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# APPRAISAL OF TEMPERATURE RANGES AND REGENERATOR EFFICIENCY OVER STIRLING ENGINES WITH MASS TRANSFER

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**Abstract.** *A world with growing population and limited natural resources requires innovative and smart solutions for energy management. Stirling engines have gained focus as an energy sustainable alternative due to its ability to convert heat from an external source into mechanical or electrical energy. The high theoretical engine efficiency of Stirling cycle can be compromised depending on the temperatures of the hot and cold sources, operating fluid, design dimensioning and regenerator efficiency. Low temperature ranges broaden the application of the Stirling engine, such as solar-powered Stirling engines and are on focus for different optimization problems and models evaluation. On the other hand, Stirling engines operating in high temperature ranges may be an opportunity to recover wasted heat with a low emission solution. On this article it is presented an evaluation of the temperatures of the cold and hot reservoir applied on the Stirling cycle based on a differential cycle, and its effect on the total work and thermal efficiency of the engine. It is also evaluated how the regenerator can affect the performance of the proposed system. Based on the patented differential cycle and its one-dimensional model it is proposed a mathematical appraisal whose objective is perceive the response of the model to distinct temperature of external sources. System considers two  $\beta$ -type Stirling engines operating in a steady state point, simultaneously, and lacked by two thermodynamic phases among each other. It is also considered to have the mass transfer already stabilized. The dimensioning of the clearance volume of the engine is one the leading input parameters that allow the differential cycle to operate with lower temperature sources. It also directly affects the total work and efficiency generated by the model, and the initial charge mass of working fluid has the potential of increasing the work produced, even though not modifying the thermal efficiency of the cycle. The regenerator efficiency can increase the thermal efficiency, without influencing the total positive work out of the system.*

**Keywords:** *Stirling Engine, Heat Transfer, Thermal Efficiency*

## 1. INTRODUCTION

Due to globalization the world energy consumption is expected to rise nearly 50% between 2018 and 2050 (Energy Information Administration, 2021). It is important to have innovative and smart solutions that are able to combine the still needed non-renewable source of fuel into an overall sustainable solution. Technologies to improve thermal efficiency of combustion systems or generate work from wasted heat sources are raising alternatives. Stirling Engines are externally closed heat engine with theoretical efficiency, low noise and emissions that are on the trend of waste heat recover applications (Kumaravelu *et al.*, 2022). Stirling engines are applicable also to low temperature sources (LTD), in the United Kingdom, 31% of all Thermal Processes that end up on waste heat present a low temperature process (Jouhara *et al.*, 2018). Therefore, for both cases of low and high temperature ranges, the patented differential thermal machine presents itself as an alternative to increase the efficiency of the original design configuration (Lockheck, 2015). The differential machine proposed for this study considers two Stirling engines operating simultaneously, and mass will be transferred among them in order to increase the total work of the system.

## 1.1 Low Temperature Difference Application

Many studies are on focus to evaluate and improve the Stirling engines application with lower delta of temperature among the reservoirs. When achieved, this can be applied into several distinct application and mainly with a focus on energy generation on areas with poor infrastructure. Hua-Ju achieved to simulate a waste heat recover device with a low-temperature-difference (LTD) engine of 12 K within a Gamma-Type Stirling Engine and associated to vibration energy harvesters (VEHs) to make the conversion of the mechanical energy into electricity (Shih, 2019). Romanelli developed a model for the dynamics and thermodynamics also for a Gamma-Type Stirling engine that operates at low temperature difference, with deltas lower than 100 C. The engine is started by chosen initial conditions that estimate a minimal initial flywheel speed (Romanelli, 2020). Takeuchi et al developed a new Alpha-Plus-Type Stirling engine applied on low-temperature-difference with indirect-heating kinematic. The heat source was burning wood biomass and it was transferred to the engine by oil heated up to 300 C. They obtained a thermal efficiency of 15% with a power output of 14.3 kW, which was tested and validated in a community center (Takeuchi *et al.*, 2021).

Solar-powered LTD Stirling engines were investigated by Kongtragool and Wongwises (Kongtragool and Wongwises, 2003) and Thombare and Verna, who evaluated the three configuration for the Stirling engine (Alpha, Beta and Gamma) using Helium as the working fluid to extensively describe the components and the theoretical equations participating on the Stirling engine, including the regenerator and heat exchangers (Thombare and Verma, 2008). For this application, with the focus on development of more representative models, studies for optimization can be taken into scene. Rao, More, Coelho and Mariani developed a multi-objective optimization of the Stirling engine using self-adaptive Jaya algorithm. The objective functions are maximum power, thermal efficiency and minimum pressure loss. The convergence behavior shows to be much better when compared to other optimization techniques (Rao *et al.*, 2017).

Egas studied the effect of appropriate configuration selection for Stirling engines into the three different configuration, with distinct driving mechanism and compression ratio. He stated that the compression ratio directly affects the applicability of the engine into low or high temperature difference (Egas and Clucas, 2018). Also came to a conclusion that Gamma-Type Stirling Engines are more suitable for LTD applications.

## 1.2 Stirling Engine Regenerator Efficiency

The high theoretical engine efficiency of Stirling cycle is primarily decreased when imperfect regenerators are considered, once the Carnot efficiency is only obtained when 100% effective regenerator is applied. The hot gas flows through the regenerator that absorbs part of the heat and release it back to the gas when it is contact with the cold chamber, therefore it affects directly on the overall engines thermal efficiency. In general a regenerator consists of small sized porous material that enhances heat transfers between the working gas, however this increases the engine's dead volume and pressure drop over the regenerative channel.

Many studies in this area are on going to develop accurate models of the regenerator, and also develop a design for a high efficiency. Chen developed a CFD code applied to a rhombic-drive  $\beta$ -Type Stirling engine to study the effects of a moving regenerator and concluded to be one option improve the engine's efficiency Chen *et al.* (2015). Dai et al. established that an effectiveness higher than 50% can only be achieved by discrete isolated components, referred as sub-regenerators (Dai *et al.*, 2018). Nielsen and York then developed a discrete one-dimensional heat transfer model to determine the influence of parameters on the effectiveness of the Stirling engine regenerators. The usage of multiple sub-regenerators, together with the thermal mass ratio, revealed to be the major influences. An effectiveness of 95% was obtained with 19 sub-regenerators (Nielsen *et al.*, 2019).

Rutczyk developed a model for the regenerator efficiency based on inputs as mass flow, temperatures and pressures, porosity and diameter of the regenerator. Also comparable to CFD models the parametric evaluation showed that the capacity of the material has little influence on the regenerator performance, however the porosity of the material has great influence since it affects both the heat transfer area, convective heat transfer and solid mass of the regenerator (Rutczyk *et al.*, 2020).

## 2. METHODOLOGY

This chapter presents the equations that define how the evaluation of the temperatures on the reservoirs can be analyzed. The two chambers will be operating at the same time and the objective is to calculate the total work of the system and the overall thermal efficiency - considering both chambers operating under the differential cycle.

### 2.1 Thermodynamic evaluation over the differential cycle

Taking as reference the points presented on Chamber 1 on Figure 1 and that for gases that follow an ideal gas model, the internal energy depends only on the temperature ( $\Delta U = 0$ ), the first law of thermodynamics can be summarized as

$$Q_{3-4} = W_{3-4} \quad (1)$$

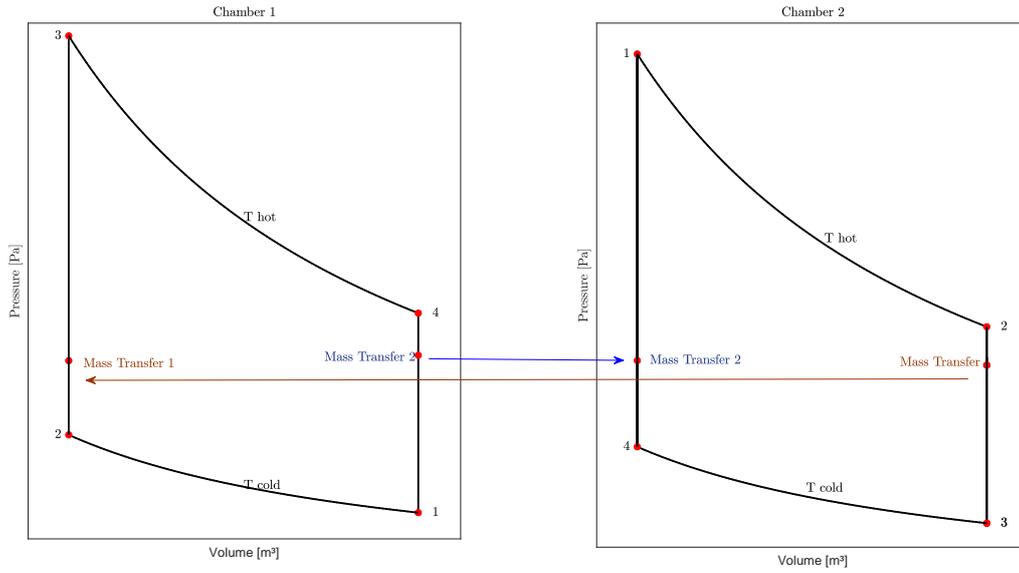


Figure 1. Pressure-Volume example of the proposed differential systems with two mass transfers.

Considering the working fluid as an ideal gas, and  $n$  = moles number at the mentioned transformation the work is calculated by

$$W_{3-4} = \int_3^4 P dv \quad (2)$$

and the ideal gas law

$$W_{3-4} = \int_3^4 \frac{n \bar{R} T}{V} dv \quad (3)$$

$$W_{3-4} = n \bar{R} T_3 \ln(V) \Big|_3^4 \quad (4)$$

Therefore the work done on the isothermal expansion is calculated by

$$W_{3-4} = n \bar{R} T_3 \ln\left(\frac{V_4}{V_3}\right) \quad (5)$$

and similarly for the isothermal compression

$$W_{1-2} = n \bar{R} T_1 \ln\left(\frac{V_2}{V_1}\right) \quad (6)$$

For isochoric processes the work is equal zero ( $W_{2-3} = W_{4-1} = 0$ ) since there is no variation on the fluid volume, consequently

$$Q_{2-3} = \Delta U_{2-3} \quad (7)$$

and

$$Q_{4-1} = \Delta U_{4-1} \quad (8)$$

where  $U$  is the internal energy of the ideal gas on a given temperature. These are the equations used by the model presented in this study. And for an ideal gas it may be also considered as

$$Q_{2-3} = m c_v (T_3 - T_2) \quad (9)$$

$$Q_{4-1} = m c_v (T_1 - T_4) \quad (10)$$

where  $T_3 = T_4 = T_h$ , and  $T_1 = T_2 = T_c$ .

Therefore the total amount of cycle work is

$$W_{cycle} = W_{3-4} + W_{1-2} \quad (11)$$

Where  $W_{3-4}$  is the outlet of work and  $W_{1-2}$  is the work rejected by the system.

The total inlet heat to the system comes from transformation 3-4, and 2-3, where the system is receiving heat from the hot source  $Q_h$  (or the heater).

$$Q_{inlet} = Q_h + Q_{3-4} \quad (12)$$

## 2.2 Regenerator influence

Stirling engines take into the design the usage of a regenerator. For this matter the heat outlet of isochoric transformation 4-1, that goes to the cold source (or cooler) is received by the regenerator.

$$Q_h = Q_{2-3} + Q_{regenerator} \quad (13)$$

If the regenerator is perfect, with no losses, all the heat of 4-1 is recovered by 2-3, on that case the total heat from the heater is considered zero for the isochorics. If that occurs,

$$Q_{regenerator} = Q_{4-1} \quad (14)$$

so,

$$Q_h = m c_v (T_h - T_c) + m c_v (T_c - T_h) = 0 \quad (15)$$

and

$$Q_{inlet} = Q_{3-4} \quad (16)$$

Since cycle efficiency can be expressed as

$$\eta_{cycle} = \frac{W_{cycle}}{Q_{inlet}} \quad (17)$$

for the ideal Stirling engine

$$\eta_{cycle} = \frac{W_{3-4} + W_{1-2}}{Q_{3-4}} \quad (18)$$

$$\eta_{cycle} = \frac{n \bar{R} T_3 \ln\left(\frac{V_4}{V_3}\right) + n \bar{R} T_1 \ln\left(\frac{V_2}{V_1}\right)}{n \bar{R} T_3 \ln\left(\frac{V_4}{V_3}\right)} \quad (19)$$

$$\eta_{cycle} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_{cold}}{T_{hot}} \quad (20)$$

which is also the efficiency of the Carnot cycle.

However if the regenerator effectiveness ( $E_r$ ) is considered during the transfer

$$E_r = \frac{T_r - T_c}{T_h - T_c} \quad (21)$$

being  $T_r$  the mean temperature of the regenerator. A perfect regenerator has  $T_r = T_h$ .

$$Q_{regenerator} = E_r Q_{4-1} \quad (22)$$

$$Q_h = Q_{2-3} + E_r Q_{4-1} \quad (23)$$

Considering the imperfect regenerator

$$\eta_{cycle} = \frac{W_{cycle}}{Q_{3-4} + Q_h} \quad (24)$$

$$\eta_{cycle} = \frac{W_{3-4} + W_{1-2}}{Q_{3-4} + Q_{2-3} + E_r Q_{4-1}} \quad (25)$$

$$\eta_{cycle} = \frac{n \bar{R} T_3 \ln\left(\frac{V_4}{V_3}\right) + n \bar{R} T_1 \ln\left(\frac{V_2}{V_1}\right)}{n \bar{R} T_3 \ln\left(\frac{V_4}{V_3}\right) + m c_v (T_3 - T_2) + E_r m c_v (T_1 - T_4)} \quad (26)$$

$$\eta_{cycle} = \frac{T_h - T_c}{T_h + \frac{c_v (T_h - T_c) (1 - E_r)}{R \ln \frac{V_1}{V_2}}} \quad (27)$$

### 3. RESULTS

It is here presented the results of work and thermal efficiency by changing the inputs of the model that will affect the usage of the proposed system on different temperature ranges of application. Low-temperature difference engines empirically operate with larger volume of the displacer (Takeuchi *et al.*, 2021), and for the engine proposed in this study it is desired to understand how the design of the chambers will affect the outputs of the model. In order to attend the thermodynamic and physical constraints of the model, there is a limit on the input variables which allow the chambers to transfer mass.

#### 3.1 Effect of initial charge mass and compression ratio

Also for Stirling engines without mass transfer, low-temperature difference engines need to have a design method that consider the temperature difference. To evaluate the effect and response of the system to the change of compression ratio and charge mass, a fixed amount of mass transfer was set to 15%, the displacement volume was fixed at 113.14 cm<sup>3</sup> and a cold source was set to be 288 K. As high as the compression ratio gets, the higher is the work generated, however the system requires an operation with a hot source with higher temperatures. When the compression ratio is reduced, the system allows an operation at lower temperatures on the hot source, but on the other hand the total work produced and the thermal efficiency is also reduced.

One alternative to increase the positive work is to set the initial fluid charge mass on the chambers to a higher value. Figure 2 shows three different configurations of charge fluid mass, where this increase on the total work is observed. It must be pointed out that the thermal efficiency is not affected by the change of initial mass (Figure 3). By raising the mass, the peak pressure inside the chamber will increase as well as seeing on Figure 4, and a linear raise on the maximum pressure also when working with higher temperatures. Modern Diesel engines, for example, operate with a cylinder pressure of 200bar, and for the simulation proposed with 2.5g of initial charge mass values over 300bar were seen. Therefore a suitable material for the inner cylinder walls must be selected accordingly.

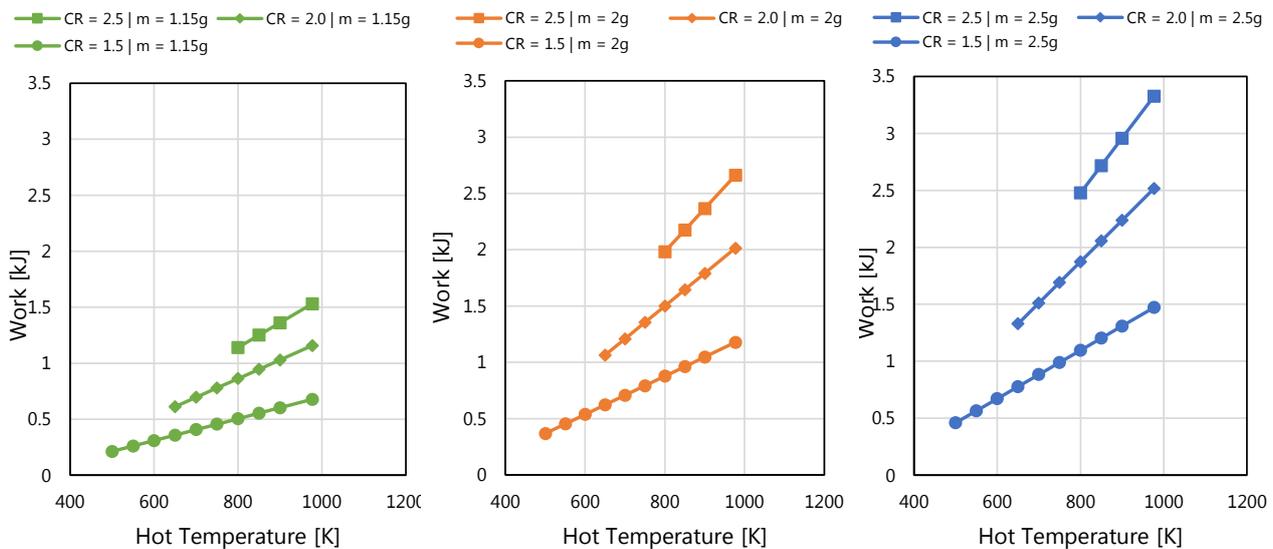


Figure 2. Influence of the hot temperature source over the work generated by the system by keeping a constant amount of mass transfer of 1.5% and 288K at the cold source.

#### 3.2 Effect of Regenerator Efficiency

The regenerator on  $\beta$ -Type Stirling engines can be considered only by the gap among the walls of the cylinder and outer face of the displacer, therefore the solid material of the core cylinder may act like a regenerator material to store and release heat. This configuration is ideal for new models development, which is the case of this study. By doing so, and modeling accordingly to equations presented on Section 2.2 the influence of the regenerator efficiency can be used as an input to the differential cycle.

To isolate the effect of the regenerator, constant temperatures were set as 288K at the cold and 977K at the hot source. Also, the compression ratio was simulated as two different situations, being 2.5 and 3.2. The higher value of Compression Ratio (CR) physically represents an engine with lower clearance volume and this engines are less susceptible to allow the mass transfer among the two engines, so the highest amount of mass transfer simulated is 1.3%. Therefore, on the case

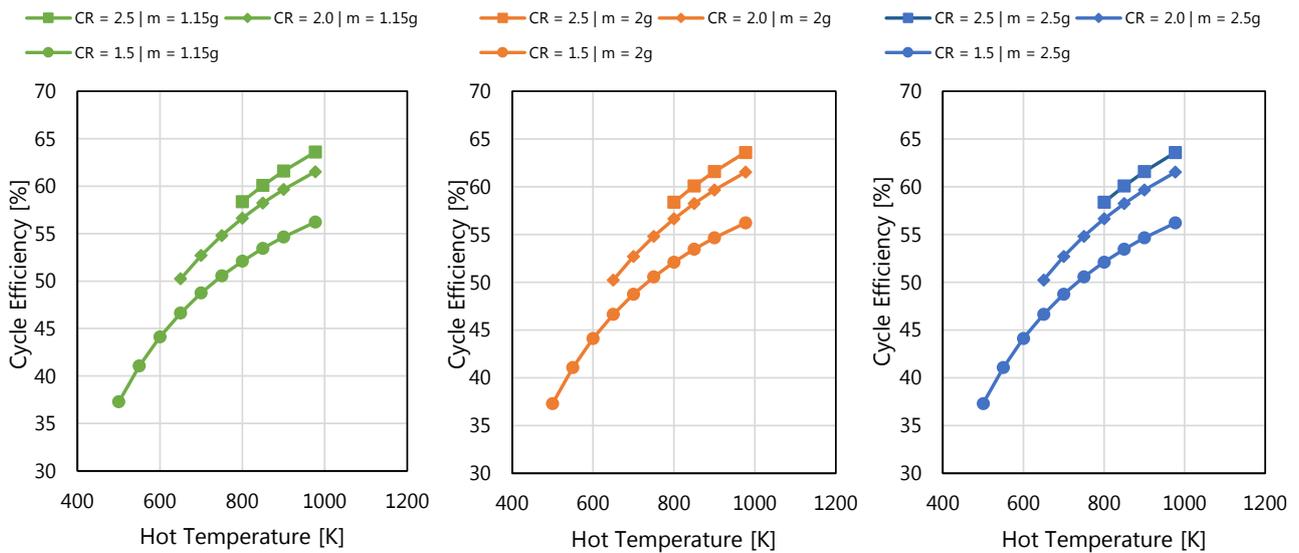


Figure 3. Influence of the hot temperature source over the thermal efficiency by keeping a constant amount of mass transfer of 1.5% and 288K at the cold source.

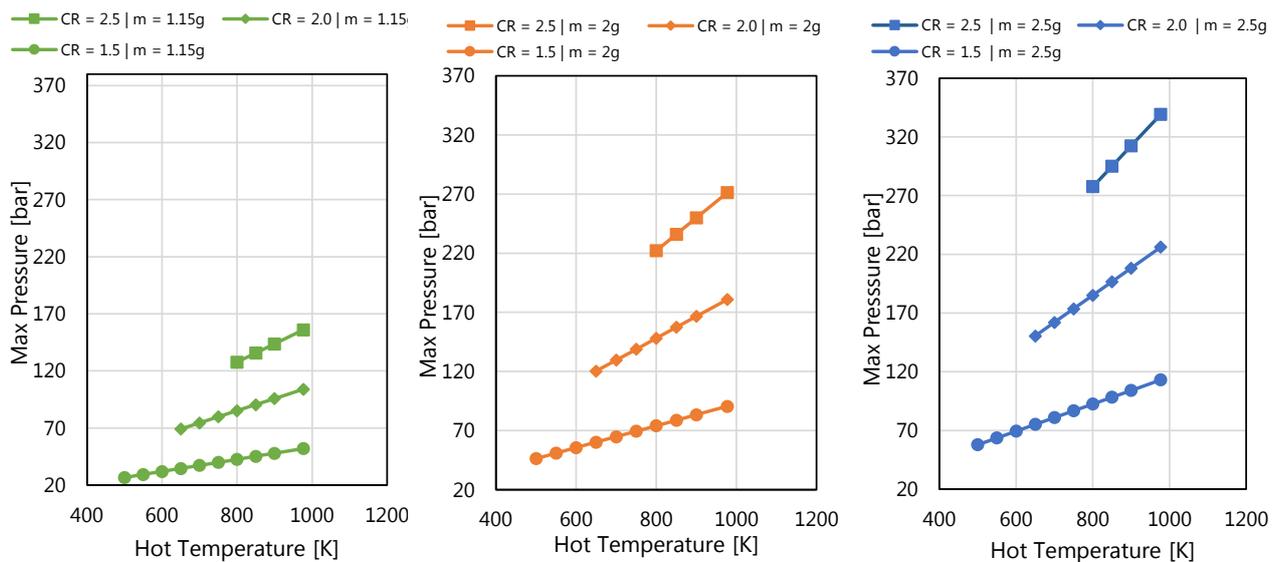


Figure 4. Influence of the hot temperature and initial charge fluid mass on the chamber maximum pressure by keeping a constant amount of mass transfer of 1.5% and 288K at the cold source.

of the compression ratio = 2.5, a higher amount of mass transfer is also simulated as 5%. The results are presented on Figure 5

The regenerator efficiency by itself does not have impact on the total work produced by the system, this can also be noticed on Figure 6. This output is dependant on the design configuration of the engine, temperatures of the reservoirs and amount of transferred mass. However the thermal efficiency of the cycle is widely affected by the regenerator, and this behaviour is the same also for Stirling engines without mass transfer. The simulation showed that for high efficiency regenerator (over 90%) a good association of mass transfer can result on higher cycle efficiency even for engines with lower compression ratio.

Figure 6 represents the output of the model when evaluating both the regenerator and the hot temperatures are used as input at the same time. On this case, the initial charge mass, cold temperature, fraction of mass exchange and compression ratio were kept constant accordingly to Table 1. The thermal efficiency has a higher range of values depending on the hot temperature for higher values of regenerator efficiency.

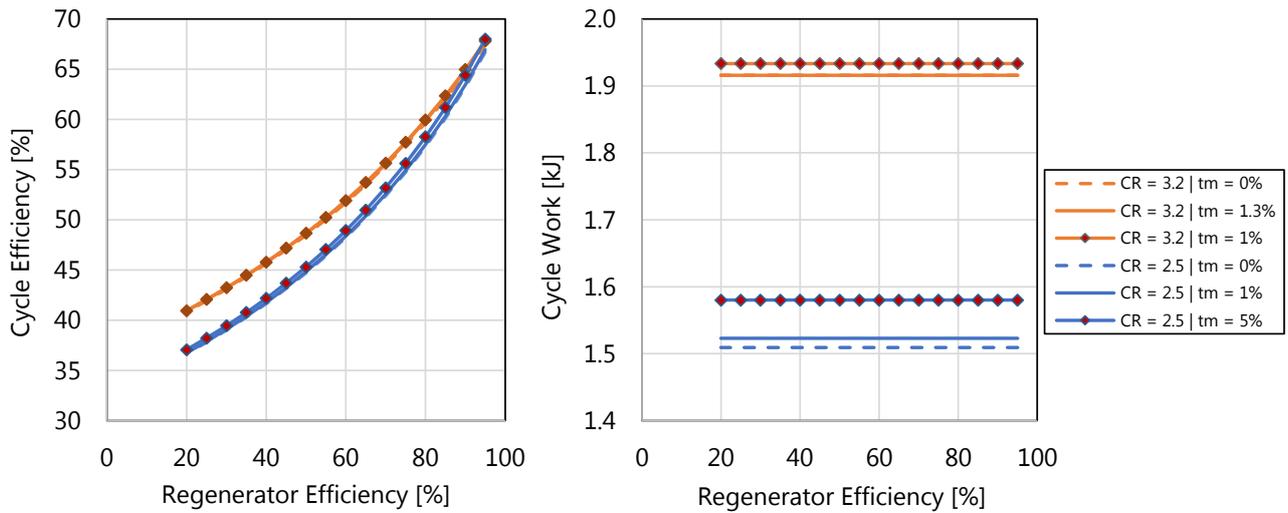


Figure 5. Influence of the regenerator efficiency over the thermal efficiency by keeping a constant temperatures at the sources.

Table 1. Fixed inputs parameters for evaluation on Figure 6.

Initial Mass [g]	Cold Temperature [K]	Transferred Mass [%]	Compression Ratio [-]
2	288	1	2

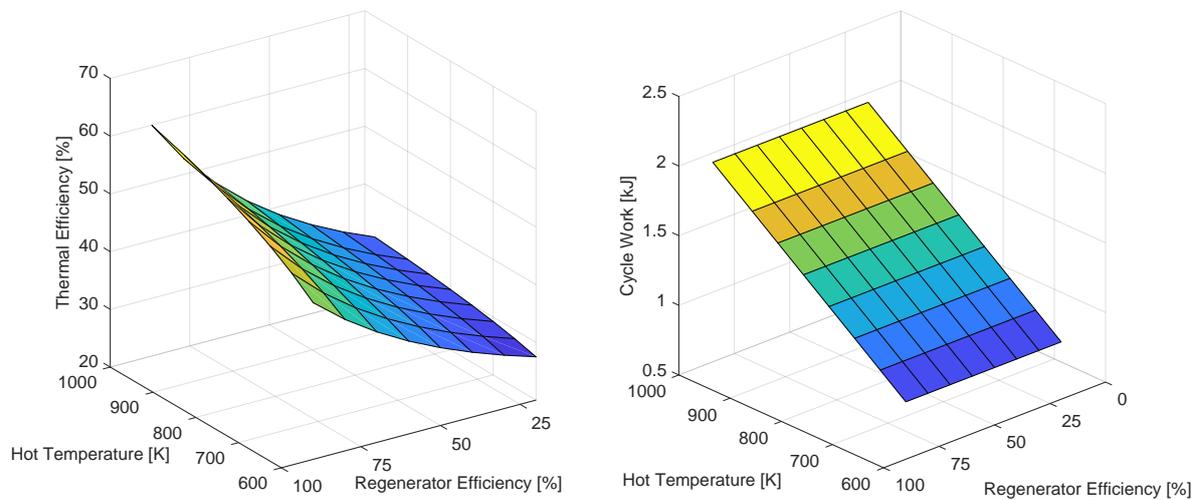


Figure 6. Model output for thermal efficiency and cycle work over the regenerator efficiency and hot temperature.

#### 4. CONCLUSIONS

This paper presents a survey on the application of the differential thermal machine based on a Stirling Engine due to different temperatures from the cold and hot sources. The system is designed to have two  $\beta$ -Type Stirling Engines operating simultaneously, lacked by two thermodynamic phases with Helium as working fluid which is considered to follow the ideal gas model. It is possible to simulate different temperature ranges of application by adapting the input parameters of the differential engine. It is possible to concluded that:

1. By keeping the design variables constant and also fixing the amount of mass transfer among the engines, higher temperatures on the hot source leads to a linear increase on the total work produced by the system, the same behaviour is also expected on Stirling engines without mass transfer.
2. Lower compression ratio allows the system to operate with lower temperatures. However there is a there is a side

effect which is seen as a reduction on the engine's work and thermal efficiency.

3. Higher amount of initial charge mass may be an alternative to increase the total work but it was observed that the peak of pressure inside the chamber is directly affected with a raise, therefore the material considered for the design of the cylinder walls needs to be taken into consideration.
4. The Stirling Engine efficiency is highly affected by the efficiency of the internal regenerators, and it was observed that the mass transfer is an option to increase the thermal efficiency of the system, even though it does not affect the total output of the positive work.

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