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ANALYTICAL METHODS TO CALCULATE RADIATION VIEW FACTORS BETWEEN DISKS – A REVIEW

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Abstract. *The article deals with the development of the equations used to calculate the radiation view factors between disks. Two methods are available: the first, using the contour integration method and intended for parallel disks, and a second method coming from spherical cavities. In such a case, the disks need to be inserted in the cavity, or one spherical cavity must be drawn around the disks to determine two independent and non-intercepting caps. The first method presents more compact equations and is the most used in the area, despite its limitation to parallel disk case. Both methods show equal numerical results for this disposition, although with very dissimilar equations. The text shows how equations and calculation methods are equivalent. Finally, the spherical cavity method has a particular advantage in solving problems when disks are not parallel, but disks must define one specific spherical cavity and special attention is necessary.*

Keywords: *thermal radiation, view factors, parallel disks, spherical cavity, contour integration method.*

1. INTRODUCTION

View factors are necessary to calculate the radiative heat transfer between surfaces and this thermal energy rate is inserted in the general energy balance that takes place in the problem to be solved. It appears in the case of surfaces separated by vacuum or transparent media, generally gaseous media in a short distance. The view factors are uniquely geometrical entities, depending on the distance, sizes, and angles, regarding the relative disposition of surfaces.

A series of methods has been used to determine them, starting from an estimation by simple inspection, followed by a more elaborated analytical approach, ending by a large quantity of numerical methods. Numerical method usually integrates the view factor in the energy equations applied to each grid element and do not need to be calculated previously.

For parallel disks the first approach was presented by Keenes (1912), as referred by Howell (1982), and the article aim was a description of a calorimeter designed to determine the Stefan-Boltzmann constant. The radiative exchange was established between a blackbody cavity and a cold water-based calorimeter. As circular apertures were used, a view factor between parallel disks was calculated considering the contour integral method, reducing area integrals to an only one contour integral. The approach is also presented by Modest (1993). This application results in a very compact equation system, and it is widespread in practically any publications that include calculation of radiative transfer.

Another approach is the “inside sphere method”, presented by Modest (1993). It is quite simple in its conception, and the restriction consists of finding a spherical cavity that includes the disks, without interference between them. Although a simple method in the conception, the resulting equation system is more complex and not so well known as the previous one. It has a particularity of solving configurations where disks are not parallel, but some restrictions are imposed, and everyone should be warned about dubious results coming from specific situations related to disks dispositions.

Concerning to parallel disks, both methods must have equal numerical results. Although the final equation systems are very distinct, it can be demonstrated that they are similar at the end, if some mathematical manipulations are employed.

2. CONTOUR INTEGRATION METHOD

The development is based on the Eq. (1), Siegel and Howell (1992), following the application of the Green Theorem, transforming two integrals on the area A_1 and A_2 (Fig. 1), in two contour integrals, cartesian coordinates. The configuration is characterized by two parallel disks with radius R_1 and R_2, separated by a distance H.

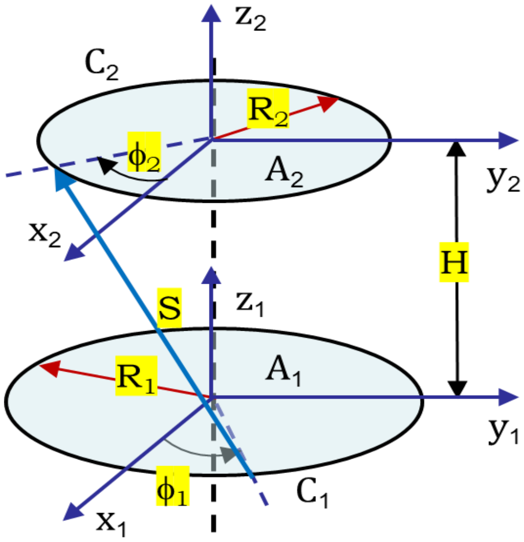

Figure 1 – Parallel disks and coordinate system.

$$F_{1-2} = 1/(2\pi A_1) \oint_{C2} \oint_{C1} \ln S\,(dx_1\,dx_2 + dy_1 dy_2); \tag{1}$$

where S is the distance between two generic points on the contours, Eq. (2):

$$S^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \tag{2}$$

As the origin of the coordinate system is coincident with the center of the disk 1, $z_1 = 0$, $z_2 = H$, then $dz_1 = dz_2 = 0$;

$$S^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + H^2 \tag{3}$$

The representation of the cartesian coordinates in the cylindrical coordinate system and respective differentials are given in Eqs. (4) to (7):

$$x_1 = R_1 \cos\phi_1\ ;\ dx_1 = -R_1 \sin\phi_1\, d\phi_1 \tag{4}$$

$$x_2 = R_2 \cos\phi_2\ ;\ dx_2 = -R_2 \sin\phi_2\, d\phi_2 \tag{5}$$

$$y_1 = R_1 \sin\phi_1\ ;\ dx_1 = R_1 \cos\phi_1\, d\phi_1 \tag{6}$$

$$y_2 = R_2 \sin\phi_2\ ;\ dx_2 = R_2 \cos\phi_2\, d\phi_2 \tag{7}$$

Substituting the coordinates x and y, a new formula for S can be obtained – Eq. (8):

$$S^2 = (R_2 \cos\phi_2 - R_1 \cos\phi_1)^2 + (R_2 \sin\phi_2 - R_1 \sin\phi_1)^2 + H^2 \tag{8}$$

Developing the square terms, Eq. (9) is the result:

$$S^2 = R_2^2 \cos^2\phi_2 + R_1^2 \cos^2\phi_1 - 2R_1R_2 \cos\phi_1 \cos\phi_2 + R_2^2 \sin^2\phi_2 + R_1^2 \sin^2\phi_1 - 2R_1R_2 \sin\phi_1 \sin\phi_2 + H^2 \tag{9}$$

and regrouping them, Eq. (10) can be achieved, as $\sin^2\phi + \cos^2\phi = 1$:

$$S^2 = R_1^2 + R_2^2 - 2R_1R_2 (\sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2) + H^2 \tag{10}$$

Considering the identity from Eq. (11), (Spiegel, 1992):

$$\sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 = \cos(\phi_1 - \phi_2), \tag{11}$$

a S new value can be represented by the difference between polar angles - Eq. (12):

$$S^2 = R_1^2 + R_2^2 + H^2 - 2R_1R_2 \cos(\phi_1 - \phi_2) \quad (12)$$

Such difference can be expressed as a simple angle ϕ, as $\phi = \phi_1 - \phi_2$, Eq. (13):

$$S^2 = R_1^2 + R_2^2 + H^2 - 2R_1R_2 \cos\phi \quad (13)$$

To reduce the equation size, new groups of variables could be used: $A = R_1^2 + R_2^2 + H^2$, and $B = 2R_1R_2$. Then a more compact equation appears, Eq. (14):

$$S^2 = A - B\cos\phi \quad (14)$$

Taking Eq. (1) and replacing the differential in cartesian coordinates, Eq. (15) is the result:

$$F_{1-2} = 1/(2\pi A_1) \oint_{C2} \oint_{C1} \ln S \,[(-R_1 \sin\phi_1\, d\phi_1)(-R_2 \sin\phi_2\, d\phi_2) + (R_1 \cos\phi_1\, d\phi_1)(R_2 \cos\phi_2\, d\phi_2)] \quad (15)$$

Multiplying the terms inside the integrals, as done in Eq. (9):

$$F_{1-2} = R_1R_2/(2\pi A_1) \oint_{C2} \oint_{C1} \ln S \,[(\sin\phi_1 \sin\phi_2) + (\cos\phi_1 \cos\phi_2)]\, d\phi_1\, d\phi_2 \quad (16)$$

Using the identity from Eq. (11) allows to rewrite Eq. (16) as Eq. (17):

$$F_{1-2} = R_1R_2/(2\pi A_1) \oint_{C2} \oint_{C1} \ln S \,\cos(\phi_1 - \phi_2)\, d\phi_1\, d\phi_2 \quad (17)$$

With the difference between polar angles $(\phi = \phi_1 - \phi_2)$, and for $\phi_2 = 0$, Eq. (18) appears:

$$F_{1-2} = R_1R_2/(2\pi A_1) \oint_{C2} \oint_{C1} \ln S \,\cos\phi\, d\phi\, d\phi_2 \quad (18)$$

Following with the integral on the contour *1* and substituting *S* from Eq. (14):

$$\oint_{C1} \ln S \cos\phi\, d\phi = \int_0^{2\pi} \ln S \cos\phi\, d\phi = \int_0^{2\pi} \ln (A - B\cos\phi)^{1/2} \cos\phi\, d\phi = 1/2 \int_0^{2\pi} \ln (A - B\cos\phi) \cos\phi\, d\phi \quad (19)$$

From Keene (1912), the last integral can be solved as:

$$\oint_{C1} \ln S \cos\phi\, d\phi = 1/2\, [\sin\phi \ln(A - B\cos\phi)]_0^{2\pi} - 1/2\, B \int_0^{2\pi} \sin^2\phi / (A - B\cos\phi)\, d\phi \quad (20)$$

As the first term is null $(\sin 0 = \sin 2\pi = 0)$, and using trigonometry to have Eq. (21):

$$\oint_{C1} \ln S \cos\phi\, d\phi = -1/2\, B \int_0^{2\pi} \sin^2\phi / (A - B\cos\phi)\, d\phi = -1/2\, B \int_0^{2\pi} (\cos^2\phi - 1)/(B\cos\phi - A)\, d\phi \quad (21)$$

The following term, Eq. (22), can be developed to obtain the term inside the last integral in Eq. (21):

$$\cos\phi/B + A/B^2 + [(A^2/B^2) - 1]/(B\cos\phi - A) = (\cos^2\phi - 1)/(B\cos\phi - A); \quad (22)$$

then, the next step is given in the Eq. (23):

$$\oint_{C1} \ln S \cos\phi\, d\phi = -1/2\, B \int_0^{2\pi} \{\cos\phi/B + A/B^2 + [(A^2/B^2) - 1]/(B\cos\phi - A)\}\, d\phi =$$

$$= -1/2\, [0 + 2\pi A/B - [(A^2 - B^2)/B] \int_0^{2\pi} 1/(B\cos\phi - A)\, d\phi \quad (23)$$

Following Spiegel (1992), the definite integral can be solved as:

$$\oint_{C1} \ln S \cos\phi\, d\phi = -1/2\, [2\pi A/B - [(A^2 - B^2)/B]\, 2\pi/(A^2 - B^2)^{1/2} = -\pi[A/B - (A^2 - B^2)^{1/2}/B] \quad (24)$$

This result for contour *1* can be substituted in Eq. (18):

$$F_{1-2} = -R_1 R_2/(2\pi A_1) \oint_{C2} \pi [A/B - (A^2 - B^2)^{½}/B]\, d\phi_2 \tag{25}$$

As ϕ_2 must turn from 2π to 0, the limits should be inverted, Eq. (26):

$$F_{1-2} = R_1 R_2/(2A_1) \int_0^{2\pi} [A/B - (A^2 - B^2)^{½}/B]\, d\phi_2 = R_1 R_2/(2\pi R_1^2)\ [A/B - (A^2 - B^2)^{½}/B] 2\pi \tag{26}$$

Performing some arithmetic operations, a more compact equation is found, Eq. (27):

$$F_{1-2} = (R_2/R_1)\ [A/B - (A^2 - B^2)^{½}/B] \tag{27}$$

Then substituting A and B, introduced after Eq. (13) in the text, a final equation is determined:

$$F_{1-2} = ½ \{R_1^2 + R_2^2 + H^2 - [(R_1^2 + R_2^2 + H^2)^2 - 4R_1^2 R_2^2]^{½}\}/R_1^2 \tag{28}$$

Using $X = 1 + (1 + R_2^2/H^2)/(R_1^2/H^2)$, the result becomes similar to that appearing in several references like Siegel and Howell (1992), Howell (1982), Incropera and DeWitt (1996), Modest (1993), etc.

$$F_{1-2} = ½ \{X - [X^2 - 4(R_2/R_1)^2]^{½}\}/R_1^2 \tag{29}$$

3. METHOD OF THE SPHERICAL CAVITY

Presented by Modest (1993), two disks (parallel or not), can be displaced inside a spherical cavity (or a sphere can be drawn containing the disks), as shown in Fig. 2. Disks *1* and *2* define spherical caps, with the respective areas $A_{1'}$ and $A_{2'}$. The view factor F_{1-2} between disks can be express function of $F_{2'-1'}$, the view factor between the spherical caps, Eq. (30):

$$F_{1-2} = F_{1-2'} = A_{2'} F_{2'-1}/A_1 = A_{2'} F_{2'-1'}/A_1 \tag{30}$$

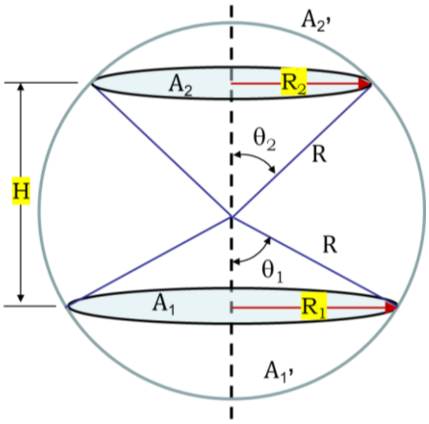

Figure 2 – Disks inside a spherical cavity of radius *R*.

The view factor between the spherical caps ($F_{2'-1'}$), depends only on the ratio area of the cap *1* to sphere area A_s, (Siegel and Howell, 1992), Eq. (31):

$$F_{2'-1'} = A_{1'}/A_s; \tag{31}$$

which can be replaced in Eq. (30) to obtain Eq. (32):

$$F_{1-2} = (A_{2'} A_{1'})/(A_1 A_s) \tag{32}$$

The spherical cap area is generically expressed by Eq. (33); R is the cavity radius:

$$A_{i'} = 2\pi R^2 (1 - \cos\theta_i) \tag{33}$$

These areas and the cavity area $(4\pi R^2)$, could be substituted into Eq. (32):

$$F_{1-2} = R^2 (1 - \cos\theta_1)(1 - \cos\theta_2)/R_1^2 \tag{34}$$

The cosines can be expressed in function of the radius of the disks and the cavity (Eq. 35), and put in the Eq. (34), to obtain Eq. (36), the usual formula associated to this method:

$$\cos \theta_i = (R^2 - R_i^2)^{½}/R = [1 - (R_i/R)^2]^{½} \quad (35)$$

$$F_{1\text{-}2} = R^2 \{1 - [1 - (R_1/R)^2]^{½}\} \{1 - [1 - (R_2/R)^2]^{½}\}/R_1^2 \quad (36)$$

4. DETERMINATION OF THE CAVITY RADIUS

As the radius R of the cavity is not initially defined in the problem, it needs to be expressed as a function of the distance H and disks radius R_1 and R_2. Observing Fig. (2), H can be expressed by Eq. (37), which can be squared to arrive in Eq. (38):

$$H = (R^2 - R_1^2)^{½} + (R^2 - R_2^2)^{½} \quad (37)$$

$$H^2 = [(R^2 - R_1^2)^{½} + (R^2 - R_2^2)^{½}]^2 \quad (38)$$

Developing the square term on the right side of Eq. (38) and performing some additional operation, from Eq. (39) to Eq. (41):

$$H^2 = (R^2 - R_1^2) + (R^2 - R_2^2) + 2[(R^2 - R_1^2)^{½} (R^2 - R_2^2)^{½}] \quad (39)$$

$$H^2 = 2 R^2 - R_1^2 - R_2^2 + 2(R^4 - R^2 R_1^2 - R^2 R_2^2 + R_1^2 R_2^2)^{½} \quad (40)$$

$$½ (H^2 - 2 R^2 + R_1^2 + R_2^2) = (R^4 - R^2 R_1^2 - R^2 R_2^2 + R_1^2 R_2^2)^{½} \quad (41)$$

Squaring Eq. (41) allows to obtain Eq. (42), whose left term can be squared to obtain Eq. (43). In Eq. (43) some terms can be cancelled, ending in the Eq. (44), which enables to isolate the cavity radius, Eq. (45), here with the inclusion of an additional quantity $(2 R_1^2 R_2^2 - 2 R_1^2 R_2^2)$:

$$¼ (H^2 - 2 R^2 + R_1^2 + R_2^2)^2 = R^4 - R^2 R_1^2 - R^2 R_2^2 + R_1^2 R_2^2 \quad (42)$$

$$¼[(H^4 - 4H^2R^2 + 4R^4 + 2H^2R_1^2 + 2H^2R_2^2 - 4R^2R_1^2 - 4R^2R_2^2 + R_1^4 + R_2^4 + 2R_1^2R_2^2] = R^4 - R^2R_1^2 - R^2R_2^2 + R_1^2R_2^2 \quad (43)$$

$$H^2R^2 = ¼ H^4 + ½ H^2 (R_1^2 + R_2^2) + ¼ (R_1^4 + R_2^4) - ½ R_1^2 R_2^2 \quad (44)$$

$$R^2 = ¼ [H^4 + 2 H^2 (R_1^2 + R_2^2) + R_1^4 + R_2^4 + 2 R_1^2 R_2^2 - 4 R_1^2 R_2^2] / H^2 \quad (45)$$

The right term of Eq. (45) could be partially expressed as the left side of Eq. (46), in a compact form, which is substituted back in Eq. (45), resulting in Eq. (47). Then, Eq. (47) can be root-squared, to get Eq. (48) for the radius R:

$$(R_1^2 + R_2^2 + H^2)^2 = R_1^4 + R_2^4 + 2 R_1^2 R_2^2 + 2 H^2 (R_1^2 + R_2^2) + H^4 \quad (46)$$

$$R^2 = [(R_1^2 + R_2^2 + H^2)^2 - 4 R_1^2 R_2^2] / 4 H^2 \quad (47)$$

$$R = [(R_1^2 + R_2^2 + H^2)^2 - 4 R_1^2 R_2^2]^{½} / 2H \quad (48)$$

The radius R of the spherical cavity can also be represented as in Eqs. (49) and (50), Modest (1993):

$$R^2 = (Y^2 - 1) (R_1R_2/H)^2 \quad (49)$$

$$Y = (R_1^2 + R_2^2 + H^2)/(2 R_1R_2) \quad (50)$$

5. EQUIVALENCE BETWEEN EQUATIONS FOR THE VIEW FACTOR

Starting from Eq. (36), that can be developed in a more appropriate form, firstly Eq. (51), then Eq. (52):

$$F_{1\text{-}2} = R^2 \{1 - [(R^2 - R_1^2)/R^2]^{½}\} \{1 - [(R^2 - R_2^2)/R^2]^{½}\} /R_1^2 \quad (51)$$

$$F_{1\text{-}2} = [R - (R^2 - R_1^2)^{½}]\ [R - (R^2 - R_2^2)^{½}]/R_1^2 \quad (52)$$

Multiplying the first group by the second group of terms in the right side of Eq. (52), permits to obtain Eq. (53) and multiplying/dividing by 2, to obtain Eq. (54):

$$F_{1\text{-}2} = [R^2 - R(R^2 - R_1^2)^{½} - R(R^2 - R_2^2)^{½} + (R^2 - R_1^2)^{½}\ (R^2 - R_2^2)^{½}]/R_1^2 \quad (53)$$

$$F_{1\text{-}2} = \{2R^2 + 2(R^2 - R_1^2)^{½}\ (R^2 - R_2^2)^{½} - 2R[(R^2 - R_1^2)^{½} + (R^2 - R_2^2)^{½}]\}/2R_1^2 \quad (54)$$

Adding and subtracting the sum $(R_1^2 + R_2^2)$, then changing the order of some parts:

$$F_{1\text{-}2} = \{R_1^2 + R_2^2 + 2R^2 - R_1^2 - R_2^2 + 2(R^2 - R_1^2)^{½}\ (R^2 - R_2^2)^{½} - 2R[(R^2 - R_1^2)^{½} + (R^2 - R_2^2)^{½}]\}/2R_1^2 \quad (55)$$

The sum of the third to sixty terms is equal to H^2, Eqs. (39) and (40), and the last term is equal to $2RH$ – comparing to Eq. (37). Then the result is expressed in Eq. (56):

$$F_{1\text{-}2} = (R_1^2 + R_2^2 + H^2 - 2\ R\ H)/2R_1^2 \quad (56)$$

As the radius R is not part of the initial problem, it can be replaced by Eq. (48), leading to Eq. (57), then to Eq. (58):

$$F_{1\text{-}2} = \{R_1^2 + R_2^2 + H^2 - 2\ H[(R_1^2 + R_2^2 + H^2)^2 - 4R_1^2 R_2^2]^{½}/2H\}/2R_1^2 \quad (57)$$

$$F_{1\text{-}2} = ½\{R_1^2 + R_2^2 + H^2 - [(R_1^2 + R_2^2 + H^2)^2 - 4R_1^2 R_2^2]^{½}\}/R_1^2 \quad (58)$$

Equation (58) is equal to Eq. (28), showing the equivalence between the methods when used to calculate the view factor for parallel disks. As the inside sphere method is valid for two disks in any position, constrained to the condition of a specific sphere can be drawn around them, Eq. (58) can be used even though disks are not parallel. However, special attention must be put on the determination of the *H* value.

6. CONSIDERATIONS ABOUT RELATIVE CONFIGURATIONS IN THE CAVITY METHOD

In this analysis, the view factor is calculated from the disk *i* $(i=1)$, to other disks *2*, *3* and *4*, as shown in Fig. 3. All the cases result in the same cavity of radius *R*. Disk $i = 1$ is considered the disk of minor diameter. Disks *2* and *4* have the same distance to the sphere center and disk *3* is coincident with this center.

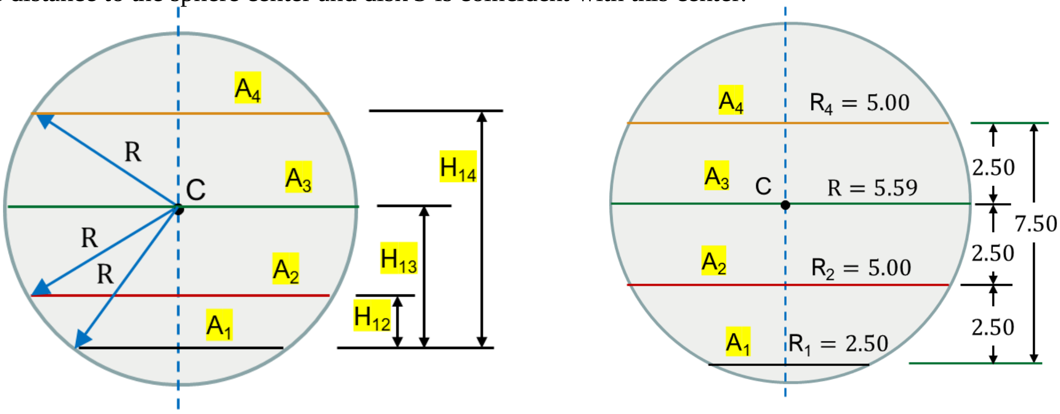

Figure 3 – Cavity method with four disks inside and respective distances: left – generic; right – undefined units.

For the view factor $F_{1\text{-}4}$, H_{14} is the distance separating them, is related to Eq. (59), which is similar to Eq. (37).

$$H_{14} = (R^2 - R_1^2)^{½} + (R^2 - R_4^2)^{½} \quad (59)$$

For $F_{1\text{-}3}$, H_{13} is also the distance between *1* and *3*, but $R_3 = R$ and the second term of Eq. (37) is null. Equation (36) as well as Eq. (58) are adequate to estimate the view factor in such configuration.

Finally, for the view factor $F_{1\text{-}2}$, H_{12} is the distance between the disks – Eq. (60) - and Eq. (36) gives dissimilar results from Eq. (28) or Eq. (58). The estimation of *R* using Eq. (48) – starting from Eq. (37) or from Eq. (60) - arrives at the same value. The minus sign disappears when the Eq. (41) is squared, leading to the Eq. (42).

$$H_{12} = (R^2 - R_1^2)^{½} - (R^2 - R_2^2)^{½} \tag{60}$$

Both view factors *($F_{1\text{-}2}$ or $F_{1\text{-}4}$)* can be estimated from Eq. (28) or Eq. (58). However, the problem arises wherever Eq. (36) is considered. There is no difference for $F_{1\text{-}4}$, but attention must be paid for $F_{1\text{-}2}$. In such a case, the sphere center in not between the disks *1* and *2*, and the spherical cap *2* is not the cap under the disk *2* in the Fig 3, but all the cavity area over the disk *2*. Starting with Eq. (32) for such case and Eq. (33) for the area of the caps, allows to write Eq. (61) and Eq. (62), after some manipulations:

$$F_{1\text{-}2} = [4\,\pi R^2 - 2\,\pi R^2\,(1 - \cos\theta_2)]\ 2\,\pi R^2\,(1 - \cos\theta_1)/(\,\pi R_1^2\ .\ 4\,\pi R^2) \tag{61}$$

$$F_{1\text{-}2} = (1 + \cos\theta_2)\,(1 - \cos\theta_1)\,R^2\,/(R_1^2) \tag{62}$$

Finally, replacing the cosine functions, an equation similar to Eq. (36) is presented specially for the case where the cavity center is situated beyond the disks involved, Eq. (63):

$$F_{1\text{-}2} = R^2\,\{1 + [1 - (R_2/R)^2]^{½}\}\,\{1 - [1 - (R_1/R)^2]^{½}\}/R_1^2 \tag{63}$$

Table 1 shows some view factors for disks defined in Fig. (3). Equations (28) and (58) can be applied to all the cases, while Eqs. (36) and (63) are restricted to the respective situations.

Table 1 – View factors for data from Fig. (3), right side.

	Eq. (28) or Eq. (58)	Eq. (36)	Eq. (63)
$F_{1\text{-}2}$	0.764		0.764
$F_{1\text{-}3}$	0.528	0.528	0.528
$F_{1\text{-}4}$	0.292	0.292	

7. CONCLUSION

Two approaches are available to calculate the radiation view factor between parallel disks. The most known comes from the integral contour method, with a very simple equation. The other method comes from the cavity method, where disks are circumscribed by a spherical cavity, leading to a less known equation. However, in this article is proved that the equations are similar, as the equation of the first method can be developed from the second.

The spherical cavity method needs special attention as in the case when the center of the cavity is not situated between the disks. A particular equation needs to be used in this case, which represents a disadvantage when compared to the integral contour equation, applied without restriction for parallel disks.

The advantage of the cavity method occurs for non-parallel disks, restricted always to the disks inside a unique sphere. In doing that, the disks could be rotated around the center of the cavity, without changing the view factor value. Consequently, the parallel position can be chosen and the simple equation from the integral contour method can be used as well. Of course, the distance between disks must be carefully determined in this situation.

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