux posterior distribution.

2. LITERATURE REVIEW

The representation of the inner heat flux is critical for the accurate determination of the temperature field during the barrel heating due to the firing of multiple rounds. As a result, several investigations are presented in the literature aiming at different heat flux distributions in gun barrels in order to better understand the thermal behavior of the weapon.

Chen and Liu (2008) investigated an inverse heat conduction problem regarding the determination of the unknown heat flux on the 2-D gun barrel based on the input estimation scheme that employed a first part with a Kalman filter and the other segment was a recursive least-squares algorithm. In addition, the finite element method was incorporated to simulate cases with a uniform and non-uniform unknown heat flux, which was varying with time and axial location related to a convective boundary condition at the outer surface. The temperature measurement data employed to inversely estimate the unknown heat flux came from a 7.62 mm gun barrel outer surface temperature. As a result, the unknown heat flux associated to different levels of input-heat flux modeled by Weibull distribution and the temperature history on the chamber, considering its time dependence, were effective estimated with accuracy, including the measurement errors effect.

Orlande et al. (2008) presented solutions of the state estimation problems using the Bayesian approach. Two Bayesian filters, Kalman filter and a Particle filter with the sampling importance resampling (SIR) algorithm, were investigated with a linear and non-linear unsteady heat conductions models that focus on predict the transient temperature field in a medium considering the temperature measurements errors. A one-dimensional linear heat conduction problem in a semi-infinite medium was initially analyzed without heat generation and containing a discretization process exposed by explicit finite difference scheme considering constant physical properties and boundary condition with temperature kept at zero Celsius. The non-linear heat conduction issue, on the other hand, presented a first insulated boundary condition and a second with a constant heat flux applied to thermophysical parameters temperature dependent and without heat generation.

Colaço et al. (2012) presented solutions of the state estimation problems using the Bayesian approach. Two Bayesian filters, Particle filter with the sampling importance resampling (SIR) algorithm and the auxiliary sampling importance resampling (ASIR) filter, were investigated to estimate an unknown heat flux at the outer surface of a square cavity containing a liquid subjected to natural convection. The direct heat transfer problem consisted of a laminar fluid inside a two-dimensional square cavity with constant temperatures on the left and right surfaces, well insulated bottom surface and a time-varying heat flux applied to the top wall. This physical problem was solved by finite volume scheme in order to obtain the estimated solution of heat fluxes from the temperature data measurements at the boundaries of the cavity. Two different heat flux profiles were estimated with both algorithm methods and showed good agreement.

Jablonski and Jablonski (2017) conducted a mathematical model evaluation in order to understand the thermal response of a temperature sensor, located inside of a 155 mm gun barrel, based on a heat conduction problem. The main purpose is to inversely estimate the measurements error of the heat flux profile relying on a measurement error of the temperature profile. The 155 mm gun barrel was modeled in a one-dimensional transient heat conduction equation and a Duhamel Principle was employed in order to solve the direct problem with a Weibull heat flux distribution. Also, the placement and the influence of the temperature sensor selection played an important role in the gun bore investigation. Therefore, the results revealed that the time response of the sensor and the possible location sensor errors are an important aspect when solving the inverse heat conduction situation with accuracy. In addition, it was observed that the accuracy of the model improves as the estimated heat flow increases, and the sensor with constant time decreases.

Noh et al. (2017) adopted a tree-dimensional inverse heat conduction problem with a multi-layered hollow cylinder in order to prevent erosion, crack, melt and wear of the gun barrel by high temperature gas flow. The direct heat conduction problem used a commercial numerical software, ANSYS Fluent, to compute the outer surface temperature of the tube in order to acquire the exact solution of the heat flux. In addition, the measured temperature at the outer surface obtained in the previously process was employed for the inverse heat transfer model, which generated the estimation of the temperature distribution in the gun barrel considering the thermal resistance network method. An unknown heat flux was estimated at the inner surface of the gun bore by the input estimation algorithm, which presented the Kalman filter and the recursive least square algorithm. The results displayed a heat flux profile close to the actual solution of the heat flux in the direct problem.

Abaci et al. (2022) utilized a thermal model based on the transient heat diffusion equation with two axisymmetric problems, which were numerically solved by employing one-dimensional and two-dimensional cases in order to calculate the temperature distribution in the gun barrel walls. During the firing process, different combustion gases parameters were computed with the focus on determination of the boundary conditions for these two different conduction models. The boundary conditions utilized for both models were taken as heat convection at the inner surface and thermal radiation effects combined with natural convection at the outer surfaced. The mathematical model was validated by measurements obtained by a thermal camera (FLIR), which temperatures were recorded at the outer wall surface. The experimental findings revealed that both models performed well when compared to the data of the camera measurement. Furthermore, the heat conduction equations results were compared and showed good agreement between the two numerical models investigates.

Within this context, the goal of this work is to numerically estimate the heat flux at the inner surface of an 81 mm mortar from synthetic measurements of the temperature at this surface for a single round. Furthermore, the number of particles and initial value employed on the heat flux will be analyzed.

3. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem involving the transient one-dimensional heat diffusion in the barrel of the 81mm mortar, for a certain firing condition, was analyzed. Initially, the barrel is considered to be in thermal equilibrium with the outer ambient air at temperature T_∞. As the firing sequence evolves, the inner surface of the weapon tube experiences a transient heat flux, $q(t)$, from the burning of the propellant that, in turn, causes an increase in the temperature of the gun barrel. During this process, the mortar is cooled by natural convection with the ambient air and therefore an external heat transfer coefficient h_∞ and outer temperature T_∞ are needed for the formulation. By establishing the geometry of the mortar barrel as a hollow cylinder of inner and outer radius R_{in} and R_{ext}, it is a simple matter to derive the equations for the transient temperature field as:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{k}{r}\frac{\partial T}{\partial r} + k\frac{\partial^2 T}{\partial r^2} \qquad R_{in} < r < R_{ext} \qquad t > 0 \qquad (1)$$

$$T = T_0 \qquad R_{in} \leq r \leq R_{ext} \qquad t = 0 \qquad (2)$$

$$-k\frac{\partial T}{\partial r} = q(t) \qquad r = R_{in} \qquad t > 0 \qquad (3)$$

$$k\frac{\partial T}{\partial r} + h_\infty T = h_\infty T_\infty \qquad r = R_{ext} \qquad t > 0 \qquad (4)$$

4. NUMERICAL ANALYSYS

The numerical simulations were performed within the MATLAB® platform and executed a computer with Windows 10 Pro, 64 bits, processor Intel® i5 6400 2.70Ghz, and 16 Gb of random-access memory. The solution of the direct problem given by Eqs. (1) to (4) was obtained with an implicit finite difference scheme (Ozisik et al., 2017). The heat flux $q(t)$ from the hot propellant gases was estimated with the Particle Filter, with the synthetic temperature measurements obtained over the inner surface of the gun barrel.

Particle filters are sequential Monte Carlo estimations most used when it comes to nonlinear evolution and observation models (Ristic et al., 2004). This Bayesian filter, given by Table 1, consists of generating particles that represent state variables and have a certain weight. In the prediction step, the particles are calculated using the evolution model. In the update step, the particle weights are calculated using the observation model, where the likelihood information is considered. Then, the weights are normalized so that the sum of all weights equals unity. In the resampling step, the importance resampling algorithm (SIR) is used, in which the most significant weight particles are replicated while eliminates low importance weight particles. At the end of the algorithm execution, the posteriori density average is provided by the particles. The summary of SIR algorithm is presented in Table 1.

Table 1. Sampling Importance Resampling (SIR) Algorithm (Ristic et al., 2004) and (Colaço et al., 2012).

1. Set the particle indicator $i = 1$, where each particle represents a set of state variables.
For $i=1, \ldots, N$ draw new particles from the prior density using the evolution model
Calculate the correspondent weights w_k of each particle based on the observation model.
2. Calculate total weight and then normalize particle weights
3. Resample particles based on the weights, as follows:
Construct the cumulative sum of weights (CSW) by computing $c_i = c_{i-1} + w^i_k$ for i = 1, …, N with $c_0=0$
Let $i=1$ and draw a starting point u_1 from the uniform distribution U[0, N^{-1}]
For $j=1, \ldots, N$
Move along the CSW by making $u_j = u_i + N^{-1}$ (j-1)
While $u_j > c_i$ make $i= i+ 1$
Assign sample $x^i_k = x^i_k$
Assign sample $w^i_k = N^{-1}$

The particles are composed by the state variables x_k, in this work, considered as the heat flux and the radial temperature, as follow:

$$x_k = [q, T_1, T_2, ..., T_M]$$

(5)

The states variables evolve with the evolution model. The heat flux is updated following a Gaussian random walk model around the current value of each particle, as presented in Eq. 6. The radial temperature is calculated from equations 1 to 4 presented before with a noise of 1% of the state variable value.

$$q_k = q_{k-1} + \sigma_q \varepsilon$$

(6)

The synthetic measurements (z_k) are taken as the temperatures at the inner surface (Y_1) evolving with time (t):

$$z_k = \left[Y_1(t_1), Y_1(t_2), ..., Y_1(t_{end}) \right]$$

(7)

The weight for each particle is calculated from the observation model, here considered as the core of a gaussian distribution, described in Eq. (5).

$$w = \exp\left[-\frac{1}{2\sigma_m^2} (T_{1k} - Y_{1k})(T_{1k} - Y_{1k})^T \right]$$

(8)

4.1 Verification of the numerical solution

The finite difference numerical solution was verified by establishing a comparison between the present results and a previously research report, Beltran et al. (2012), that proposed an analytical approach for a single round situation and established an external cooling by natural convection. The one-dimensional heat flow model is adopted to simulate the transient temperature field and the exponential heat flux formulation, described in Eq. (9), is considered with parameter b set to 210.97 s^{-1} together with an initial heat flux of magnitude $q_0 = 192.7$ MW/m².

$$q(t) = q_0 \exp(-bt)$$

(9)

Here, the thickness of the wall is considered to be 30 mm and the barrel is supposed to be manufactured from special steel which is typically used in weaponry. Table 2 describes the parameters and thermophysical properties employed in the verification phase.

Table 2. Parameters of the 155 mm howitzer.

Parameters	Value	Unit
Thermal conductivity, k	40	W/(m.K)
Specific heat, c_p	460	J/(kg.K)
Density, ρ	7833	kg/m³
Heat transfer coefficient, h_∞	40	W/(m².K)
External temperature, T_∞	27	°C
Initial temperature, T_0	27	°C
Inner radius, R_{in}	77.5	mm
Outer radius, R_{ext}	107.5	mm

Figure 1 displays a comparison of the present evaluations with previously published results for a single round. The heating period for one round is defined to be 0.02 s with a magnitude time step of 0.1 ms. It can be observed that the inner surface reaches a maximum bore temperature of 700.73°C in 4.07 ms during the heating phase, followed by a rapid decline, while the outer temperature of the barrel remains practically unchanged. An analysis of these results indicates good agreement for both the inner and outer surfaces of the gun and therefore, the numerical code and the numerical solution are considered to be validated.
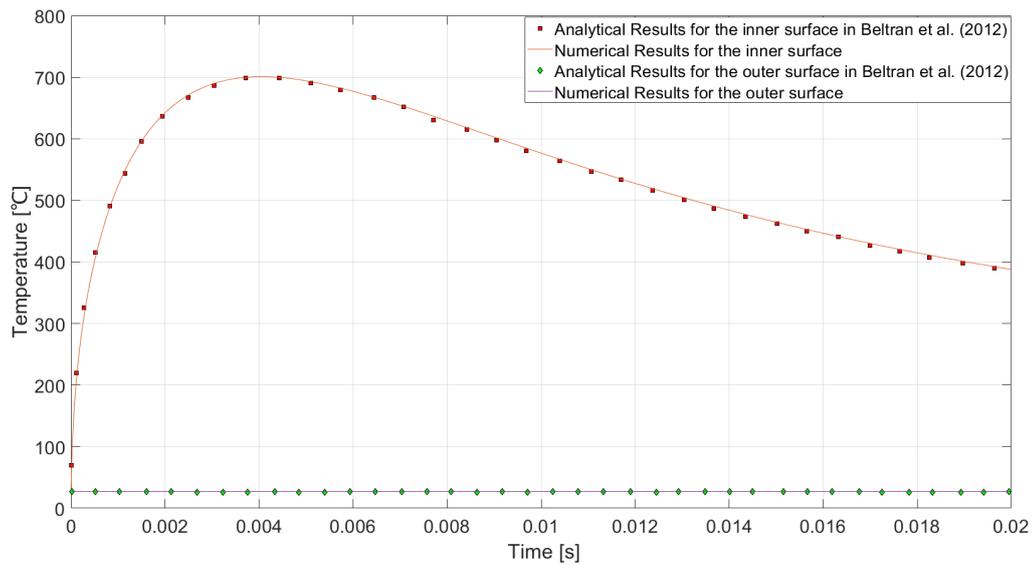
Figure 1. Comparison between the numerical and the analytical solutions for a single round.

5. RESULTS AND DISCUSSION

To generate the simulated temperatures measurements, two distinct heat flux distributions were investigated, indicated in Eqs. 10 and 11, together with the parameters shown in Table 3. Equations 1 to 4 were discretized by means of an implicit finite difference scheme and solved using Thomas algorithm (Ozisik et al., 2017). The geometrical and thermal parameters used to produce the simulated temperature measurements are provided in Table 4. A noise of 1°C was added to the temperature measurements to represent the associated uncertainties. The heating period of a single round was taken as 0.02 s with a computational grid of $\Delta r = 3.5$ μm and a time step size of $\Delta t = 1.0$ ms.

Case 1	$q(t) = q_{01} \exp(-bt)$	(10)
Case 2	$q(t) = q_{02} \dfrac{\eta}{\lambda}\left(\dfrac{t}{\lambda}\right)^{\eta-1} \exp\left[-\left(\dfrac{t}{\lambda}\right)^{\eta}\right]$	(11)

Table 3. Parameters of heat flux to generate the synthetic measurements.

Parameters	Value	Unit
b	0.0055	s^{-1}
λ	0.0055	-
η	2	-
q_{01}	21.79	MW/m²
q_{02}	0.12	MW/m²

Table 4. Parameters of the direct problem.

Parameters	Value	Unit
Thermal conductivity, k	50	W/(m.K)
Specific heat, c_p	470	J/(kg.K)
Density, ρ	7800	kg/m³
Heat transfer coefficient, h_∞	28	W/(m².K)
External temperature, T_∞	27	°C
Initial temperature, T_0	27	°C
Inner radius, R_{in}	40.5	mm
Outer radius, R_{ext}	47.5	mm

The inverse problem used the same parameters presented in Table 4. However, in order to avoid the inverse crime, the computational domain was changed to $\Delta r = 4.67$ μm. The initial values investigated to the heat flux are indicated in Table 5. In both cases, the random walk parameter to the heat flux evolution model σ_q was adopted as 1 MW/m². The states variables have a non-informative prior, considering that the heat flux could not assume negative values.

Table 5. Heat flux initial values investigated in the numerical analysis.

Case	Parameters	Value	Unit
1	$q\,(t = 0)$	10	MW/m²
	$q\,(t = 0)$	21.8	MW/m²
	$q\,(t = 0)$	30	MW/m²
2	$q\,(t = 0)$	0.01	MW/m²
	$q\,(t = 0)$	0.12	MW/m²
	$q\,(t = 0)$	10	MW/m²

An inspection of Fig. 2 to Fig. 4 shows that using 20 particles the temperature and the heat flux are already recovered. However, due to the small standard deviation of the particles, the 95% credible interval did not contain the measurements of temperature. Increasing the number of particles solves this issue, as can be seen in the figures using 1000 particles. Furthermore, in Fig. 2, due to the initial heat flux value adopted be smaller than the real, the curve tends to have an increase of the magnitude before have the exponential decay behavior. For Figs. 3 and 4, the exponential decay is observed since the beginning of the estimations.
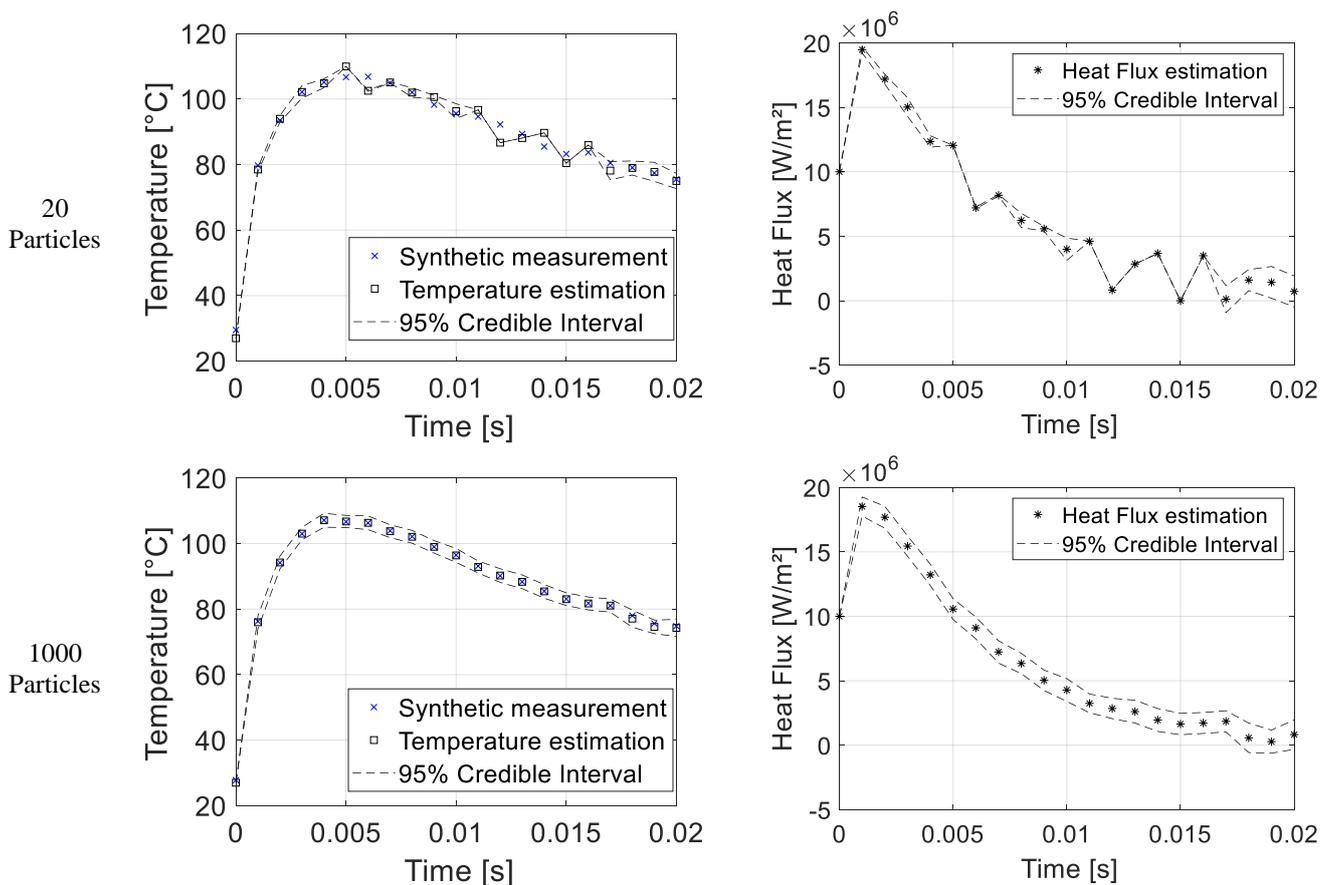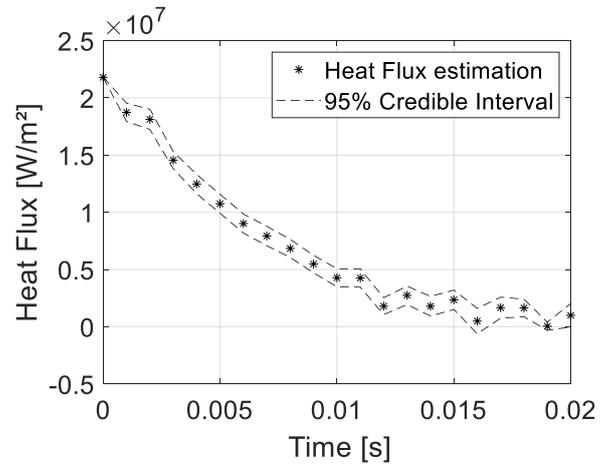

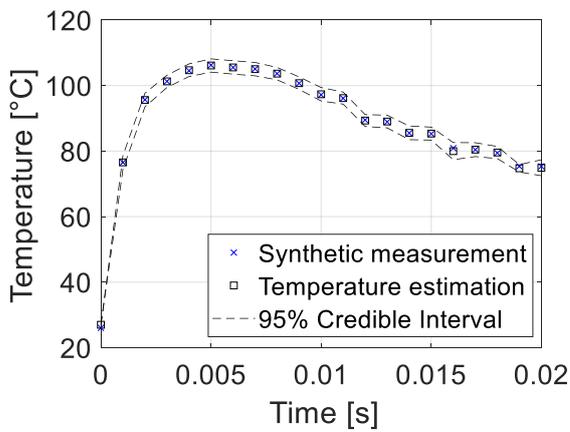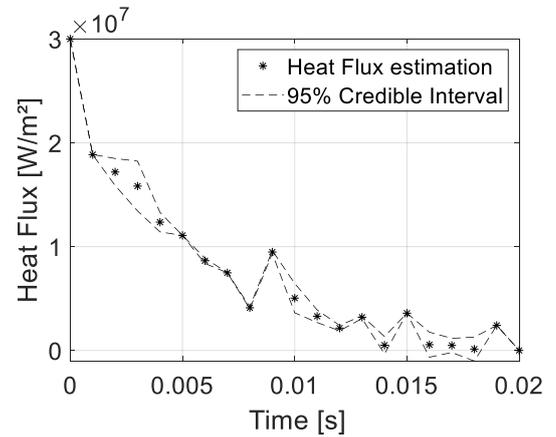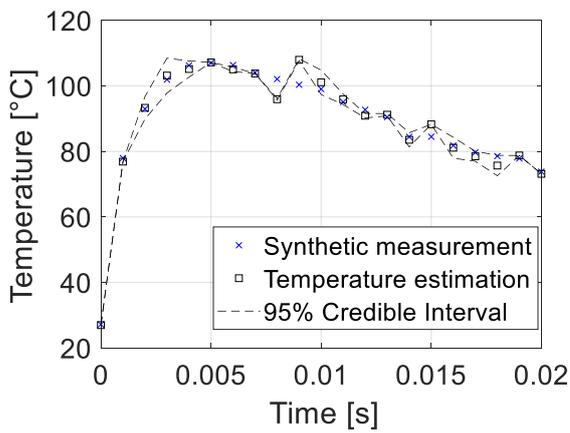Figure 2. State variables estimation considering an initial value of 10 MW/m² – Case 1.
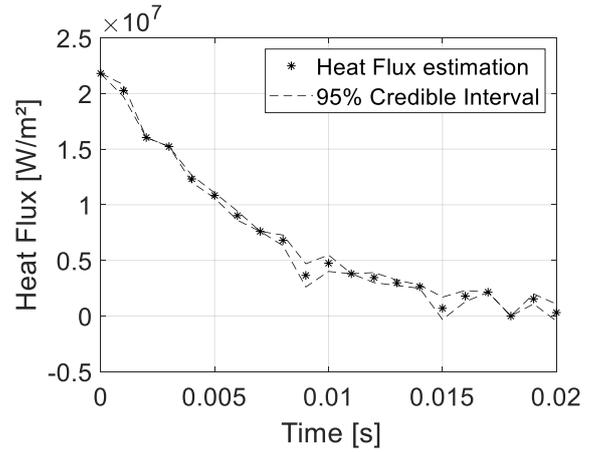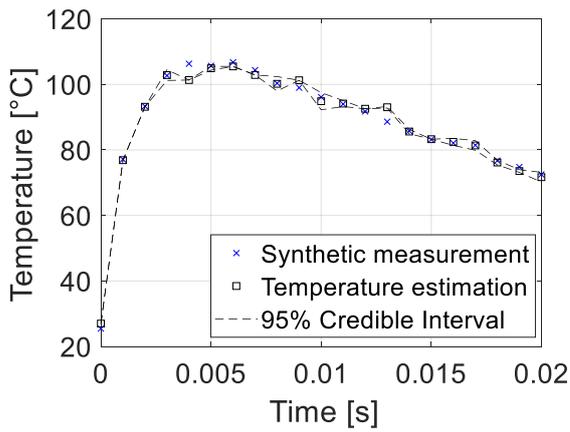
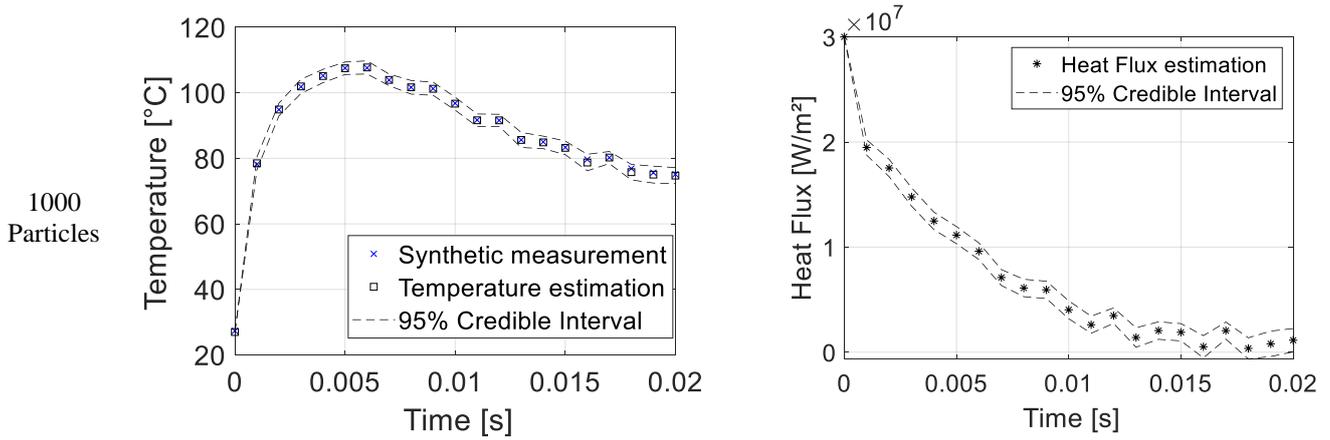Figure 3. State variables estimation considering an initial value of 21.8 MW/m² – Case 1.

Figure 4. State variables estimation considering an initial value of 30 MW/m² – Case 1.

In Figs. 5 to 7, the Weibull distribution was used to generate the synthetic measurements. The heat flux distribution was recovered from a random walk model. As analyzed for case 1, the increase in the number of particles provides a better estimation. The influence of the starting value to the heat flux is also noticeable, in this case, having an initial heat flux value higher than the real one provides a distinct curve associated to the Weibull distribution, but this does not influence the filtering of temperature measurements.
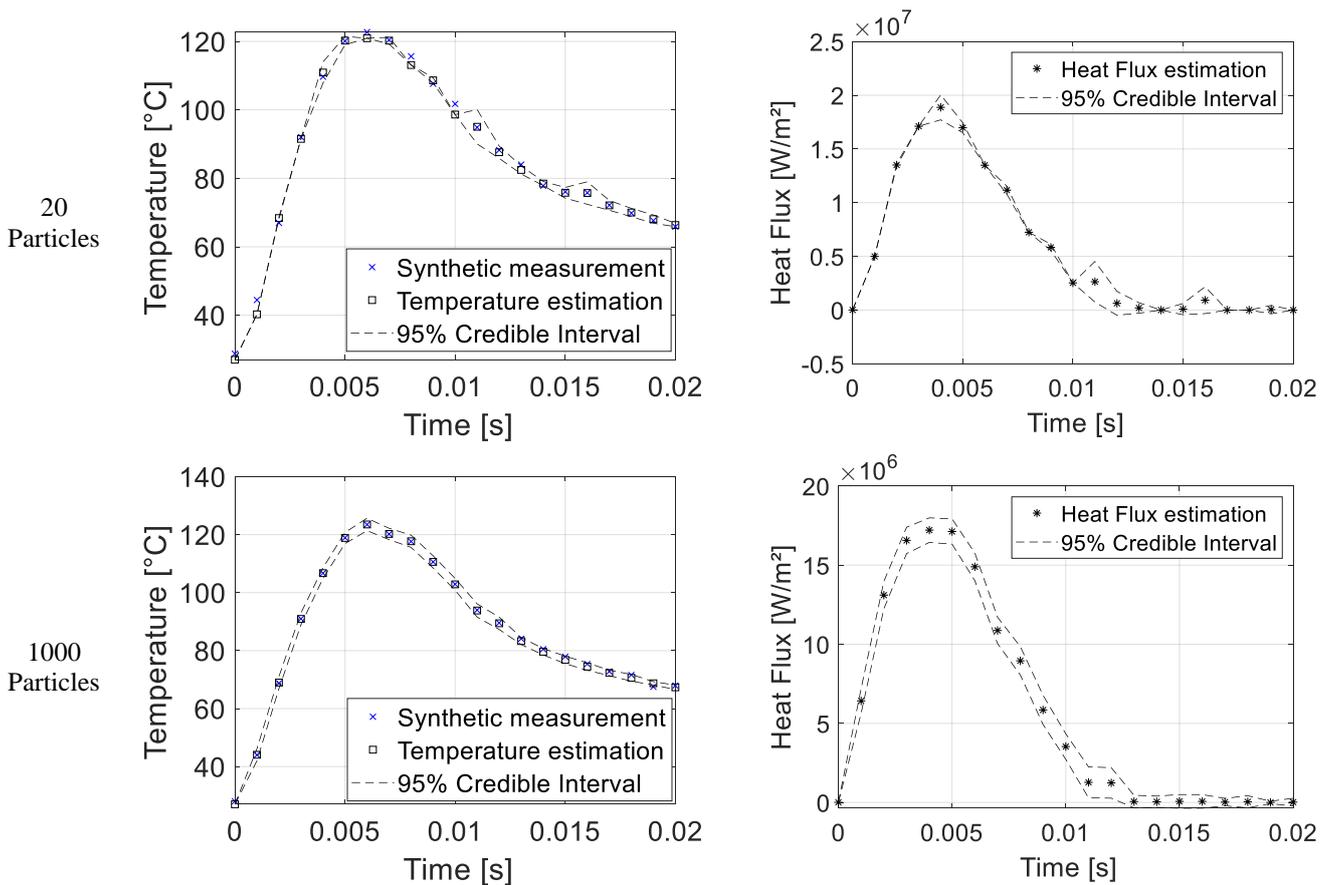

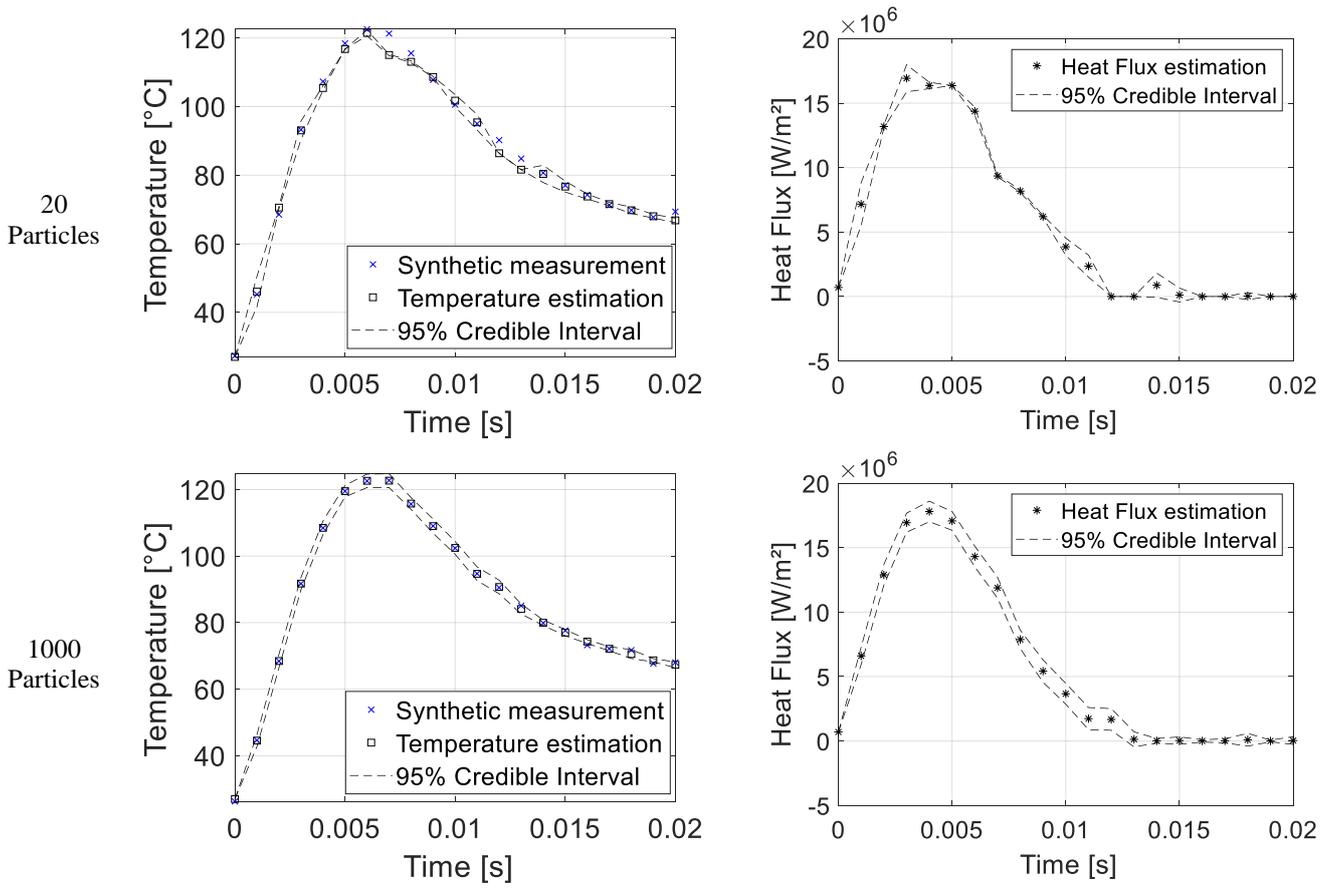Figure 5.State variables estimation considering an initial value of 0.01 MW/m² – Case 2.

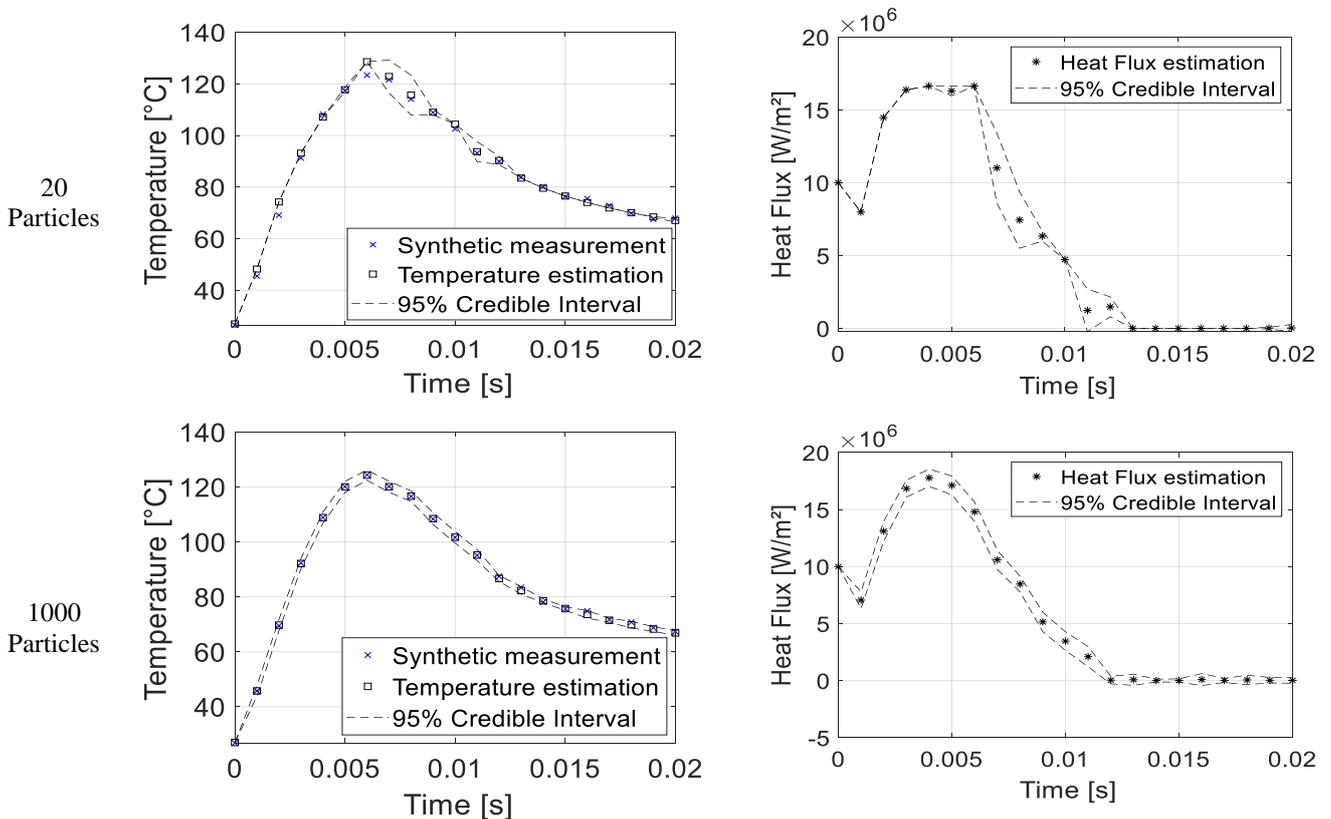Figure 6. State variables estimation considering an initial value of 0.12 MW/m² – Case 2.

Figure 7. State variables estimation considering an initial value of 10 MW/m² – Case 2.

In conclusion, the present contribution addressed an inverse analysis to the heat flux associated to the energy released by the propellant gases in an 81 mm mortar. The Sampling Importance Resampling (SIR) algorithm is applied for the state estimation problem. Synthetic measurements of the inner surface temperature of the weapon are employed considering different types of heat flux related to gun heating situations. The results obtained so far indicate that the estimations performed by the proposed algorithm are within a desirable pattern of accuracy for simulations using 1000 particles. Our current research efforts are now focused on the evaluation of the inner heat flux for multiples rounds and using temperature measurements from the outer surface of the gun barrel.

6. ACKNOWLEDGEMENTS

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