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PARAMETRIC ANALYSIS OF A RECUPERATIVE HEAT EXCHANGER

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Abstract. As time passes, the level of complexity of the systems that are part of cars, aircraft and other elements that make up the lives of citizens in large cities grows in order to meet the new stricter rules in the environmental appeal. with low cost and high energy efficiency. Mathematical modeling together with simulation and optimization emerge as an extremely important tool in this scenario, as it can provide good results from low initial investments when compared to a purely experimental approach in the design of new devices. Heat exchangers are widely used in the most diverse areas of society, as it consists of equipment whose main objective is to facilitate the thermal exchange between two or more fluids that can be used in a range of different applications. In this way, the design and parametric optimization of such devices in order to obtain maximum performance, from the maximum reduction of entropy generation and maximization of energy efficiency, are of paramount importance when it comes to the production and development of new heat exchanger projects that adapt to new trends in terms of the environment and energy efficiency. The simulation of these systems takes place under a broad set of operating parameters, such as mass flow and line pressure, as well as geometric parameters, such as pipe diameter and pipe position. Thus, the optimization step may become unfeasible if the number of parameters is too high, which implies computational times that are impractical for the purposes of the project. Given the above, it is often necessary to introduce simplifying considerations and hypotheses in order to reduce the number of variables or terms in the equations without significant loss of translation of the physics of the problem. Thus, reduced-order models, which constitute an intermediate approach when compared to low- and high-order methods, can be used as a mathematical modeling tool without a significant loss of precision in the results. The present work presents an optimization and parametric analysis of a recuperative heat exchanger using a reduced-order approach employing the volume element model (VEM) as a discretization method, which is capable of providing accurate results at low costs. computational. The Laws of Conservation of Mass and Energy are applied to volume elements in combination with empirical correlations in order to quantify the quantities of interest, such as the convection heat transfer coefficient and temperature distribution. A parametric analysis was performed in order to observe the behavior of entropy generation in order to find its minimum points. The mass flow of water varied from 0.001 kg/s to 0.0085 kg/s with the mass flow of hot gases remaining constant at 0.14 kg/s. A local minimum was found at a mass flow rate of 0.0015 kg/s.

Keywords: Volume element model, optimization, parametric analysis, recuperative heat exchanger

1. INTRODUCTION

With each passing day, society is becoming more dependent on electronic equipment, either because of the comfort that comes from it, or because of the possibility and convenience of carrying out a certain process using only the touch of your fingers and even your voice. Nowadays, books are exchanged for tablets, magazines and newspapers for laptops and conversations and dialogues on social networks using smartphones, for example. All of this costs a price: the increase in electricity consumption and the increase in the levels of waste that are produced and discarded each day.

It is important to emphasize that the increase in energy consumption, as well as the increase in the production of waste, is closely related to the level of economic activity of a society and because of this, it is a reflection of its global behavior, as well as of industrial, commercial activity, and services.

The search for more sustainable ways of reusing and / or treating MSW has become increasingly a global trend, and with this, new technologies are incorporated into existing systems with the aim of reversing the unfavorable scenario of increased consumption for next years. Studies indicate that there is a propensity that in 2050 it will be produced around 3.5 billions of tons of solid urban waste as a result of population growth and greater population urbanization (Kaza *et al.*, 2018). This requires measures to be taken in order to have better management regarding the reuse of products, awareness of the population and a more efficient and sustainable use of our energy reserves.

A possible alternative for the use of the material to be discarded is the possibility of enjoying the potential energy that is present in them. One type of energy recovery procedure is to exploit the energy capacity that has been potentially stored in the waste, transforming them into electrical energy, heat and / or fuel from the treatment of waste. This process known as waste-to-energy (WtE), allows to reduce the volume of material that would be deposited in landfills or other storage modality, allowing the conversion of the energy that was accumulated in the RSU, and would be wasted, in other forms of energy such as heat and / or electrical energy (Magnaleli *et al.*, 2020).

Several studies are carried out every year in order to better understand the processes and energy conversion in the biomass burning procedure that occurs inside the system (Santos and Ceribeli, 2013), as well as the search for improvement in efficiency and the reduction of pollutant production, as well as the gases that are released in the combustion process.

In view of the above, it is clear that one of the great challenges that exists today, is to be able to increase energy production efficiently, preserving the environment. Thus, the development of equipment such as heat exchangers that can use energy with minimal waste, becomes something substantial, thus allowing the improvement of industrial processes and contributing to the reduction of emissions polluting gases (Kuruneru *et al.*, 2021).

The implementation of studies in more complex systems with the objective of improving energy efficiency, reducing production costs and environmental problems is directly related to the ability to obtain answers in a reasonable time. Thus, the modeling and simulation of physical, biological, electrical, thermal, mechanical elements, among others, is an important technique for the improvement and optimization of equipment and devices in general.

In general, mathematical modeling can be applied both at the system and component levels (Sage, 1992). It is possible to find in the literature several classifications for these two elements, for example: i) qualitative modeling: where it is possible to accurately observe the trends in responses, but has low precision of absolute values and in local variables; quantitative modeling, that is, that has precision in response trends and in the values of local variables (Woods and Lawrence, 1997; Vargas *et al.*, 2001); ii) Modeling of high order (high order) and low order (low order) (Shapiro, 2003); and iii) Concentrated and distributed modeling (Trivelato, 2003; Kaiser, 2004).

Thermodynamic optimization is of paramount importance in the search for better performances and/or performances from the constraints associated with environmental and geometric issues (BEJAN, A.; MAMUT, E., 1998). The optimization step in the design of heat exchangers requires an extreme knowledge of thermodynamics, fluid dynamics and how much or in which steps there are higher costs related to the design or physical construction of the same (RAJA, B.D. *et al.*, 2017).

Thus, the present article develops a mathematical model in a quasi-permanent and transient regime, which involves the system in a global way, using the volume element method, VEM, in order to build a simplified mathematical model that allows to provide with precision and low computational time system responses. A parametric analysis was performed in order to observe the behavior of entropy generation in order to find its minimum points.

2. SYSTEM IN ANALYSIS

Several operational characteristics of the equipment under study are obtained from the manufacturer of the same. A schematic representation of the various subsystems that make up the bioenergetic engineering system installed at the Center for Research and Development in Self-Sustainable Energy at the Federal University of Paraná can be seen in "Figure 1".

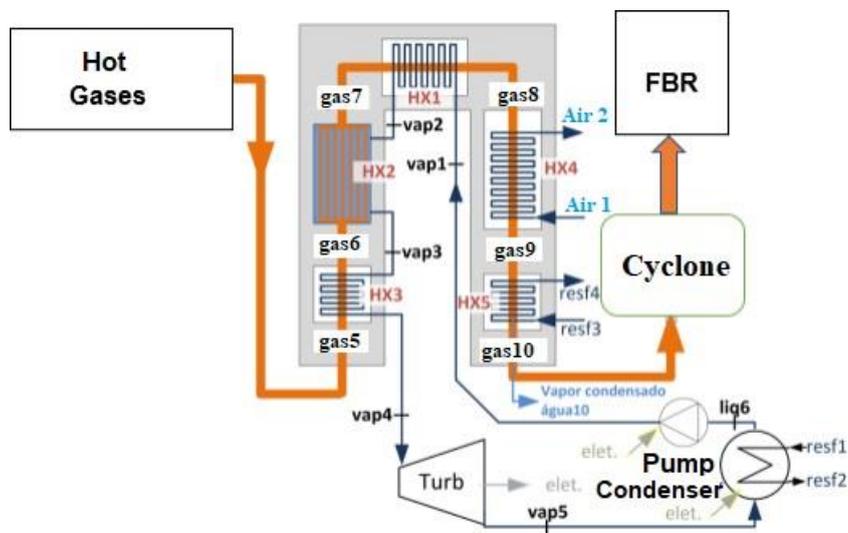


Figure 1. Illustration of hot gases, heat exchangers, Rankine cycle and photobioreactors.
Available from: Galante, 2019.

Hot gases are produced from the processing of MSW originating from UFPR. In order to obtain complete combustion, preheating from the use of liquefied petroleum gas (LPG) fuel in the main combustion chamber is necessary. Just as the supply of quality and distribution of air for combustion promotes the burning of MSW, however the material that does not undergo complete combustion is directed to the lower part below the grid, originating most of the ashes generated, around 80%. Then the gases pass through the post-combustion chamber, where the material comes in contact with the LPG again to guarantee the complete oxidation process of the material. In this step, the use of fuel, liquefied petroleum gas, is made to ensure that the temperature is above 900 ° C and thus the release of dioxins and the formation of nitrous oxide is avoided.

The main function of the incinerator is to be able to provide a correct destination for the residues by converting the chemical energy associated with them into the thermal energy of the combustion products. As an important part of this stage, the gases with high temperature are sent to a set composed of three heat exchangers called, respectively, HX3, HX2 and HX1. In this phase, the water flowing in the opposite direction to the gases is initially heated in the TR HX1 and directed to the HX2 where it will undergo the phase change process. To ensure that only steam enters the turbine, the water accesses a third heat exchanger, HX3, where it will be overheated.

The cyclone is installed before the FBR to reduce the amount of ash in the flue gases. In this model, around 20% of the ash mass is removed in this device. The plant's latest equipment is the FBR, which allows the cultivation of microalgae with the help of flue gases rich in CO_2 and agro-industrial waste diluted in water as a culture medium. The FBR set represented here is composed of 6 units of 10 m³ of culture medium.

3. MATHEMATICAL MODELING IN A REDUCED ORDER FOR HEAT EXCHANGERS

The mathematical model of reduced order of the heat exchangers, HX1, HX2 and HX3 proposed in this article, is based on the application of the principles of classical thermodynamics explained in the laws of conservation of mass, energy and concepts of heat transfer. The discretization process is done by dividing the domain of the system under analysis into cells of finite centered volumes called volume elements. This step is important, as the size of each volume element does not necessarily have to be small to ensure stability numerical and the precision of the results as it happens with methods like finite differences, finite elements and finite volumes. Here is described the mathematical modeling for the three heat exchangers under analysis in this article and the simplifications associated with the model.

3.1 Mathematical Modeling of Heat Exchangers HX1, HX2 and HX3

Figures 2, 3 and 4 illustrate the schematic diagrams for each of the three heat exchangers. In Figure 2, the division of the volume elements is also exposed, which will be omitted for the others, as it consists of an exactly the same step. The principles of conservation of mass and energy are applied to the system to obtain the mathematical model.

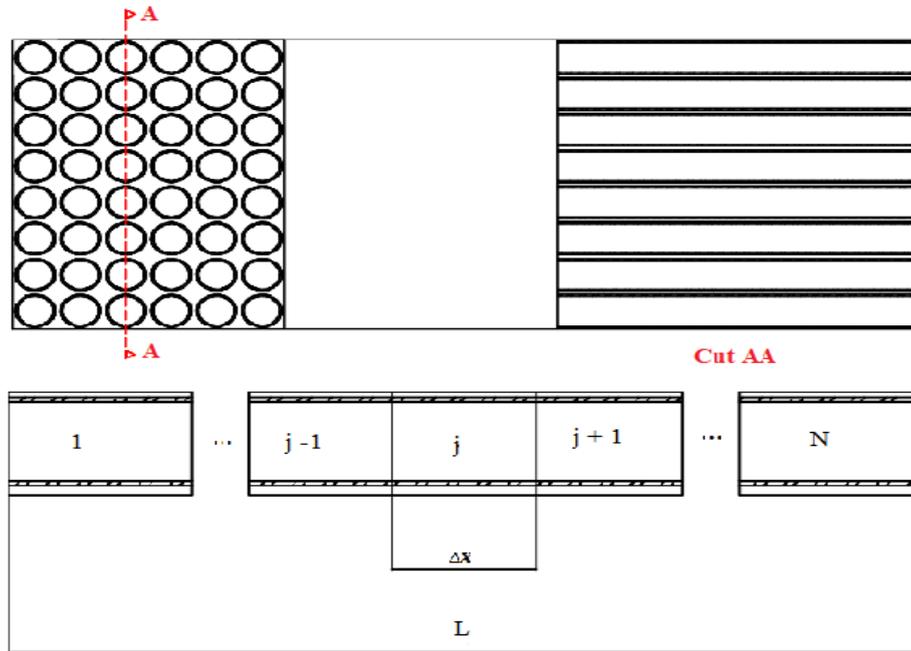


Figure 2. Schematic diagram of recuperative heat exchangers (XH1) and the division of the system into volume elements.

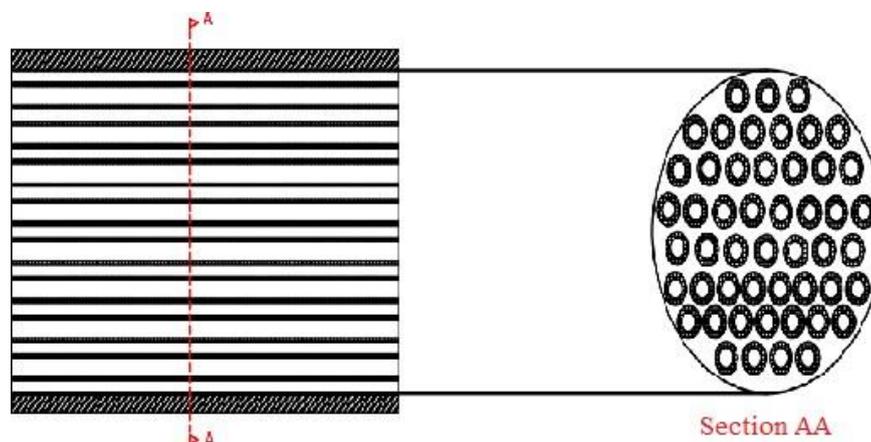


Figure 3. Schematic diagram of recuperative heat exchangers (XH2).

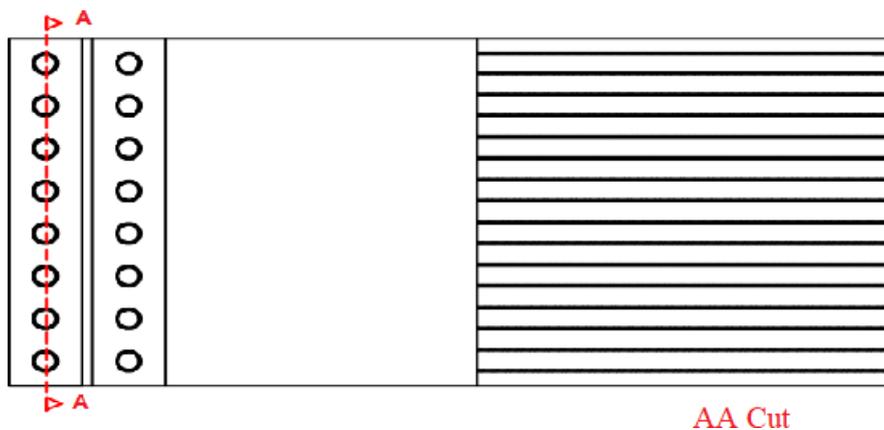


Figure 4. Schematic diagram of recuperative heat exchangers (XH3).

At first, the equipment was divided into volume elements as seen at the bottom of Figure 2, as well as its details can be seen in Figure 5 and the mass and energy transfers, which elucidates a mixed behavior (fluid and solid) of each volume element. In order to characterize the type of exchanger in terms of functionality and performance and to be able

to map the phase in which the fluid, in this case water, is in each heat exchanger, that is, if it is subcooled, superheated and in phase change, each volume element will be separated into five subsystems, namely: subsystem 1: Tube (solid part); subsystem 2: Hot gases; subsystem 3: Subcooled liquid; subsystem 4: phase change; subsystem 5: superheated steam.

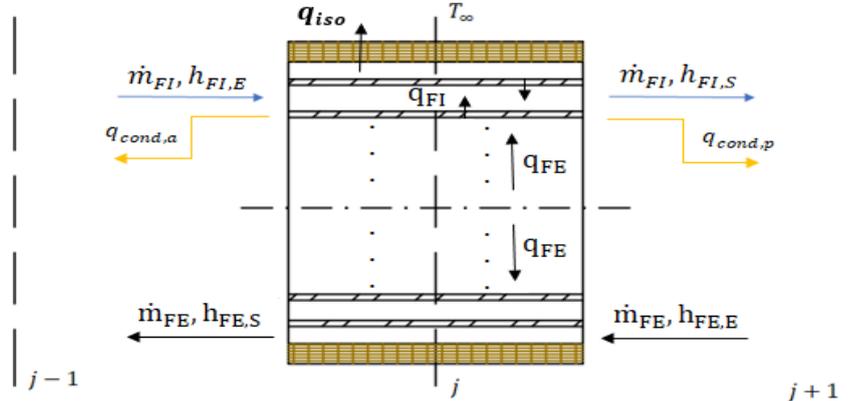


Figure 5. Detail of the volume element, where the mass and heat interactions between subsystems can be observed.

It is important to note that each subsystem permeates all heat exchangers, HX1, HX2 and HX3, but does not necessarily coexist in the same heat exchanger. For example, the HX1 exchanger has characteristics of an economizer, so that, theoretically, it will use energy from the waste combustion gases and heat the water to temperatures close to saturation. Thus, subsystems 4 and 5 do not make sense at this stage, as only liquid will be present. The division into subsystems is necessary for the three exchangers, as it allows the development of a strategic mapping of the point where the phase change will occur, that is, at which point the liquid will assume the saturation temperature and the title, X, will start to vary until reaching one value and all liquid to vapor.

The next step lies in the mathematical modeling of each subsystem within the volume element illustrated in Figure 6. In this analysis, the principles of combined mass and energy conservation are used, as well as evaluating the energy interactions between all subsystems a from the appropriate equations to quantify them.

Subsystem 1: Tube (solid part): The 1st Law of Thermodynamics is applied to system 1, according to the diagram in Figure 6, the following equation is obtained:

$$m_{T,j} \cdot c_T \cdot \frac{dT_{T,j}}{dt} = q_{FE,j} + q_{cond,a,j} + q_{cond,p,j} - q_{FI,j} \quad (1)$$

Where

$$q_{FE,j} = h_{FE,j} \cdot A_{ET,j} \cdot (T_{FE,j} - T_{T,j}) \quad (2)$$

$$q_{cond,a,j} = -k_{T,j} \cdot A_{AT,j} \cdot \left(\frac{T_{T,j} - T_{T,j-1}}{\Delta x} \right) \quad (3)$$

$$q_{cond,p,j} = -k_{T,j} \cdot A_{AT,j} \cdot \left(\frac{T_{T,j} - T_{T,j+1}}{\Delta x} \right) \quad (4)$$

$$q_{FI,j} = h_{FI,j} \cdot A_{IT,j} \cdot (T_{T,j} - T_{FI,j}) \quad (5)$$

The boundary conditions are defined as

$$\text{First volume element (cell): } q_{cond,a} = q_{cond,p} = 0 \quad (6)$$

$$\text{Last volume element (cell): } q_{cond,p} = q_{cond,a} = 0 \quad (7)$$

Equations 6 and 7 assume that heat transfers through the ends of the heat exchanger can be neglected when compared to the loss through the outer sidewall.

Subsystem 2: Hot gases (external flow): Applying the first law of thermodynamics to subsystem two, which consists of the hot gases that were processed in the incinerator and in the post-combustion chamber, as shown in Figure 6, and assuming variables without a subscript for the hot fluid, we have:

$$m_{FE,j} \cdot c_{v,FE} \cdot \frac{dT_{T,j}}{dt} = \dot{m}_{FE,j} \cdot c_{p,FE} \cdot (T_{FE,j+1} - T_{FE,j}) - q_{FE,j} - q_{ISO,j} \quad (8)$$

Where

$$q_{ISO,j} = (U \cdot A)_{ISO,j} \cdot (T_{FE,j} - T_{\infty}) \quad (9)$$

$$(U \cdot A)_{ISO,j} = \left[\frac{1}{h_{FE,j} \cdot A_{iISO,j}} + \frac{\ln\left(\frac{d_{eISO,j}}{d_{iISO,j}}\right)}{2 \cdot \pi \cdot k_{ISO} \cdot L} + \frac{1}{h_{\infty,j} \cdot A_{eISO,j}} \right] \quad (10)$$

$$A_{eISO} = \pi \cdot d_{eISO,j} \cdot \Delta x \quad (11)$$

$$A_{iISO} = \pi \cdot d_{iISO,j} \cdot \Delta x \quad (12)$$

The boundary conditions are

$$T_0 = T_E = 0 \text{ and } \frac{\partial T_n}{\partial x} = 0 \quad (13)$$

Subsystem 3: Subcooled liquid: Again the 1st law of thermodynamics is applied to subsystem 3, as seen in Figure 6, providing the following equation:

$$m_{FL,j} \cdot c_{FL} \cdot \frac{dT_{FL,j}}{dt} = \dot{m}_{FL,j} \cdot c_{p,FL} \cdot (T_{FL,j-1} - T_{FE,j}) + q_{Fi,j} \quad (14)$$

In which

$$q_{Fi,j} = h_{Fi,j} \cdot A_{FL,j} \cdot (T_{T,j} - T_{FL,j}) \quad (15)$$

With

$$A_{FL,j} = \pi \cdot \frac{d_{FL,j}^2}{4} \quad (16)$$

The boundary conditions are

$$T_{r,n} = T_{r,E} \text{ (known parameter) and } \frac{\partial T_{r,0}}{\partial x} = 0 \quad (17)$$

Subsystem 4: Phase change: For the mathematical modeling of the phase change process, a quasi-permanent regime will be considered, that is, the deviations that a given property undergoes in time is considered to be infinitesimal when comparing their changes in space. Thus, applying energy conservation, one has to $\frac{dE}{dt} = 0$, during the integration interval, Δt , which is justified for small amounts of Δx .

$$0 = \dot{m} \cdot X_{j-1} \cdot h_v + \dot{m} \cdot (1 - X_{j-1}) \cdot h_l - \dot{m} \cdot X_j \cdot h_v - \dot{m} \cdot (1 - X_j) \cdot h_l + q_{FL,j} \quad (18)$$

On what

$$q_{FL,j} = h_{FL,j} \cdot A_{FL,j} \cdot (T_{T,j} - T_{FL,sat,j}) \quad (19)$$

Subsystem 5: Superheated steam: The modeling for subsystem 5 follows the same process as for subsystem 3, however, now the physical properties needed to quantify the quantities will be done for superheated steam.

$$m_{FL,v,j} \cdot c_{v,FL} \cdot \frac{dT_{FL,v,j}}{dt} = \dot{m}_{FL,v,j} \cdot c_{p,FL} \cdot (T_{FL,j-1} - T_{FE,j}) + q_{FL,v,j} \quad (20)$$

In which

$$q_{FL,v,j} = h_{FL,v,j} \cdot A_{FL,j} \cdot (T_{FL,v,j-1} - T_{FL,v,j}) \quad (21)$$

Thermodynamic objective function is formulated by considering the effectiveness and entropy generation rate (S). Equation 22 elucidates the methodology proposed by Bejan, 1977, for two counter-current flows expressed as a function of temperature and pressure.

$$\dot{S} = \dot{m}_H \left[c_{p,H} \left(\frac{T_{H,2}}{T_{H,1}} \right) - R_H \left(\frac{P_{H,2}}{P_{H,1}} \right) \right] + \dot{m}_C \left[c_{p,C} \ln \left(\frac{T_{C,2}}{T_{C,1}} \right) - R_C \ln \left(\frac{P_{C,2}}{P_{C,1}} \right) \right] \quad (22)$$

The mathematical model was computationally implemented using MATLAB® (developed by MathWorks inc.). The problem is equivalent to obtaining the approximate solution of the system of basic equations presented above. It is important to note that this set of equations has a mixed characteristic, that is, for subsystem 4, which was considered to operate in an almost permanent regime, the resulting equation for each EV is algebraic, so the iterative method of Gauss Seidel was applied with the objective of enabling the approximate solution of these equations. In addition, the heat transfer coefficients for water and hot gases were considered constant, that is, a priori, changes in them will not be taken into account as, for example, the liquid water is in phase change.

The equations developed for the other subsystems can be explicitly integrated in relation to time using an adaptive time step with the fourth and fifth order Runge-Kutta method (KINCAID and CHENEY, 1991). The time step is automatically traversed according to the local truncation error, which is kept below a compatible tolerance of 10^{-6} .

4. RESULTS AND DISCUSSION

The initial conditions used in the simulations were $T_\infty = 298.15$ K for $1 \leq j \leq n$. The physical parameters used in this work to simulate the system shown in figures 2, 3, 4 and 5 were $n = 20$ for the converged mesh, the choice of the number of volume elements was made following the work done by Dilay et al. (2014), $L = 5.5$ m, $\dot{m}_w = 1.01$ kg.s⁻¹, $\dot{m}_g = 3.5$ kg.s⁻¹, $c_{p,g} = 1.004$ kJ.kg⁻¹.K⁻¹, $c_{v,g} = 0.717$ kJ.kg⁻¹.K⁻¹, $h_g = 2.2$ W.m⁻².K, $h_w = 5.1$ W.m⁻².K, $\rho_{w,L} = 1000$ kg/m³, $r_i = 0.5$ m, $r_e = 0.55$ m, $c_{p,sh} = 2$ kJ.kg⁻¹.K⁻¹, $c_{v,sh} = 1.5$ kJ.kg⁻¹.K⁻¹, $P_{sat} = 101325$ Pa, $R_w = 0.46152$ kJ.kg.K, $c_{w,L} = 4.18$ kJ.kg⁻¹.K⁻¹, $\rho_{steel} = 7854$ kg/m³, $c_{steel} = 0.434$ kJ.kg⁻¹.K⁻¹, $\rho_g = 1.225$ kg/m³, $\rho_{w,L} = 1000$ kg/m³, $\rho_{w,vap} = 0.59$ kg/m³, $k_{steel} = 0.036$ kW/m.K, $h_{w,L} = 417.46$ kJ/kg, $h_{w,vap} = 2675.5$ kJ/kg, $u_{w,L} = 417.46$ kJ/kg, $u_{w,vap} = 2675.5$ kJ/kg, $T_{in,air} = 1200$ °C and $T_{sat,w} = 100$ °C.

Figure 6 illustrates the steady-state behavior of the water temperature variation along the flow in the countercurrent heat exchanger as a function of its length.

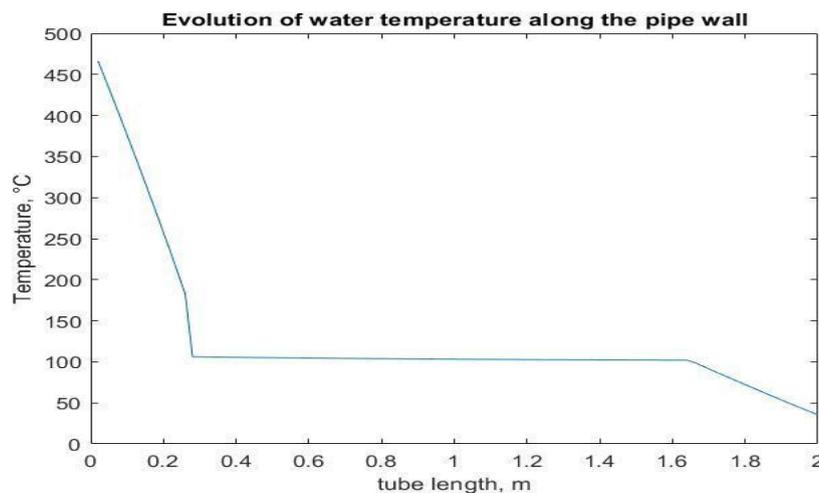


Figure 6. Evolution of water temperature in the heat exchanger for a tube length of 2 m.

The water enters at a temperature of 25 °C and leaves at approximately 416.37 °C as it flows countercurrent to the flow of hot gases along the length of the tube.

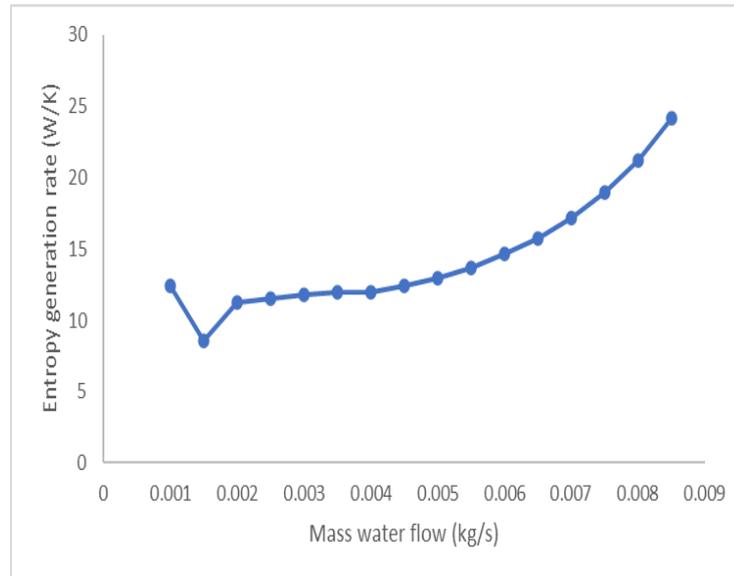


Figure 7. Entropy generation rate as a function of water mass flow.

Observing the behavior of the graph in Figure 7, it is possible to find a point of minimum entropy generation rate as the mass flow of hot gases remains constant and the mass flow of water varies. That is, there is a maximum mass flow of water where as it decreases or exceeds its value, the rate of entropy generation begins to increase. This happens because as the water flow is increased too much for a constant flow of hot gases, the heat from the gases begins to not be able to supply the energy demand. On the other hand, the low water flow suggested a waste of energy associated with hot gases.

5. CONCLUSIONS

In this work, the element of volume method (VEM) was used for mathematical modeling in an engineering system, that is, a recuperative heat exchanger. This system is formed by multiple subsystems, each with characteristics intrinsic to each one. Furthermore, this system interacts with each other by heat and mass transfer.

Thus, a mathematical model was developed for a recuperative heat exchanger that consists of equipment widely used in engineering, so that the use of VEM allowed to demonstrate for a small number of volume elements (VE), in the case $n=20$ and, consequently, low simulation time, the answer to the evolution of temperatures for the countercurrent flow of water and hot gas for constant heat transfer coefficients. Thus, it is expected that VEM can be applied as an efficient methodology with low computational cost for the design, simulation, control and optimization of engineering systems.

6. ACKNOWLEDGEMENTS

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