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NUMERICAL APPROXIMATION FOR POROUS PARALLEL FINS WITH RADIATION

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Abstract. *This work aims to find the temperature distribution of porous parallel fins with insulated tips in a dominant radiation situation. Using the prevalent mathematical dimensionless models in the literature, the problem becomes similar to the solid fin case. However, due to the consideration of radiation from another fin and from the base, the model gives rise to an Integro-Differential problem. In order to solve the differential equations, the Finite Difference Method (FDM) is employed, along with the usage of a weighting parameter α which gives rise to a gradually increasing sequence of elements that numerically converge. For evaluating the integrals, the high-speed Trapezoidal Rule method approximates the heat transfer between the parallel porous fins due to radiation and the heat transfer between the fin and the fin's base, which is considered to be at a constant temperature. It must be noted that the integrals must be solved first. Then by using the previously described method alongside Gauss-Seidel iterations, the problem can be solved for each element of the sequence. After this procedure, the sequence is built (for each element) until it converges according to an established tolerance. The obtained results are in good agreement with the literature, such as, for example, a rise in the emissivity of the porous fins causing the decrease of the temperature distribution even when considering radiation from the fin's base and from the other parallel fin. The heat transfer from the porous fin to the environment still surpasses both of them in many situations. Another interesting result is the impact of the fin's base on the temperature distribution. As the length of the base increases, the temperature distribution on the porous fin is also greatly augmented, and the fin's base influence becomes higher than the effect from the other parallel fin.*

Keywords: *nonlinear heat transfer, porous fin, finite difference, numerical simulation*

1. INTRODUCTION

Transport in porous media presents an enormous variety of relevant engineering applications. For instance, in Nield and Bejan (2006), the balance equations were highlighted in various conditions; Kaviany (1991) used a widespread method to solve flows through porous media, the volume averaging method. Another method thoroughly employed for dealing with such problems is the mixture theory approach. Examples of this methodology are the packed-bed heat exchanger, analyzed by Martins-Costa *et al.* (1996), or the flow in a channel with two distinct regions Martins-Costa and Saldanha da Gama (1994). In this latter work, a mixture was employed in the porous matrix region, while in the other region, a pure fluid flew.

Due to the many porous media applications, an area of great interest in employing such materials is the heat exchangers, more specifically, porous fins. Examples are the rectangular porous fin Gorla and Bakier (2011) studied. The momentum equation is simplified by Darcy's model and the energy equation by considering local thermal equilibrium, with temperature variating only along the fin's length without radiant surface exchange. Among porous fins, other works could be quoted. Ma *et al.* (2016) studied a fin with temperature-dependent parameters and solved the problem using Spectral Methods. Torabi and Yaghoobi (2013) proposed a series solution for a porous fin under convection and radiation by using the differential transformation method, which according to the study, allowed the results to be obtained faster and with more precision than conventional numerical methods.

More studies of porous fins include the case study of Daradji and Bouaziz (2018). The authors analyzed porous fins in a spiral shape, with both temperature-dependent and independent conductivity, while obtaining the influence of geometric parameters of the spiral on the temperature distribution. Das (2014) conducted a similar study where a cylindrical porous fin with conduction, convection, and radiation is considered. An interesting aspect of this particular study is that after numerically obtaining the temperature distribution, the author uses inverse methods to estimate values for the heat exchange parameters.

It is worthy of notice that most of the literature does not consider the influence of the fin's base or other fins. In this paper, such effects are considered for the case of infinite parallel porous fins and the impact of thermal properties values when such effects are taken into account.

2. METHODOLOGY

The problem investigated in this study is illustrated in figure 1:

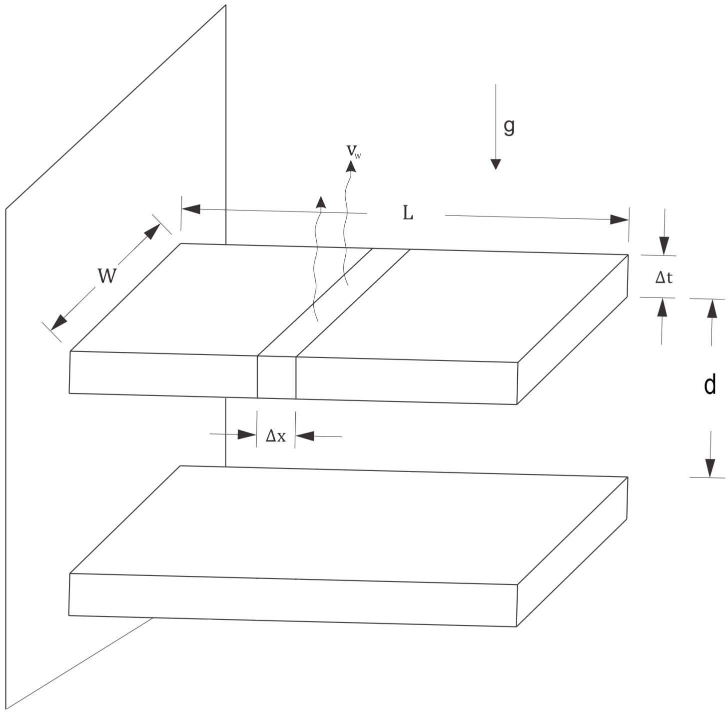

Figure 1. Illustration of parallel fins

The method used in this paper is similar to the one employed in Martins-Costa *et al.* (2022), in which a large positive constant (denoted by α) is used to create a non-decreasing sequence that gradually converges numerically. In the explored problem, in order to simplify view factor calculations, the fins are assumed to be mathematically infinite in their width (W). In real terms, the fins have a very large width when compared to all other dimensions. The first step to implementing the methodology is to establish the energy balance equation to be employed, which takes a similar form as can be seen in both Martins-Costa *et al.* (2022) as well as in Gorla and Bakier (2011):

$$\frac{d^2T}{dx^2} - \frac{v_W \rho c_P}{k_{eff} t}\left(T - T_F\right) - \frac{2\sigma\varepsilon}{k_{eff} t}\left(T^4 - T_F^4\right) = 0 \tag{1}$$

where T represents the porous fin's temperature, T_F the fluid's temperature far from the porous fin, v_W the fluid velocity, ρ the specific mass, c_P the heat transfer coefficient at constant pressure, k_{eff} the effective heat conductivity ($k_{eff} = k_S(1-\phi) + k_F\phi$), ϕ the porosity of the material, k_S the conductivity of the solid material, k_F the fluid's thermal conductivity, t the thickness of the rectangular porous fin, σ the Stefan-Boltzmann parameter and ε the emissivity.

Using Darcy's law, the equation can be rewritten as:

$$\frac{d^2T}{dx^2}-\frac{gK\beta\rho c_P}{\nu k_{eff}t}\left(T-T_F\right)-\frac{2\sigma\varepsilon}{k_{eff}t}\left(T^4-T_F^4\right)=0 \tag{2}$$

where g represents the gravity, K the permeability of the porous material, β is the thermal expansion coefficient and ν the dynamic viscosity.

However, equation (2) can only describe a single fin, now by introducing the effect of the porous fin's base, the energy equation becomes:

$$\frac{d^2T}{dx^2}-\frac{gK\beta\rho c_P}{\nu k_{eff}t}\left(T-T_F\right)^2-\frac{\sigma}{k_{eff}t}\left[2\left|T\right|^3T-2\int\left|T\right|_b^3T_bF_{1b}dS_b\right]=0 \tag{3}$$

where T_b represents the base's temperature, F_{1b} is the view factor of the base in relation to the investigated fin and dS_b is the differential of base's radiant surface.

It must be noted that the environment was considered a non-participant element in terms of heat exchange through radiation. Finally, by adding the effect arising from the other parallel fin, the energy equation is:

$$\frac{d^2T}{dx^2}-\frac{gK\beta\rho c_P}{\nu k_{eff}t}\left(T-T_F\right)^2-\frac{\sigma}{k_{eff}t}\left[2\left|T\right|^3T-\frac{1}{2}\int\left|T\right|^3T\;F_{12}dS_2-2\int\left|T\right|_b^3T_bF_{1b}dS_b\right]=0 \tag{4}$$

where F_{12} is the view factor of another parallel fin in relation to the investigated fin and dS_2 is the differential of another fin's radiant surface.

In order to turn Eq. (4) into a dimensionless one, the following parameters are used:

$$X=\frac{x}{L} \tag{5}$$

$$\theta=\frac{T}{T_B} \tag{6}$$

$$\theta_F=\frac{T_F}{T_B};\qquad \theta_b=1; \tag{7}$$

$$S_h=\frac{Da\quad Ra}{k_R}c_P\left(\frac{L}{t}\right)^2 \tag{8}$$

$$Da=\frac{K}{t^2};\qquad k_R=\frac{k_{eff}}{k_F}; \tag{9}$$

$$Ra=\frac{g\beta}{k_F\nu}\left(T_B\right)t^3 \tag{10}$$

$$G=\frac{2\sigma\varepsilon}{k_{eff}t}L^2\left(T_B\right)^3 \tag{11}$$

At this point, it is important to mention that besides the scaled spatial variable *X*, the scaled temperatures θ represents the dimensionless temperature along the fin's length (*L*), θ_F represents the fluid's dimensionless temperature far from the fins, θ_b represents the base's dimensionless temperature which is assumed to be constant at 1, S_h is a porous parameter (accounting for permeability and buoyancy effects), *G* is a radiation parameter, *Da* is Darcy number and *Ra* is Rayleigh number.

Applying Eqs. (5)-(11) into Eq. (4):

$$\frac{d^2\theta}{dX^2} - S_h\left(\theta - \theta_F\right)^2 - G\left[2\left|\theta\right|^3\theta - \frac{1}{2}\int_0^1\left|\theta\left(\xi\right)\right|^3\theta\left(\xi\right)F_{12}\left(\xi\right)d\xi - 2\left|\theta_b\right|^3\theta_b\left(\int_0^1 F_{1b}\left(\varsigma\right)d\varsigma\right)\right] = 0 \tag{12}$$

where the new variables ξ and ς are the position along the other fin's length and the position along the fin's base.

These view factors are obtained by using a 2D approach due to the fin's very long width, as it can be seen in Howell *et al.* (2015):

$$F_{12}\left(X,\xi\right) = \frac{\left(\frac{d}{L}\right)^2}{2\left(\left(X-\xi\right)^2 + \left(\frac{d}{L}\right)^2\right)^{3/2}} \tag{13}$$

$$\int_0^d F_{1b}\left(X,\varsigma\right)d\varsigma = 1 - \frac{X}{2\left(X^2 + \left(\frac{d}{L}\right)^2\right)^{1/2}} \tag{14}$$

Making use of the procedure presented in Martins-Costa *et al.* (2022) in order to solve the problem proposed by Eq. (12), the following scheme is used:

$$\frac{d^2\varphi^{i+1}}{dx^2} - \alpha\varphi^{i+1} + \gamma = 0 \tag{15}$$

$$\gamma = \alpha\varphi^i - f\left(\varphi^i\right) \tag{16}$$

where i represents the iteration number.

The function f is given by:

$$f\left(\varphi^i\right) = S_h\left(\varphi_j^i - \theta_F\right)^2 + G\left[2\left|\varphi_j^i\right|^3\varphi_j^i - \frac{1}{2}\int_0^1\left|\varphi^i\left(\xi\right)\right|^3\varphi^i\left(\xi\right)F_{12}\left(X,\xi\right)d\xi - 2\left|\theta_b\right|^3\theta_b\left(\int_0^1 F_{1b}\left(X,\varsigma\right)d\varsigma\right)\right] \tag{17}$$

where j represents the node's spatial position number.

Applying finite differences into Eq. (15), it will change into:

$$\frac{\varphi_{j+1}^{i+1} - 2\varphi_j^{i+1} + \varphi_{j-1}^{i+1}}{\Delta X^2} - \alpha\varphi_j^{i+1} + \gamma_j^i = 0 \tag{18}$$

With this proposed scheme, as seen in Martins-Costa *et al.* (2022), the boundary conditions at the fin's base as well as the boundary condition of the insulated tip can be written as:

$$\varphi_1^{i+1} = 1 \tag{19}$$

$$\frac{-2\varphi_N^{i+1} + 2\varphi_{j-1}^{i+1}}{\Delta X^2} - \alpha\varphi_N^{i+1} + \gamma_N^i = 0 \tag{20}$$

Rearranging these equations:

$$\varphi_j^{i+1} = \frac{\varphi_{j+1}^{i+1} + \varphi_{j-1}^{i+1} + \Delta X^2 \gamma_j^i}{2 + \alpha \Delta X^2} \quad (21)$$

$$\varphi_N^{i+1} = \frac{2\varphi_{j-1}^{i+1} + \Delta X^2 \gamma_N^i}{2 + \alpha \Delta X^2} \quad (22)$$

3. RESULTS

Using the methodology previously explored and a convergence criterion of 10^{-6}, the first significant result of this study was obtained. It is the temperature distribution for a single pair of parallel porous fins. Figure 2 compares these results with those of a single fin.

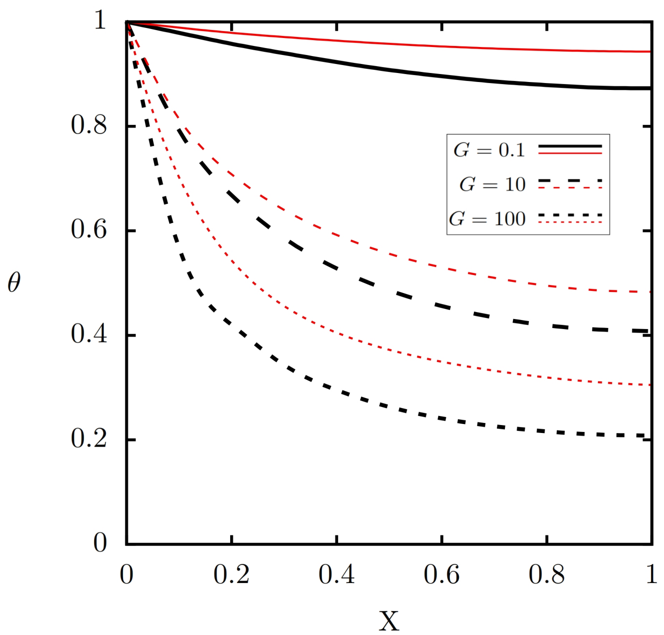

Figure 2. Comparison between a single fin and a pair of fins

Figure 2 shows the impact of radiation parameter G in the heat exchange between a pair of porous fins without the influence of neither the fin base nor convective effects. The only effect accounted for in these problems is the radiation effect in a single fin or between two parallel fins. As the radiation parameter G increases, the temperature strongly decreases along the fin in both cases. In Fig. 2, the black lines represent a single fin, whereas the red lines represent a pair of parallel porous fins exchanging heat through radiation. The mere addition of another fin significantly increases the temperature along the fin's length.

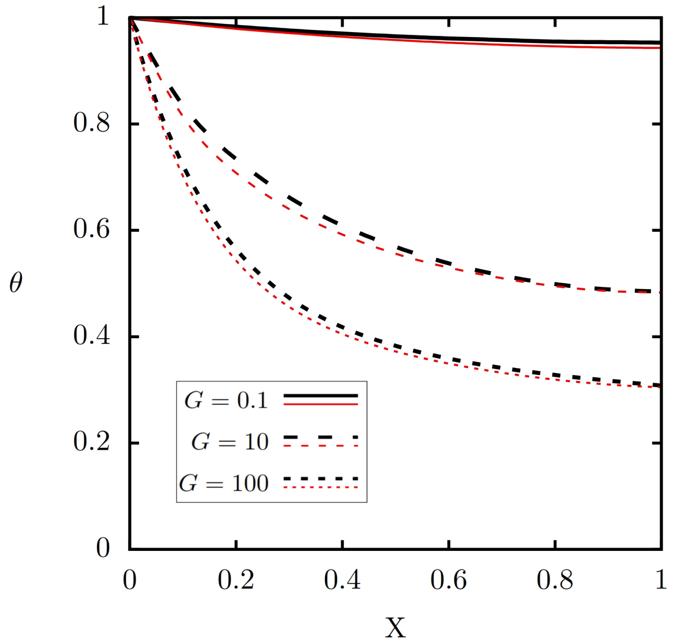

Figure 3. Effect of radiation between a pair of porous fins

Figure 3 shows the impact of radiation parameter G in the heat exchange between a pair of porous fins, again without the influence of neither the fin base nor convective effects of the parameter S_h. In short, only the radiation effect between two parallel fins is considered. The temperature strongly decreases along the fin with the increase of the radiation parameter G. In Fig. 3, the black lines represent the ratio of the distance between fins and the fins' length d/L=0.1. In contrast, the red lines represent a ratio of d/L=0.5. So, it can be noted that a lower ratio of d/L increases the temperature distribution very slightly.

Another relevant result for a pair of porous fins is the impact of the fin base, shown in Fig. 4.

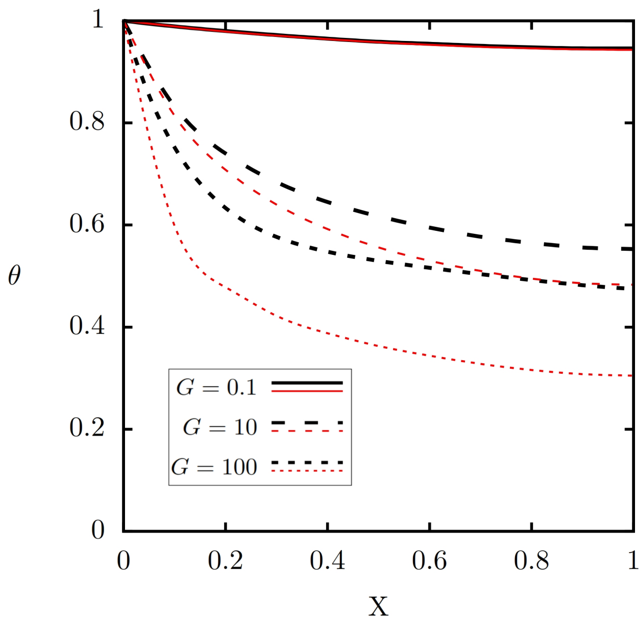

Figure 4. Effect of the base's radiation

Figure 4 was constructed neglecting convection effects (accounted by considering the parameters *Sh*), using distinct values of the radiation parameter G. The temperature distribution along the fin depicted in black considers both the fin base effect and the radiation effect between two parallel fins. The red results are obtained, neglecting the fin base effect (as shown in Fig. 3). All the results in Fig. 4 employ a ratio of d/L = 0.5. It can be observed that the base influence in the temperature distribution strongly increases as the radiation parameter G is increased. (For small values of the radiation parameter G, the fin base has almost no influence.)

However, a hypothesis closer to reality would not be a single pair of fins but an array of many parallel fins, as represented in Fig. 5.

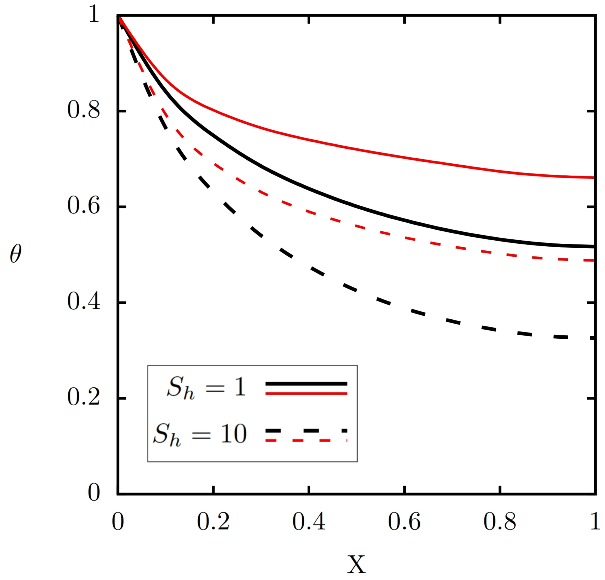

Figure 5. Effect of convection on an array of a large number of parallel fins

Figure 5 represents in black d/L = 0.1 while in red d/L = 0.5, using a radiation parameter *G*=10 and accounting for effects of the fin base radiation and two distinct convection parameters S_h, for an array of a large number of parallel fins. It is important to note that compared to the previous results, the fins in an array of many fins (in such a large number that it can be mathematically considered an infinite number of fins) show higher temperature values with S_h =1. Also, in Fig. 5, an interesting observation can be made: the effects of radiation from the fin's base mitigates the effects of convection on the temperatures, especially for higher ratios of the distance between fins (d) to fin's length (L). Additionally, unlike Fig. 3, it is not the lower ratio d/L that possesses higher temperature distribution along the fin's length. Instead, the higher ratio is due to the base's influence, as previously discussed.

4. CONCLUSIONS

The obtained results show good agreement with the literature, such as those presented by Martins-Costa et al. (2022), Ma et al. (2016) and Gorla and Bakier (2011) in terms of behavior when analyzing the dimensionless parameters of radiation (*G*) and convection (S_h).

However, due to the lack of literature exploring the effects of radiation from other fins and the base, further studies in this area are recommended, especially for higher optimization of the employment of porous fins in heat exchangers.

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