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ON THE WATER HAMMER IN ELASTO-VISCOPLASTIC PIPES

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Abstract. *This paper presents an extension of the water hammer analysis to include the elasto-viscoplastic behavior of the pipe material. To accomplish this goal, first, a recently validated 2D unsteady fluid flow model is applied as the basic framework. Then, the elasto-viscoplastic behavior of the pipe material is incorporated according to an internal variable constitutive theory. The proposed model results in a nonlinear hyperbolic system of partial differential equations whose solution is approximated by using the method of characteristics followed by a Newton-Raphson method. Taking as reference the transient responses obtained from a classic reservoir-pipe-valve installation at a high-temperature environment, the proposed model limits the pressure fluctuations compared to those found by the modeling that disregards the elasto-viscoplasticity. Such behavior can be implied by the plastic yielding and creep found in the pipe material.*

Keywords: *water hammer, elasto-viscoplasticity, thermodynamic of irreversible processes, internal variable theory*

1. INTRODUCTION

The presence of fluid transients in liquid transmission lines is unavoidable. Normal operational circumstances such as stopping of pumps and valve closures cause sudden changes in the fluid momentum that lead to flow unsteadiness. As liquid transmission lines are, in general, composed of flexible pipes, significant deformation of the pipe wall occurs in the course of the transient due to the cyclic passage of liquid pressure waves. As a consequence, local and instantaneous mechanical interactions between the liquid and pipe wall arise and determine the behavior of the fluid-pipe system. Hence, understanding not only the fluid flow dynamics but also the mechanical behavior of the pipe submitted to internal pressure loads is fundamental to the proper design and operation of piping systems.

Traditionally, the theory of fluid transients assumes the pipe has linear elastic behavior. However, in many industrial contexts, the pipe material is most likely subjected to anelastic deformations rather than elastic ones. A classic example of that is the wide use of polymeric materials, like polyvinyl chloride (PVC), polyethylene (PE), and high-density polyethylene (HDPE), to produce liquid transmission lines for sanitary sewer and water supply systems. These polymeric materials respond to hydraulic loadings with a viscoelastic mechanical response. As the responses of fluid transients in viscoelastic pipes are poorly described by the classical theory, intense research towards this topic has been developed throughout the last five decades. To mention some of the vast literature in this field, one may cite the essential works of Gally et al. (1979), Covas et al. (2004), and Pezzinga et al. (2014).

Beyond viscoelasticity found in these polymeric pipes, metallic pipes are likely to be subjected to anelastic deformations when they are employed in adverse operating conditions. Severe loading magnitudes, high-temperature environments or even a combination of both may lead to stresses far above the pipe wall yield stress that turns the material to exhibit an elasto-viscoplastic mechanical behavior. Among many engineering applications that deal with piping structures undergoing viscoplastic deformations, thermal-hydraulic systems of nuclear power plants has been a constant topic of research (Alwar et al., 1993, Krausz and Krausz, 1996, Krempl, 2000, Costa Mattos et al., 2015). More recently, some attention has also been concerned with the proper analysis of the elasto-viscoplastic characteristics of metallic and polymeric materials applied in compliant pipes employed in the Oil & Gas industry (De Souza et al., 2019, Motta et al., 2019).

Contrarily to the case of where the pipe-wall deforms viscoelastically, few works contemplating the analysis of fluid transients with plastic deformation are found in the literature. The attempts to model this phenomenon were developed in the works of Youngdahl and Kot (1975) and Romander et al. (1980). Nevertheless, none of these models have made use of constitutive relations capable of fully describing the elasto-viscoplastic behavior of metals.

In addition to this limited description of the pipe's mechanical behavior, they have assumed the flow to be one-dimensional, in which the fluid friction is assumed to be described by the same shear stress relation found in fully-developed flows (Darcy-Weisbach equation). Despite the usefulness of this assumption when the first modelings were developed, this approach has been proved to be inappropriate when the transient flow has to be described accurately (Duan et al., 2020). Due to the no-slip condition at the pipe walls, the flow inversion after the onset of the transient generates fluid velocity profiles that are highly distinct from those found in a fully-developed flow. Such a local and two-dimensional effect causes friction mechanisms that the Darcy-Weisbach approximation cannot completely assemble. Thus, proper modeling of fluid transient requires more sound fluid flow modeling. A diverse range of quasi-one-dimensional and quasi-two-dimensional fluid models that better capture unsteady flow mechanics can be found in the literature.

To introduce a mechanical formulation for transient flows that incorporates the elasto-viscoplastic mechanical response of the pipe into the modern analysis of fluid transients, this work takes advantage of a recently validated quasi-2D fluid-flow model to add the pipe's elasto-viscoplastic behavior by employing a constitutive theory established over the bases thermodynamics of irreversible processes (TIP) with internal variables (Germain et al. 1983).

A simple numerical example concerning the water hammer in a liquid-filled compliant pipe illustrates some capabilities of the proposed model. The model's responses show that the viscoplastic deformation plays a crucial role in the unsteady responses of the fluid-pipe system.

2. MODELLING

The fluid transient model for elasto-viscoplastic pipes developed herein is derived from a three-step procedure. First, an unsteady fluid flow model in deformable circular pipe is presented, and then the constitutive equations for the elasto-viscoplasticity are introduced. In the last step, final governing equations are achieved by combining the fluid model with the pipe constitutive equations.

The quasi-2D model developed by Andrade and Freitas Rachid (2022) is employed to achieve the basic equations for an unsteady fluid flow in a deformable circular pipe. This model applies the theory of mixtures (Atkin and Crane, 1976) to conceive the fluid flow as a structured pseudo-mixture comprised by a set of n shell-shaped constituents (with $n \geq 2$). They are assumed to hold the same fluid properties and be concentrically-distributed in the radial direction r . Each of these structured flow layers has a fixed volume fraction α_j and can only slip over and/or beneath each other with an independent velocity v_j along the direction of the center-line of the pipe x . In this kinematic framework, each j -th constituent is supposed to be subjected to a surface force per unit of volume m_j and a reactive contact friction force per unit of lateral area a_j acting on the pipe wall-fluid interface. In addition, each of them can also be completely described by its radius $R_j = R_{j-1} + \Delta R_{j-1}/2 + \Delta R_j/2$, thickness ΔR_j , and fixed volumetric fraction $\alpha_j = 2R_j\Delta R_j/R^2$, being R the undisturbed internal radius of the pipe. This geometrical description of the pseudo-mixture admits the definitions of the mass density, pressure, and velocity for the mixture as a whole as $\rho = \sum_{j=1}^n \rho_j$, $p = \sum_{j=1}^n p_j$, and $v = \sum_{j=1}^n \rho_j v_j / \rho$, respectively, in which $\rho_j = \alpha_j \rho$, $p_j = \alpha_j p$.

The pipe deformation must be taken into account to describe the mechanics of fluid transients properly. In the present context, the pipe has negligible radial and axial inertia, and it is assumed to be subjected to small deformations and axisymmetrical plane-stress distribution. By considering that the fluid flow is in the low Mach number range and that the fluid is slightly compressible, the mechanical balances of the model are given by (Andrade and Freitas Rachid, 2022):

$$\frac{1}{K} \frac{\partial p}{\partial t} + \rho_0 \frac{\partial v}{\partial x} + 2 \frac{\partial \varepsilon_\theta|_{r=R}}{\partial t} = 0, \quad (1)$$

$$\rho_0 \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{2}{R} \sum_{j=1}^n a_j = 0, \quad (2)$$

$$\alpha_j \rho_0 \frac{\partial v_j}{\partial t} + \alpha_j \frac{\partial p}{\partial x} + m_j + \frac{2}{R} a_j = 0, \quad j = 2, \dots, n, \quad (3)$$

where ε_θ is the circumferential strain of the pipe. As a result of the virtual mixture arrangement in which each constituent slides on each other, the shear stress parcel responsible for the momentum diffusion in the radial direction in the virtual mixture is fully represented by the interaction force, m_j , and reactive force, a_j , acting upon the constituents. The nature of the mixture structure and momentum transfer principle implies that the surface forces must be related to the velocity

difference of adjacent constituents of the mixture. Invoking the principle of the material frame-indifference, a constitutive expression that represents the force m_j of each constituent may be postulated as follows:

$$m_j = C_{j,j-1} (v_j - v_{j-1}) + C_{j,j+1} (v_j - v_{j+1}) \text{ for } j = 1, \dots, n, \text{ with } C_{1,0} \equiv 0 \text{ and } C_{n,n+1} \equiv 0, \quad (4)$$

in which $C_{j,j-1}$ and $C_{j,j+1}$ are material constants of the model.

Appealing to the same core idea used to develop the constitutive equations for the force m_j , it is possible to postulate a similar expression for the friction reactive force a_j . By taking into account the mixture structure, a_j is non-null only for $j = n$ since this constituent is the only one in contact with the pipe wall (see Fig. 1). As the pipe wall in this modeling is assumed to be standstill in the x -direction, the resulting expressions for a_j are:

$$a_j = \begin{cases} 0, & \text{for } j = 1, \dots, n-1 \\ Cv_n, & \text{for } j = n \end{cases} \quad (5)$$

in which C is another material constant of the model to be determined.

By assuming that changes in the viscous and turbulent structure during the transient event can be neglected (Vardy et al., 2015), the material constants can be achieved by solving the momentum balance equation for each constituent (Eq. (3)) under the assumption of a fully-developed steady flow condition. Under this context, this balance equation is simplified to:

$$\alpha_j \frac{\partial p}{\partial x} + m_j + \frac{2}{R} a_j = 0, \quad j = 1, \dots, n. \quad (6)$$

When taking Eqs. (4-5) into account, the set of n equations in Eq. (6) form a linear system in terms of the material constants C and $C_{j,j+1}$, for $j = 1, \dots, n-1$, which may be solved with the aid of a known steady-state velocity profile.

The approach above is general for laminar or turbulent flows. Applying the discrete parabolic-shape laminar velocity profile, the material constants of the model are given by

$$C_{j,j+1} = \frac{4\rho_0\nu}{R_{j+1}^2 - R_j^2} \left(\sum_{i=1}^j \alpha_i \right) \quad (7)$$

$$C = \frac{2\rho_0\nu R}{R^2 - R_n^2}, \quad (8)$$

in which ν is the fluid kinematic viscosity. Meanwhile, in the case of turbulent flows, the algebraic turbulence model described in Vardy and Brown (2007) is applied to obtain an expression for the velocity profile. This approach can be characterized by a core region with constant viscosity ν_c that ranges from the pipe centerline to $r = 0.8R \equiv R_M$ followed by an annular region with thickness $b = 0.2R$, where the viscosity displays a continuous decreasing linear behavior to a minimum constant value ν_w at the wall. The resulting material constants are given by:

$$C_{j,j+1} = \frac{4\rho_0\nu_c}{R_{j+1}^2 - R_j^2} \left(\sum_{i=1}^j \alpha_i \right), \text{ for } 0 \leq R_j < R_M, \quad (9)$$

for the core and

$$C_{j,j+1} = \frac{2\nu_w\rho_0(1-\sigma_{cw})}{b^2 \left\{ \frac{R_{j+1} - R_j}{b} + \frac{(-4 + 5\sigma_{cw})}{(1-\sigma_{cw})} \ln \left[\frac{\left(\frac{1-\sigma_{cw}}{b}\right)R_j - 4 + 5\sigma_{cw}}{\left(\frac{1-\sigma_{cw}}{b}\right)R_{j+1} - 4 + 5\sigma_{cw}} \right] \right\}} \left(\sum_{i=1}^j \alpha_i \right), \text{ for } R_M \leq R_j \leq R_{n-1}, \quad (10)$$

for the annulus, while the reactive material constant C , which is associated with the constituent at $R_j = R_n$, is given by,

$$C = \frac{v_w \rho_0 R (1 - \sigma_{cw})}{b^2 \left\{ \frac{R - R_n}{b} + \frac{(-4 + 5\sigma_{cw})}{(1 - \sigma_{cw})} \ln \left[\left(\frac{1 - \sigma_{cw}}{b} \right) R_n - 4 + 5\sigma_{cw} \right] \right\}}. \quad (11)$$

In Eqs. (9-11), $\sigma_{cw} = v_c/v_w$ is the ratio of the kinematic eddy viscosities.

2.1 Pipe constitutive equations

The elasto-viscoplastic response of the pipe is described by constitutive equations that derive from the formalism of the thermodynamics of irreversible processes (Germain et al., 1983). Fundamentally, this theory is constructed based on the axiom of the local accompanying state. This axiom dictates that a finite set of state variables defines the state of a medium at any given material point and time. Within this thermodynamic framework, relations of these state variables with the Helmholtz free energy potential and a pseudo-potential of dissipation describe the diverse processes involved in the deformation of a material. State laws that are sufficient to portray all reversible processes are achieved by the relationship between the Helmholtz free energy potential and the state variables. Meanwhile, evolution laws, which are the relationships between these variables with the pseudo-potential of dissipation, are introduced to describe any irreversible processes. Together, the state and evolution laws define the complete set of constitutive equations of the material.

Classically, the thermodynamic state of an isothermal anelastic solid is supposed to be identified by the total strain and anelastic strain tensors $\boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon}^a$, in addition to a set of internal variables $\boldsymbol{\beta}$ (Germain et al., 1983). These variables are associated with the macroscopic description of irreversible structural rearrangements that may occur in the medium. Thus, they are introduced to the modeling as a constitutive choice that depends on the phenomenological aspects of a specific type of material. In the present work, the pipe is assumed to be an isotropic elasto-viscoplastic material. This mechanical behavior is characterized by a diverse range of mechanical phenomena: elasticity, plasticity (e.g., hardening/softening, Bauschinger and ratcheting effects), creep, and relaxation phenomena. In the framework of small deformations and isothermal processes, the set of state variables that comprise all these features in an elasto-viscoplastic solid under non-monotonic loadings are:

$$(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^a, \boldsymbol{\beta}) \text{ with } \boldsymbol{\beta} = (q, \mathbf{c}), \quad (12)$$

in which q stands for a scalar internal variable, so-called cumulated plastic strain, associated with the isotropic hardening; and \mathbf{c} is a tensorial internal variable related to the kinematic hardening. Considering that the elasto-viscoplastic solid follows the Odqvist's creep law when an initial yield stress σ^y , isotropic hardening, and nonlinear kinematic hardening are present, the Helmholtz free energy potential can be assumed to be a differentiable scalar function of the state variables given as follows:

$$\rho_p \Psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^a, q, \mathbf{c}) = W_a + W_e, \quad (13)$$

in which ρ_p is the mass density of the pipe material, which is supposed to be invariant. W_e represents the elastic strain energy density expressed as

$$W_e = \frac{1}{2} \frac{E_0}{(1 + \nu_0)} \left\{ \frac{\nu_0}{1 - 2\nu_0} [\text{tr}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a)]^2 + (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a) : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a) \right\}. \quad (14)$$

Where ν_0 and E_0 are the young modulus and Poisson's ratio, and the symbol “:” represents the inner product of tensors, i.e. $\mathbf{A}:\mathbf{B} = \text{tr}(\mathbf{A}^T \mathbf{B})$. Meanwhile, the anelastic strain energy density W_a can be taken in the following form

$$W_a(q, \mathbf{c}) = W_a^q + W_a^c, \quad (15)$$

in which

$$W_a^q = b \left[q + \frac{1}{d} \exp(-dq) \right] \text{ and } W_a^c = \frac{1}{2} a \mathbf{c}:\mathbf{c} \quad (16)$$

represent the anelastic strain energy due to the isotropic hardening and its counterpart related to the kinematic hardening. The scalars b, d, a are simply material coefficients.

Taking as reference the aforementioned strain energy densities, the so-called state laws are expressed by:

$$\boldsymbol{\sigma} = \rho_p \frac{\partial \Psi}{\partial \boldsymbol{\varepsilon}} = \frac{\nu_0}{(1 + \nu_0)} \text{tr } \boldsymbol{\sigma} + \frac{E_0}{(1 + \nu_0)} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^a), \quad (17)$$

$$B^q = -\rho_p \frac{\partial \Psi}{\partial q} = -b [1 - \exp(-dq)], \quad (18)$$

$$\mathbf{B}^c = -\rho_p \frac{\partial \Psi}{\partial \mathbf{c}} = -a\mathbf{c}, \quad (19)$$

Where ν_0 and E_0 are the Young's modulus and Poisson's ratio. Equation (17) defines the thermodynamic force $\boldsymbol{\sigma}$, which represents the definition of the stress tensor of a generalized Hookean solid. Meanwhile, B^q is the thermodynamic force connect to the isotropic hardening variable and describes how the yield stress varies with plastic deformation; and \mathbf{B}^c is the thermodynamic force (eventually called back stress tensor) that characterized the kinematic hardening by accounting the anisotropy induced by the plastic deformation.

By following the same assumptions, the pseudo-potential of dissipation can be introduced as

$$\Phi = \frac{k}{n+1} \left\langle \frac{f + \frac{\phi}{2a} (\mathbf{B}^c : \mathbf{B}^c + a^2 \mathbf{c} : \mathbf{c})}{k} \right\rangle^{n+1}, \quad (20)$$

in which $\langle \Xi \rangle$ (usually called McCauley bracket) is defined as $\langle \Xi \rangle = \max\{0, \Xi\}$, and

$$f = J + B^q - \sigma^y, \text{ with } J^2 = \frac{3}{2} (\boldsymbol{\sigma} + \mathbf{B}^c)_{dev} : (\boldsymbol{\sigma} + \mathbf{B}^c)_{dev}, \quad (21)$$

is the yield function that defines the elastic domain. In Eqs. (20-21), k , n , σ^y , ϕ are material parameters. The preceding pseudo-potential of dissipation give rise to the following set of evolution laws:

$$\dot{\boldsymbol{\varepsilon}}^a = \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} = \frac{3}{2} \left\langle \frac{f}{k} \right\rangle^n \frac{(\boldsymbol{\sigma} + \mathbf{B}^c)_{dev}}{J}, \quad (22)$$

$$\dot{q} = \frac{\partial \Phi}{\partial B^q} = \left\langle \frac{f}{k} \right\rangle^n = \left[\frac{2}{3} \boldsymbol{\varepsilon}^a : \dot{\boldsymbol{\varepsilon}}^a \right]^{\frac{1}{2}}, \quad (23)$$

$$\dot{\mathbf{c}} = \frac{\partial \Phi}{\partial \mathbf{B}^c} = \boldsymbol{\varepsilon}^a - \phi \mathbf{c} \dot{q}. \quad (24)$$

For a complete discussion of the definition and conception of these state and evolution laws and the required experimental procedures to identify the material parameters a , b , d , σ^y , k , n , ϕ , one should report to the works of Lemaitre and Chaboche (1990).

2.2 Governing Equations

The governing equations of the model are obtained by employing the constitutive equations of an elasto-viscoplastic pipe (Eqs. (17-24)) into the mechanical balances developed for unsteady flows in deformable pipes described at the beginning of section 2. Nevertheless, the postulation of the final model equations requires some additional hypotheses regarding the state of stress in the pipe wall.

In the present fluid transient phenomenon, the pipe is assumed to be subjected to only internal pressure loads such that the circumferential stress component σ_θ turn to be the only independent one among the principal stresses of $\boldsymbol{\sigma}$. In addition, the pipe is considered to be thick-walled in which the mean circumferential, axial, and radial stress components are approximated by the quasi-static stress distribution achieved by Tijsseling (2007). As a consequence, the evolution laws of the material turn out to be averaged-based. To adjust the anelastic strain term found in the mass balance with this new averaged-based perspective, the mean anelastic strain is assumed to be equal to $\Pi \varepsilon_\theta^a|_{r=R}$, where $\Pi = \left(R + \frac{1}{2}e\right)/R$. Finally, gathering all these assumptions, the governing equations of the mechanical model for the unknowns p , v , v_2 , ..., v_n , ε_θ^a , c_θ , q can be expressed as:

$$\begin{aligned}
 & \left[\frac{1}{\rho_0 c_f^2} \right] \frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} + 2\Pi \frac{\partial \varepsilon_\theta^a}{\partial t} = 0, \\
 & \frac{\partial v}{\partial t} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{2}{R\rho_0} a_n = 0, \\
 & \frac{\partial v_j}{\partial t} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\alpha_j \rho_0} \left\{ m_j + \frac{2}{R} a_j \right\} = 0, \quad j = 2, \dots, n \\
 & \frac{d\varepsilon_\theta^a}{dt} - \frac{3}{2J} \left(\frac{1}{3} \Omega p - \frac{1}{3} v_0 \Gamma p - \frac{2}{3} a c_\theta \right) \dot{q} = 0, \\
 & \frac{dc_\theta}{dt} - \frac{3}{2J} \left(\frac{1}{3} \Omega p - \frac{1}{3} v_0 \Gamma p - \frac{2}{3} a c_\theta \right) \dot{q} + \phi c_\theta \left(\frac{f}{k} \right)^n = 0, \\
 & \frac{dq}{dt} - \left(\frac{f}{k} \right)^n = 0.
 \end{aligned} \tag{25}$$

Where

$$c_f = \left\{ \rho_0 \left[\frac{1}{K} + \frac{2}{E_0} \left(\frac{R}{e} + \frac{(1 + \frac{e}{R})}{(2 + \frac{e}{R})} + v_0 - v_0^2 \Gamma \right) \right] \right\}^{-1/2} \tag{26}$$

is the pressure wavefront speed and

$$\begin{aligned}
 J^2 &= \frac{3}{2} \left[\frac{2}{3} v_0 \Gamma p - \frac{1}{3} \Gamma p + \frac{1}{3} a c_\theta \right]^2 + \left[\frac{1}{3} \Omega p - \frac{1}{3} v_0 \Gamma p - \frac{2}{3} a c_\theta \right]^2 + \left[\frac{2}{3} \xi p - \frac{1}{3} v_0 \Gamma p + \frac{1}{3} a c_\theta \right]^2, \\
 \Gamma &= (\sigma_\theta + \sigma_r)/p = \left(\frac{R}{e} \frac{1}{1 + \frac{1}{2} \frac{e}{R}} \right), \quad \Omega = (2\sigma_\theta - \sigma_r)/p = \frac{R^2}{e(e+2R)} + \frac{6R^2(e+R)^2 \ln \left[1 + \frac{e}{R} \right]}{e^2(e+2R)^2}, \\
 \xi &= (2\sigma_r - \sigma_\theta)/p = \frac{R^2}{e(e+2R)} - \frac{6R^2(e+R)^2 \ln \left[1 + \frac{e}{R} \right]}{e^2(e+2R)^2},
 \end{aligned} \tag{27}$$

Equations (25a-c) stand for the averaged balances of mass, the balance of linear momentum for the mixture as a whole, and the balance of linear momentum for each constituent, respectively. The Eqs. (25d-f) are the evolution laws of the material. Even though the present model is conceived primarily to describe fluid transients in elasto-viscoplastic pipes, simpler modeling for fluid transients in elastic pipes is readily achieved in the present framework by eliminating the anelastic strain term of the continuity equation and the evolutions laws of the material.

3. NUMERICAL PROCEDURE

The governing equations of the model (Eq. 25) form a nonlinear hyperbolic system of partial differential equations. The method of characteristics is applied to give rise to a system of ordinary differential equations comprised by the following set of compatibility equations:

$$\begin{aligned}
 & \frac{1}{\rho_0 c_f} \frac{dp}{dt} + \frac{dv}{dt} = - \left(\frac{2}{R\rho_0} C v_n + 2c_f \Pi g_\theta \right), \quad C_f^+ \equiv \frac{dx}{dt} = c_f \\
 & - \frac{1}{\rho_0 c_f} \frac{dp}{dt} + \frac{dv}{dt} = - \left(\frac{2}{R\rho_0} C v_n - 2c_f \Pi g_\theta \right), \quad C_f^- \equiv \frac{dx}{dt} = -c_f
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 -\frac{dv}{dt} + \frac{dv_2}{dt} &= -\left(\frac{m_2}{\rho_0 \alpha_{n-1}} - \frac{2}{R\rho_0} C v_n\right), \quad C^0 \equiv \frac{dx}{dt} = 0, \\
 &\vdots \\
 -\frac{dv}{dt} + \frac{dv_{n-1}}{dt} &= -\left(\frac{m_{n-1}}{\rho_0 \alpha_j} - \frac{2}{R\rho_0} C v_n\right), \quad C^0 \equiv \frac{dx}{dt} = 0, \\
 -\frac{dv}{dt} + \frac{dv_n}{dt} &= -\left(\frac{m_n + \frac{2}{R} C v_n}{\rho_0 \alpha_n} - \frac{2}{R\rho_0} C v_n\right), \quad C^0 \equiv \frac{dx}{dt} = 0, \\
 \frac{d\varepsilon_\theta^a}{dt} &= \frac{3}{2J} \left(\frac{1}{3} \Omega p - \frac{1}{3} \nu_0 \Gamma p - \frac{2}{3} a c_\theta\right) \dot{q}, \quad C^0 \equiv \frac{dx}{dt} = 0, \\
 \frac{dc_\theta}{dt} &= + \frac{3}{2J} \left(\frac{1}{3} \Omega p - \frac{1}{3} \nu_0 \Gamma p - \frac{2}{3} a c_\theta\right) \dot{q} - \phi c_\theta \left(\frac{f}{k}\right)^n, \quad C^0 \equiv \frac{dx}{dt} = 0, \\
 \frac{dq}{dt} &= \left(\frac{f}{k}\right)^n, \quad C^0 \equiv \frac{dx}{dt} = 0.
 \end{aligned}$$

in which the characteristic equations of the model C^+ , C^- , C^0 are equal to the eigenvalues of the problem $+c_f$, $-c_f$, 0 , respectively.

The non-vanishing characteristics C_f^\pm are related to the perturbations that propagates in the domain. Meanwhile, the stationary characteristics (C^0) are associated with dispersive and/or dissipative effects. These effects occurs in both fluid and pipe and are caused by the momentum transfer in the fluid and anelastic deformation in the pipe.

To set the numerical approximation procedure, the domain of the independent variables must be discretized. The spatial domain $[0, L]$ is then divided into an integer number of N equally spaced grid of size Δx , giving rise to $N + 1$ spatial grid points $x_k = k\Delta x$, for $k = 0, \dots, N$. Based on a prior report given by Andrade and Freitas Rachid (2022), the number of spatial nodes in all numerical simulations is fixed and equal to 121. The grid choice of the constituents also is a constitutive choice of the modeler being defined a priori. To fulfill this model requirement, the number and length of the radius of each constituent must also be specified. We have used a distribution characterized by 5 constituents with the same thickness in the core ($0 < R_j < R_M$) and others 5 constituents in the annular region ($R_M < R_j < R$), obeying a geometric regressive ratio as the radius of the constituent approaches the pipe radius.

The integration procedure of the left-hand sides of the compatibility equations are exact. The integration of the right-hand sides of Eq. (28) are evaluated by employing a implicit second-order (Crank-Nicholson) approximations. The final discretized problem with the appropriated initial and boundary conditions forms a nonlinear system whose the solution, the vector of unknowns $[p \ v \ v_2 \ \dots \ v_n \ \varepsilon_\theta^a \ c_\theta \ q]$ at each discretized time-space, is obtained by employing the Newton-Raphson method. Following a long tradition of techniques to find the responses of solid materials subjected to anelastic deformations, the initial guess for Newton's method is given by the solution of the problem assuming that the pipe behaves elastically (Simo and Hughes, 1998).

4. NUMERICAL EXAMPLE

The heat transport piping system of liquid metal fast breeder reactors (LMFBR) is known to be susceptible to viscoplastic deformations due to a combination of high-temperature operating systems and severe transient pressure loads. Motivated by such installations, an investigation is carried out to analyze the transient responses of a simple liquid-filled piping system under a hypothetical LMFBR scenario, where the elevated temperature and a fast transient induce the pipe material to deform viscoplastically.

The installation is comprised by a reservoir with constant pressure $p_R = 1.25 \times 10^5$ from which liquid sodium at 600°C ($\rho_0 = 832 \text{ kg m}^{-3}$, $\mu = 0.0002 \text{ Pa.s}$, $K = 2.31 \text{ GPa}$) flows at steady-state velocity $v_0 = 5 \text{ m/s}$ in a stainless steel pipe of length L that reaches a downstream end valve. The fast transient is created by a valve closure maneuver taking place in $t_c = 0.01\text{s}$. The following Dirichlet boundary conditions represent such settlement:

$$p(x = 0, t) = p_R, \tag{29}$$

$$v(L,t) = \begin{cases} v_0 \left(1 - \frac{t}{t_c}\right) & \text{if } 0 \leq t < t_c, \\ 0 & \text{if } t \geq t_c \end{cases} \quad (30)$$

The pipe has length of $L = 30m$, inside diameter and wall thickness of $0.033 m$ and $0.001 m$, respectively. The pipe material is an AISI 316 L stainless steel, which is assumed that has never been experienced viscoplastic deformation. The elasto-viscoplastic coefficients of such material at $600^\circ C$ are: $E_0 = 130GPa$, $\nu_0 = 0.3$, $\sigma_y = 6MPa$, $k = 150 GPa$, $b = 80GPa$, $a = 16.5GPa$, $\phi = 300$.

Figure 1 shows the transient pressure histories close to the valve obtained by the present approach. To enrich the analysis, the proposed model responses disregarding the effects of the elasto-viscoplastic of the pipe (linear elastic pipe model) are also displayed in Fig. 1. By comparing the elastic and elasto-viscoplastic responses, one can observe that the latter limits the magnitude of the pressure oscillations. The pressure load induced by the valve slam is sufficient to cause significant viscoplastic deformation of the pipe wall at the first moments of the water hammer phenomenon, as shown in Fig. 2. Such deformation is intrinsically related to energy dissipation as heat (Lemaitre and Chaboche, 1990). Such energy loss causes an attenuation of the whole fluid-pipe system when compared to the model in which the pipe is assumed to be linear elastic. One may note that the viscoplastic deformation is restricted to the first cycles of the transient due to the action of two interrelated phenomena. In addition to damped pressure oscillations subject the pipe wall to less stringent stress states, the hardening phenomena of the pipe material occur as it deforms anelastically, then, the viscoplastic deformations decrease as the transient goes. At later stages of the transient phenomenon, one can observe that almost no anelastic deformations occur, such that the pipe material behaves almost elastically.

Nevertheless, plastic strain is still significantly present (Fig. 2) and cumulates throughout the cycles of the fluid transient as shown in Fig. 3. If the transient source is strong enough and/or it can occur with a regular frequency in the piping operational lifetime, the pipe can be subjected to excessive plastic strain, which is one of the main mechanisms of mechanical damage in metallic materials (Lemaitre, 2005). Then, even though the pressure oscillation results seem to be on the safe side of the pipeline integrity (reduced pressure magnitudes), the cumulated plastic strain which results from transient sources may be an concern for the proper design and maintenance of pipelines.

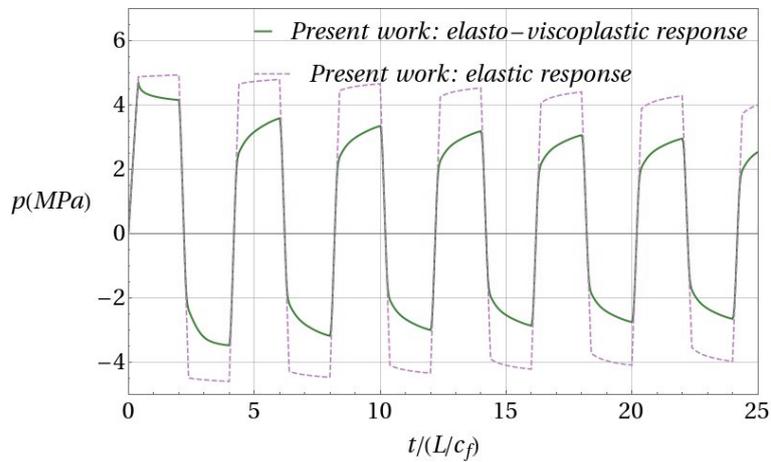


Figure 1 – Pressure histories located right next the downstream valve. Both elastic and elasto-viscoplastic responses are shown.

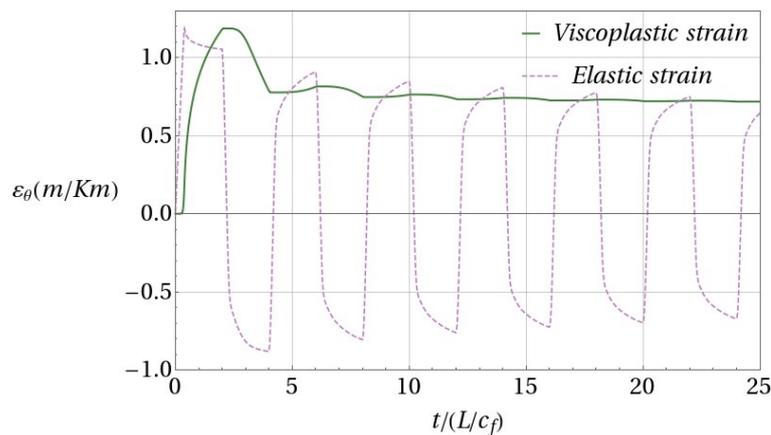


Figure 2 – Plastic circumferential strain histories located right next the downstream valve.

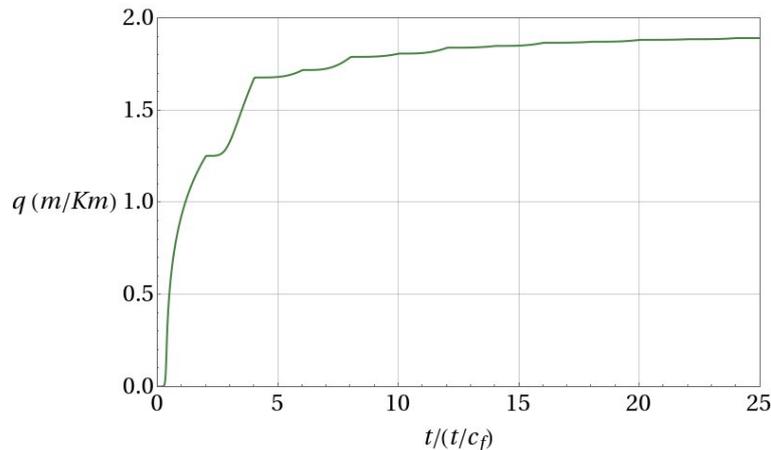


Figure 2 – Cumulated plastic circumferential strain histories located right next the downstream valve.

5. CONCLUSION

This work presents a water hammer model that extends the analysis to handle the elasto-viscoplastic behavior of the pipe. The model is constructed over a quasi-2D unsteady fluid flow model, in which the elasto-viscoplastic nature of the pipe wall is incorporated following an internal variable constitutive theory. The approximated solutions of the final governing equations are solved by the method of characteristics followed by the Newton-Raphson method. The model capabilities have been evaluated in a fluid transient caused by a valve slam in a simple reservoir-pipe-valve installation. The results show that the viscoplastic deformation limits the pressure oscillations of the phenomenon. In addition, the present work highlights the importance of the present analysis as throughout the transient, plastic deformation is cumulated such that the water hammer phenomenon can be source of damage, fatigue, and even structural failures of pipeline systems.

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