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NATURAL CONVECTION IN A VERTICAL ANNULUS FILLED WITH POROUS MEDIUM WITH INTERNAL HEAT GENERATION

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Abstract. *This work used the generalized integral transform technique (GITT) to investigate steady-state natural convection in a vertical annulus containing heat-generating porous medium. The porous medium is considered homogeneous, isotropic and saturated with an incompressible fluid. The vertical walls of the annular enclosure are kept at constant temperatures, with the temperature of the inner wall being higher than that of the outer wall, and the horizontal walls are insulated. The governing equations in stream-function formulation are integral transformed in the vertical direction, with the resulting system of nonlinear ordinary differential equations numerically solved by finite difference method. Convergence analysis and results for different aspect ratios, radius ratios and a wide range of Rayleigh numbers are presented. The GITT solutions are validated by comparison with fully numerical solutions by finite difference method, showing excellent agreement and convergence with low computational cost. The calculations were performed in the computational software Mathematica. It was shown that as the radius ratio increases, isotherms shift towards the inner (hot) wall. It was also concluded that the flow patterns and isotherms exhibit a significant influence of curvature.*

Keywords: *Natural convection, Porous media, Internal heat generation, Vertical annulus, Integral Transform*

1. INTRODUCTION

Natural convection in porous cavities is of great interest for industry and engineering applications (Lewis and Schrefler, 1998; Lewis *et al.*, 2004), such as nuclear reactor safety and dry storage of nuclear spent fuel (Lisboa *et al.*, 2018; Fu *et al.*, 2018; Mohammadian and Zhang, 2019). Natural convection in cavities of different geometries was investigated by using numerical, analytical and experimental methods , such as rectangular cavities (Vlasuk, 1972; Walker and Homsy, 1978; Lee and Goldstein, 1988; Joshi *et al.*, 2006), vertical cylinders (Martin, 1967; Holzbecher and Steiff, 1995), and horizontal annulus (Shekar *et al.*, 1984; Yuan *et al.*, 2015). Natural convection in cavities containing heat-generating porous medium has received attention , for example vertical cavities (Haajizadeh *et al.*, 1984; Blythe *et al.*, 1985; Du and Bilgen, 1990), horizontal annulus (Vasseur *et al.*, 1984), vertical annulus (Prasad and Kulacki, 1984), and domed-shaped enclosure (Das and Morsi, 2005).

As an alternative to purely numerical methods such as the finite element method (Lewis and Schrefler, 1998), an alternative hybrid numerical-analytical approach, the generalized integral transform technique (GITT), has been developed in recent decades as a powerful computational tool for the solution of non-transformable heat transfer and diffusion-convection problems (Cotta, 1993), with moving boundaries (Guerrero and Cotta, 1992; Guigon *et al.*, 2007), nonlinear boundary conditions (Cotta *et al.*, 2016), source terms (An *et al.*, 2013; Fu *et al.*, 2018) and cavities completely (Alves and Cotta, 2000; Machado dos Santos *et al.*, 2022) or partially (Lisboa *et al.*, 2018) filled with porous media. Baohua and Cotta (Baohua and Cotta, 1993) applied the GITT to study steady state natural convection in a saturated porous vertical rectangular enclosure subjected to uniform internal heat generation. The solutions were obtained in the stream function-temperature formulation and the results were compared with purely numerical solutions previously reported on the literature. Recently, the steady-state natural convection in a vertical cylinder under different boundary condition with internal heat generation was investigated using the GITT technique (Fu *et al.*, 2018).

There are a large number of publications on natural convection in annular geometries, with or without porous medium, for example, Caltagirone (1976); Burns and Tien (1979); Vasseur *et al.* (1984); Himasekhar and Bau (1988); Barbosa Mota and Saatdjian (1995); Scurtu *et al.* (2001); Roy and Gorla (2018). In this work, GITT is applied to solve the governing equations in stream-function, combining integral transform and finite difference method. Excellent convergence is shown for both stream-function and temperature. Significant effects of the introduction of curvature effects on the natural convection flow pattern are observed, as well as on the values of temperature and stream function fields.

2. MATHEMATICAL FORMULATION

Consider two-dimensional, axisymmetric, steady-state natural convection in a vertical annulus, bounded by concentric cylindrical surfaces at inner radius r_i, outer radius r_o, and horizontal end surfaces at $z = 0$ and $z = H$, as shown in Fig. 1.

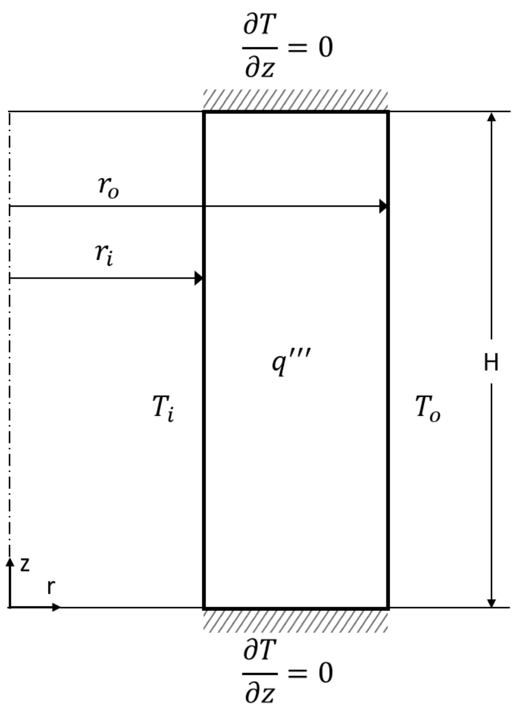


Figure 1. A vertical annulus and coordinate system.

The enclosure is filled with a porous medium, subjected to uniform internal heating, with volumetric heat generation rate q'''. The porous medium is isotropic, homogeneous, and saturated with an incompressible fluid. The inner and outer walls are kept at uniform temperatures T_i and T_o respectively, and the horizontal end walls at $z = 0$ and $z = H$ are kept adiabatic. All thermophysical properties are considered constant.

The governing equations for steady-state natural convection in a homogeneous, isotropic porous medium, based on Darcy's law and Boussinesq approximation, are as follows:

$$\nabla \cdot \vec{v} = 0, \tag{1a}$$

$$\nabla p + \rho \vec{g} - \frac{\mu}{K}\vec{v} = 0, \tag{1b}$$

$$\rho_0 c_p (\vec{v} \cdot \nabla) T = k \nabla^2 T + q''', \tag{1c}$$

with

$$\rho = \rho_0 [1 - \beta (T - T_0)], \tag{2}$$

where $\vec{v}$ is the fluid velocity, p the pressure, $\vec{g}$ the gravitational acceleration, μ the fluid viscocity, K the permeability of the porous medium, ρ the fluid density, c_p the specific heat, and k the thermal conductivity.

For two dimensional axisymmetric natural convection, equations (1) are written in cylindrical coordinates as

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rv)}{\partial z} = 0, \tag{3a}$$

$$\frac{\partial p}{\partial r} + \frac{\mu}{K}u = 0, \tag{3b}$$

$$\frac{\partial p}{\partial z} + \rho g + \frac{\mu}{K}v = 0, \tag{3b}$$

$$u\frac{\partial T}{\partial r} + v\frac{\partial T}{\partial z} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] + \frac{q'''}{\rho_0 c_p}, \tag{3c}$$

with α being the thermal diffusivity.

The stream-function ψ is introduced, which satisfies the continuity equation (3a)

$$u = -\frac{1}{r}\frac{\partial\psi'}{\partial z} = 0, \tag{4a}$$

$$v = \frac{1}{r}\frac{\partial\psi'}{\partial r} = 0, \tag{4b}$$

The governing equations in stream function form are as follows:

$$\frac{\mu}{K}v\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi'}{\partial r}\right) + \frac{1}{r}\frac{\partial^2\psi'}{\partial z^2}\right] = \rho g\beta\frac{\partial T}{\partial r}, \tag{5a}$$

$$\frac{\partial\psi'}{\partial r}\frac{\partial T}{\partial z} - \frac{\partial\psi'}{\partial z}\frac{\partial T}{\partial r} = \alpha\left[\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + r\frac{\partial^2 T}{\partial z^2}\right] + \frac{rq'''}{\rho_0 c_p}. \tag{5b}$$

The following dimensionless variables and parameters are defined

$$R = \frac{r - r_i}{r_o - r_i}, \quad A = \frac{H}{r_o - r_i}, \quad \gamma = \frac{r_o - r_i}{r_i}, \quad \psi = \frac{\psi'(r_o - r_i)}{\alpha r_i H}, \tag{6a-d}$$

$$T = \frac{T - T_o}{\Delta T_0}, \quad Ra = \frac{g\,\beta\,(r_o - r_i)\,K\,\Delta T_0}{\alpha\nu} \quad Ra_i = \frac{g\,\beta\,(r_o - r_i)^3\,K\,q'''}{\alpha\nu k}, \tag{6e-g}$$

where A is the aspect ratio and γ is the ratio of gap width over the inner radius, $\gamma = \kappa - 1$, with $\kappa = r_o/r_i$ being the radius ratio.

For the general problem of different temperatures at the inner and outer walls, the reference temperature difference is naturally taken as $\Delta T_0 = T_i - T_o$. For the particular problem of both inner and outer walls being at same temperature $T_i = T_o$, the reference temperature difference is based on the volumetric heat generation rate q''', $\Delta T_0 = q'''(r_o - r_i)^2/k$; in this case, $Ra = Ra_i$.

The dimensionless governing equations in cylindrical coordinates are given by:

$$A^2\frac{\partial}{\partial R}\left(\frac{1}{1+\gamma R}\frac{\partial\psi}{\partial R}\right) + \frac{1}{1+\gamma R}\frac{\partial^2\psi}{\partial Z^2} = Ra^*\frac{\partial\theta}{\partial R}, \quad \text{in} \quad 0 < R < 1, \quad 0 < Z < 1, \tag{7a}$$

$$\frac{\partial\psi}{\partial R}\frac{\partial\theta}{\partial Z} - \frac{\partial\psi}{\partial Z}\frac{\partial\theta}{\partial R} = \frac{\partial}{\partial R}\left((1+\gamma R)\frac{\partial\theta}{\partial R}\right) + \frac{1+\gamma R}{A^2}\frac{\partial^2\theta}{\partial z^2} + (1+\gamma R)\frac{Ra_i}{Ra^*}, \quad \text{in} \quad 0 < R < 1, \quad 0 < Z < 1. \tag{7b}$$

For the general problem of differential heating, the dimensionless boundary conditions are:

$$\psi(0,Z) = \psi(1,Z) = 0, \quad \theta(0,Z) = 1, \quad \theta(1,Z) = 0, \tag{8a-d}$$

$$\psi(R,0) = \psi(R,1) = 0, \quad \frac{\partial\theta(R,0)}{\partial Z} = \frac{\partial\theta(R,1)}{\partial Z} = 0. \tag{8e-h}$$

It can be seen that the general problem of differential heating with volumetric heat generation, there are two modified Rayleigh numbers Ra^* and Ra_i^*, representing respectively the differential heating and volumetric heat generation.

For the particular problem of equal inner and outer wall temperatures, the boundary condition for the dimensionless temperature at the inner radius is replaced by

$$\theta(0,Z) = 0. \tag{9}$$

For this problem, there is only one Rayleigh number, as $Ra = Ra_i$. The heat source term becomes $1 + \gamma R$ in the energy equation (7b).

2.1 Auxiliary problems

For the application of the generalized integral transform technique, the following auxiliary problems are chosen in the Z-coordinate:

$$\frac{\mathrm{d}^2\chi_i(Z)}{\mathrm{d}Z^2} + \mu_i^2\chi_i(Z) = 0, \quad 0 < Z < 1, \tag{10a}$$

$$\chi_i(0) = 0, \quad \chi_i(1) = 0, \quad i = 1, 2, 3, .. \tag{10b,c}$$

for the stream function ψ, and

$$\frac{\mathrm{d}^2\Gamma_n(Z)}{\mathrm{d}Z^2} + \lambda_n^2\Gamma_n(Z) = 0, \quad 0 < Z < 1, \tag{11a}$$

$$\frac{\mathrm{d}\Gamma_n(0)}{\mathrm{d}Z} = 0, \quad \frac{\mathrm{d}\Gamma_n(1)}{\mathrm{d}Z} = 0, \quad n = 1, 2, 3, ... \tag{11b,c}$$

for the temperature θ, with $\chi_i(Z)$ and $\Gamma_n(Z)$ being the eigenfunctions, and μ_i and λ_n the corresponding eigenvalues.

The eigenfunctions $\chi_i(Z)$ are given by

$$\chi_i(Z) = \sqrt{2}\sin(\mu_i Z), \quad \mu_i = i\,\pi, \quad i = 1, 2, 3, ... \tag{12}$$

The eigenfunctions $\Gamma_n(Z)$ are given by

$$\Gamma_1(Z) = 1, \quad \lambda_1 = 0, \tag{13a}$$

$$\Gamma_n(Z) = \sqrt{2}\cos(\lambda_n Z), \quad \lambda_n = (n-1)\pi, \; n = 2, 3, ... \tag{13b}$$

2.2 Integral transform pairs

The auxiliary problems allow the definition of the following integral transform pair the streamfunction and the temperature respectively:

$$\overline{\psi_i}(R) = \int_0^1 \chi_i(Z)\psi(R, Z)\mathrm{d}Z, \quad \text{transform,} \tag{14a}$$

$$\psi(R, Z) = \sum_{i=1}^{\infty} \chi_i(Z)\overline{\psi_i}(R), \quad \text{inversion,} \tag{14b}$$

$$\overline{\theta_n}(R) = \int_0^1 \Gamma_n(\theta)\theta(R, Z)\mathrm{d}Z, \quad \text{transform,} \tag{15a}$$

$$\theta(R, Z) = \sum_{n=1}^{\infty} \Gamma_n(Z)\overline{\theta_n}(R). \quad \text{inversion,} \tag{15b}$$

2.3 Transformed equations

Performing the integral transform in the 'Z' direction, Eq. (7a) is operated by $\int_0^1 \chi_i(Z)\_\mathrm{d}Z$, with the inverse formula (14b,15b) applied:

$$\frac{A^2}{1+\gamma R}\frac{\mathrm{d}^2\overline{\psi_i}(R)}{\mathrm{d}^2 R} - \frac{\gamma A^2}{(1+\gamma R)^2}\frac{\mathrm{d}\overline{\psi_i}(R)}{\mathrm{d}R} - \frac{\mu_i^2}{1+\gamma R}\overline{\psi_i}(R) = Ra^* \sum_{j=1}^{\infty} a_{ij}\frac{\mathrm{d}\overline{\theta_j}(R)}{\mathrm{d}R}, \quad i = 1, 2, 3, ... \tag{16}$$

where the coefficients a_{ij} and a_{ij}^* are given by

$$a_{ij} = \int_0^1 \chi_i(Z)\Gamma_j(Z)\mathrm{d}Z. \tag{17}$$

Similarly, Eq. (7a) is operated by $\int_0^1 \Gamma_n(Z)\_\mathrm{d}Z$, with the inverse formula (14b,15b) applied:

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty} B_{njk}\overline{\theta_k}(R)\frac{\mathrm{d}\overline{\psi_j}(R)}{\mathrm{d}R} - \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} B_{njk}^*\overline{\psi_k}(R)\frac{\mathrm{d}\overline{\theta_j}(R)}{\mathrm{d}R}$$
$$= (1+\gamma R)\frac{\mathrm{d}^2\overline{\theta_n}(R)}{\mathrm{d}R^2} + \gamma\frac{\mathrm{d}\overline{\theta_n}(R)}{\mathrm{d}R} - \frac{(1+\gamma R)\lambda_n^2}{A^2}\overline{\theta_n}(R) + (1+\gamma R)\frac{Ra_i^*}{Ra^*}\overline{f_n}, \quad n = 1, 2, 3, ... \tag{18}$$

where the coefficients $\overline{f_n}$, B_{njk} and B^*_{njk} are given by

$$\overline{f_n} = \int_0^1 \Gamma_n(Z)\mathrm{d}Z, \tag{19a}$$

$$B_{njk} = \int_0^1 \Gamma_n(Z)\chi_j(Z)\Gamma_k^{'}(Z)\mathrm{d}Z, \tag{19b}$$

$$B^*_{njk} = \int_0^1 \Gamma_n(Z)\Gamma_j(Z)\chi_k^{'}(Z)\mathrm{d}\theta. \tag{19c}$$

The transformed equations (16) and (18) are solved together with the following transformed boundary conditions:

$$\overline{\psi_i}(0) = 0, \quad \overline{\psi_i}(1) = 0, \quad i = 1, 2, 3, ... \tag{20a,b}$$

$$\overline{T_n}(0) = 0, \quad \overline{T_n}(1) = 0, \quad n = 1, 2, 3, ... \tag{20c,d}$$

For computational purpose, the eigenvalue expansions are truncated at a finite order N.

The transformed equations (16) and (18) together with the transformed boundary conditions (20) are solved by a fourth-order finite difference method in the R-direction. The inversion formulas (14b,15b) are then applied to obtain the final results for the originals temperature and stream-function. All calculations are implemented by using the Mathematica software.

3. RESULTS

The convergence for temperature and stream function is presented in Tab. 1,2, for $Ra* = 50, 100, 250, 500, 750$ and 1000, γ = 1, A=1, and $Ra_i = 500$, where the truncation order for the expansions N, varies from 10 to 25. The values of temperature and stream function are presented at different locations of the geometry, where $r = 0.25, 0.50$ and 0.75 and $z = 0.25, 0.50$ and 0.75. It can be seen in Table 1 that the temperature converged to four digits with the truncation order $N = 15$. Table 2 presents a similar behaviour to the stream function. Figure 2 presents a comparison of the calculated average internal Nusselt number with results obtained from the literature. It can be noticed that the results presented in this work are in excellent agreement with the data obtained from the literature.

Table 1. Convergence analysis of temperature to different positions in R and Z coordinates.

		T(r,z)								
Ra	N	r = 0.25			r=0.5			r=0.75		
		z			z			z		
		0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
50	10	1.0101	1.1760	1.3403	0.7714	1.0621	1.3568	0.4895	0.7886	1.1738
	15	1.0102	1.1760	1.3401	0.7715	1.0621	1.3566	0.4897	0.7885	1.1734
	20	1.0102	1.1760	1.3401	0.7715	1.0621	1.3566	0.4896	0.7885	1.1734
	25	1.0102	1.1760	1.3401	0.7715	1.0621	1.3566	0.4896	0.7885	1.1733
100	10	0.5382	0.7632	0.9654	0.3952	0.6300	0.8983	0.2585	0.4734	0.7726
	15	0.5383	0.7632	0.9653	0.3952	0.6300	0.8982	0.2586	0.4733	0.7722
	20	0.5383	0.7632	0.9653	0.3952	0.6300	0.8982	0.2585	0.4733	0.7722
	25	0.5383	0.7632	0.9653	0.3952	0.6299	0.8982	0.2585	0.4733	0.7722
250	10	0.2557	0.4849	0.7715	0.1948	0.3958	0.6872	0.1369	0.3154	0.5945
	15	0.2558	0.4849	0.7713	0.1948	0.3957	0.6869	0.1369	0.3152	0.5939
	20	0.2558	0.4849	0.7713	0.1948	0.3957	0.6869	0.1369	0.3152	0.5938
	25	0.2558	0.4849	0.7713	0.1948	0.3957	0.6869	0.1369	0.3152	0.5938
500	10	0.1756	0.3829	0.6947	0.1393	0.3246	0.6235	0.1061	0.2752	0.5474
	15	0.1759	0.3826	0.6941	0.1394	0.3243	0.6227	0.1061	0.2749	0.5462
	20	0.1760	0.3826	0.6940	0.1394	0.3243	0.6225	0.1061	0.2749	0.5460
	25	0.1760	0.3826	0.6939	0.1395	0.3243	0.6225	0.1061	0.2749	0.5458
750	10	0.1519	0.3461	0.6602	0.1233	0.3006	0.5969	0.0985	0.2638	0.5295
	15	0.1522	0.3456	0.6591	0.1234	0.3003	0.5957	0.0986	0.2634	0.5279
	20	0.1524	0.3455	0.6590	0.1235	0.3002	0.5954	0.0986	0.2634	0.5275
	25	0.1524	0.3455	0.6589	0.1235	0.3002	0.5952	0.0986	0.2633	0.5272
1000	10	0.1407	0.3265	0.6388	0.1159	0.2885	0.5804	0.0956	0.2586	0.5193
	15	0.1411	0.3258	0.6373	0.1162	0.2881	0.5788	0.0957	0.2581	0.5172
	20	0.1413	0.3257	0.6370	0.1163	0.2880	0.5783	0.0957	0.2581	0.5166
	25	0.1414	0.3257	0.6368	0.1163	0.2880	0.5780	0.0957	0.2580	0.5162

Table 2. Convergence analysis of stream function to different positions in R and Z coordinates.

$\psi(r, z)$										
Ra	N	r = 0.25			r=0.5			r=0.75		
		z			z			z		
		0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
50	10	1.962	1.606	0.408	4.052	4.585	2.846	4.817	6.593	5.523
	15	1.959	1.605	0.411	4.050	4.582	2.849	4.817	6.588	5.526
	20	1.961	1.606	0.410	4.053	4.583	2.849	4.820	6.589	5.528
	25	1.960	1.606	0.411	4.052	4.583	2.849	4.818	6.589	5.528
100	10	4.438	4.786	2.726	5.525	7.085	4.937	5.689	8.326	7.216
	15	4.438	4.785	2.730	5.525	7.083	4.940	5.689	8.321	7.221
	20	4.440	4.785	2.730	5.527	7.084	4.942	5.691	8.322	7.223
	25	4.439	4.786	2.730	5.526	7.084	4.942	5.690	8.323	7.223
250	10	8.604	11.003	7.571	8.565	12.324	9.694	8.080	12.879	11.872
	15	8.620	11.010	7.578	8.575	12.331	9.707	8.085	12.881	11.892
	20	8.626	11.014	7.582	8.579	12.335	9.712	8.089	12.884	11.900
	25	8.626	11.015	7.582	8.579	12.337	9.712	8.089	12.886	11.900
500	10	12.928	17.637	13.122	12.242	18.433	15.360	11.433	18.759	17.858
	15	12.979	17.669	13.149	12.275	18.464	15.402	11.453	18.780	17.918
	20	12.997	17.682	13.162	12.287	18.478	15.420	11.463	18.792	17.943
	25	13.003	17.689	13.164	12.291	18.485	15.424	11.464	18.799	17.949
750	10	16.180	22.566	17.444	15.152	23.147	19.837	14.228	23.393	22.588
	15	16.265	22.623	17.495	15.207	23.202	19.912	14.265	23.437	22.689
	20	16.300	22.652	17.522	15.232	23.231	19.948	14.283	23.461	22.736
	25	16.315	22.667	17.530	15.241	23.246	19.960	14.288	23.475	22.752
1000	10	18.901	26.641	21.112	17.635	27.098	23.651	16.657	27.296	26.577
	15	19.020	26.725	21.190	17.715	27.180	23.763	16.712	27.364	26.721
	20	19.073	26.770	21.234	17.752	27.224	23.819	16.740	27.402	26.793
	25	19.099	26.795	21.249	17.769	27.248	23.840	16.750	27.424	26.821

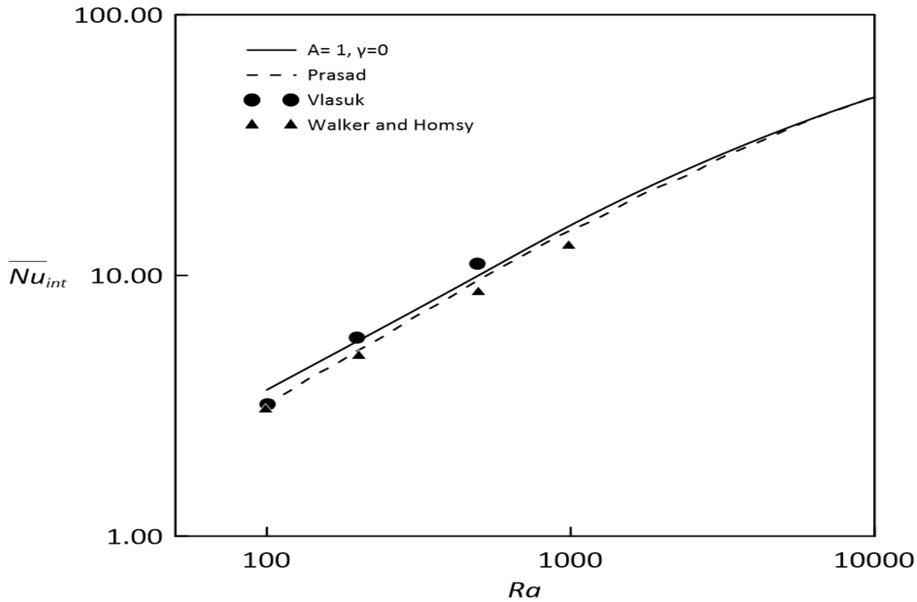
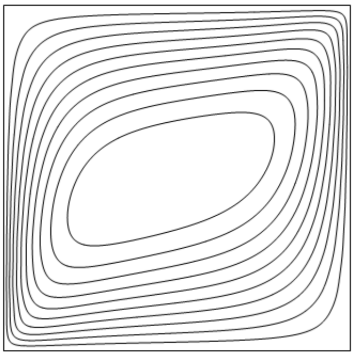
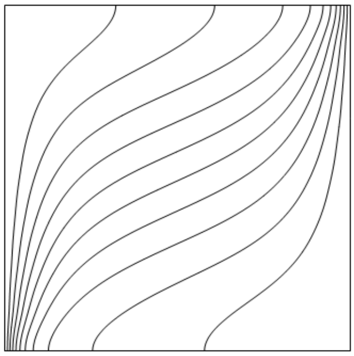
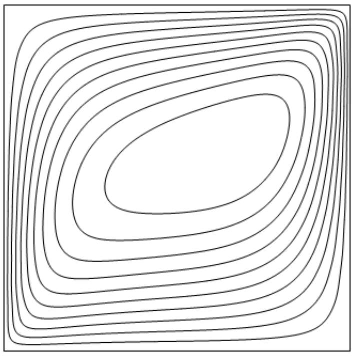


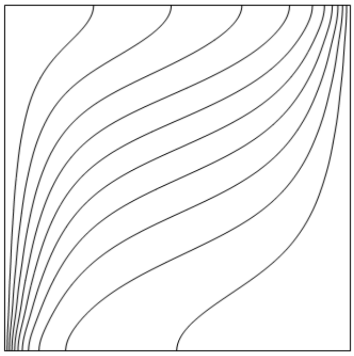
Figure 2. Comparison of average internal nusselt results with literature.

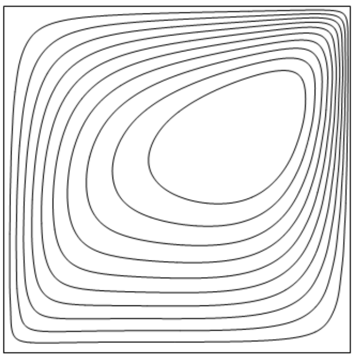
Figure 3 shows the streamlines and isotherms to fixed values of aspect ratio ($A = 1$) and Rayleigh number ($Ra* = 100$), and varying values of Ra_i and ratio of gap (γ). It was observed that as the ratio of gap and consequently the radius ratio increases, isotherms shift towards the inner wall. The opposite behavior is observed when the Ra_i increases from 0 to 500 to a fixed value of radius ratio, so that the isotherms move towards the outer wall. It was also observed that curvature effects ($\gamma > 0$) on temperature and velocity fields disturb significantly the centro-symmetrical nature found in the vertical cavity case.

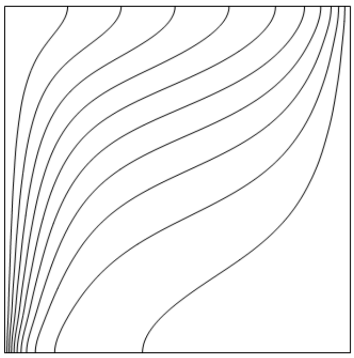

(a) Streamline, $Ra_i = 0, \gamma = 0$

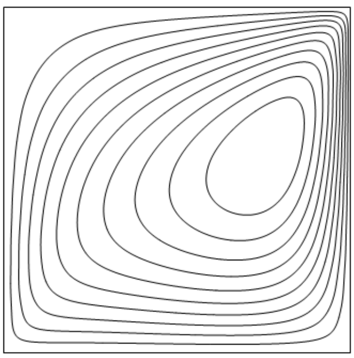

(b) Isotherms, $Ra_i = 0, \gamma = 0$


(c) Streamline, $Ra_i = 0, \gamma = 1$


(d) Isotherms, $Ra_i = 0, \gamma = 1$


(e) Streamline, $Ra_i = 0, \gamma = 5$


(f) Isotherms,$Ra_i = 0, \gamma = 5$


(g) Streamline,$Ra_i = 500, \gamma = 1$

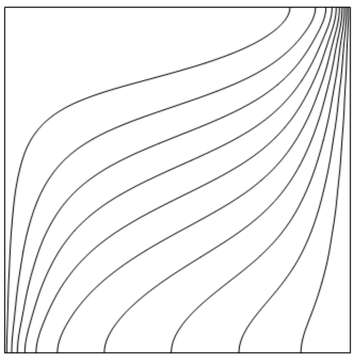

(h) Isotherms,$Ra_i = 500, \gamma = 1$

Figure 3. Streamlines and isotherms for A=1, $Ra* = 100$, Ra_i and γ varying.

Figure 4 presents the temperature distribution at mid-height of annulus to different aspect ratios and radius ratio parameters. The temperature decreases more rapidly as the radius ratio parameter increases. With the reduction of the aspect ratio, the temperature drops initially faster until it reaches an approximately constant level and then drops again, while for a higher value of aspect ratio the variation is less intense and more uniform.

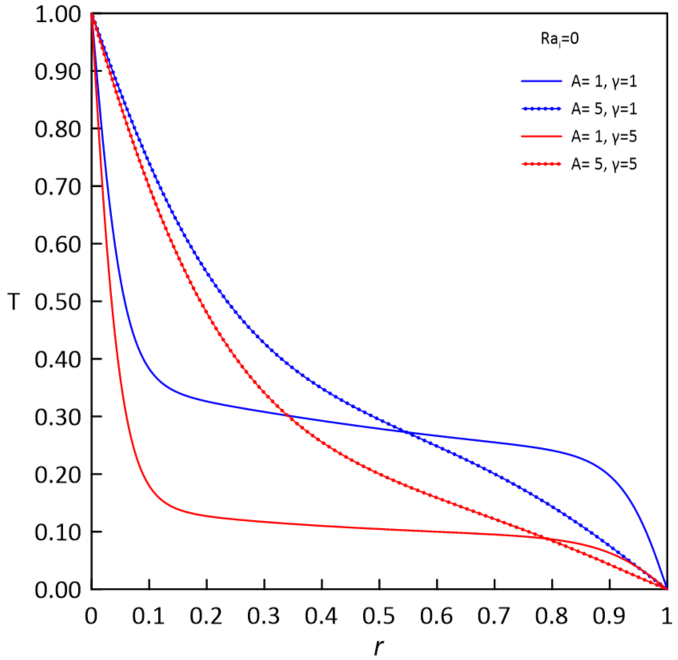
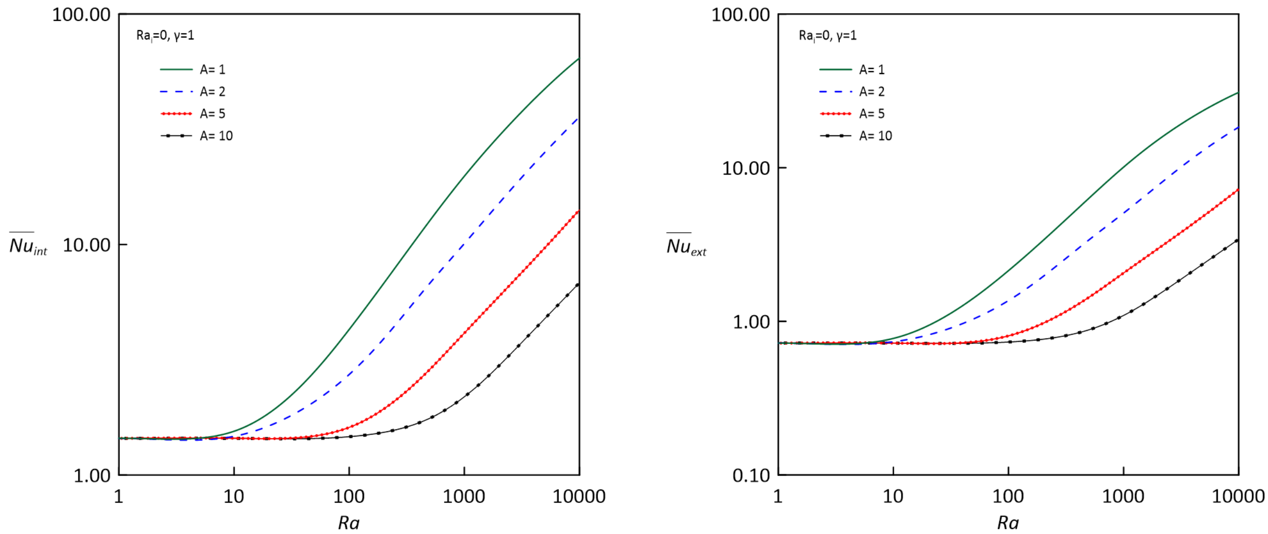


Figure 4. Temperature distribution at mid-height of annulus (Z=0.5) for Ra*=1000

Average values of Nusselt number on the inner ($\overline{Nu}_{int}$) and outer ($\overline{Nu}_{ext}$) wall are presented for a wide range of Ra* and A in Fig. 5.It can be seen that Nusselt number increases with increasing Rayleigh number, and decreases with increasing aspect ratio.



(a) Inner wall (b) Outer wall

Figure 5. Average Nusselt number variation to different Rayleigh numbers and aspect ratios.

4. CONCLUSIONS

Natural convection in a two-dimensional vertical annulus filled with saturated porous medium was investigated. The porous medium is considered homogeneous, isotropic and saturated with an incompressible fluid. The mathematical formulation through the generalized integral transform technique, a powerful hybrid numerical-analytical approach, was presented and the results were compared with literature showing excellent agreement with low computational cost. A fourth-order finite difference method in the radial direction was applied to solve the transformed equations. A convergence study to the temperature and stream function was carried out to define the value of truncation order ($N = 15$). It was observed that the introduction of curvature effects on temperature and velocity is significant and completely disturbs the central-symmetry of the vertical cavity. The internal heat generation, represented by Ra_i, also disturbs the central-symmetry of the cavity, but the isotherms move towards the outer wall instead of the inner wall as observed with the curvature effects. The average Nusselt number on the inner and outer walls always increases as the aspect ratio decreases and the Rayleigh number increases.

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