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NONLINEAR DYNAMICS OF AN ADAPTIVE ENERGY HARVESTER WITH MAGNETIC INTERACTION

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Abstract: *This work investigates the mechanical energy harvesting from smart and adaptive devices using magnetic interaction. The energy harvesting device is built from a magneto-elastic beam that interacts with magnets, being connected to the electrical circuit by magnetostrictive materials that promotes energy conversion. Magnetic interactions are modeled by considering a modified single point magnet dipole method that employs multiple points to represent the magnetic dipole. Based on the magnetic interaction model, an adaptive energy harvesting device is proposed by assuming that magnets can change their position with respect to the beam. The adaptive behavior allows one to alter the system stability and therefore, its dynamical response. A nonlinear dynamics analysis is performed showing the possibilities to enhance energy harvesting capacity from the magnet position change. The strategy is to perform a dynamical characterization and afterward control the energy barrier based on dynamical characteristics of the system. Results show interesting conditions where energy harvesting capacity is dramatically increased by changing the system characteristics passing through mono, bi and tri stable systems.*

Keywords: *Mechanical energy harvesting; nonlinear dynamics; smart materials; adaptive systems; magnetic interaction; Duffing.*

1. INTRODUCTION

The technology for power generation is a subject of essential importance in the 21st century due to either the everyday needs or the urgent search for renewable energies. In particular, these technologies have an increasing importance due to the growing use of mobile devices and the cell phone is an emblematic example with the increasing demand for phone signal, the use of internet, and mainly, batteries. The internet of things is another emblematic example that demands energy in remote places.

In this regard, there is an incessant search for solutions that can provide energy in a decentralized way. A solution that has been extensively studied is the energy harvesting, which is capable to collect available environmental energy, converting it for use (Erturk and Inman, 2011a). Different kinds of energies are available in the environment, including solar, thermal and wind. Mechanical energy is an essential kind that can be exploited such as vibrations, deformations and pressures, including the human movement itself.

Energy harvesting is performed by some harvester device that converts some type of energy available in the environment into electrical energy. Smart materials are the essential component to perform the energy conversion. In essence, these materials have multiphysics coupling that confers adaptive behavior to engineering systems (Oliveira and Savi, 2013). Typically, smart materials present coupling of different physical fields, being able to convert mechanical energy into electrical energy, for example. Piezoelectric and magnetostrictive materials are examples of smart materials usually employed for energy harvesting purposes.

Examples of energy harvesting systems are vast in the literature. A shoe equipped with a piezoelectric element in its insole can generate electrical energy from walking (Paradiso *et al.*, 1998). Gym equipments that collect energy from human exercises are possible, as verified in some Fitness Club as Rochester (New York, NY) and Greenasium (San Diego, CA). Another interesting application is the use of smart materials in the floor to collect energy, which includes night clubs (Watt, Rotterdam; Surya, London) and roads.

One of the challenges for the design of energy harvesting systems is to deal with uncertainty and variabilities associated with environmental energy sources. The linear energy harvester has a good performance under resonant conditions, but this can dramatically vary due to external variabilities. Therefore, it is important to promote system adaptation to have a good performance and make the system feasible for the energy harvesting goals. The literature presents different configurations of vibration-based energy harvesting devices (Caetano & Savi, 2021, 2022; Erturk &

Inman, 2009; Kita *et al.*, 2015). Nonlinearities are usually exploited to improve the performance of these systems (Cellular *et al.*, 2018; Liu *et al.*, 2018). Among these nonlinearities, it should be pointed out impacts aiming to increase the frequency range of the devices (Ai *et al.*, 2019). Multisability is another nonlinearity usually exploited being fundamentally associated with magnetic interactions (Lan & Qin, 2017). Bistable or tristable systems are effective but needs energy source high magnitudes for a good performance (Cottone *et al.*, 2012; Erturk & Inman, 2011b). Low energy levels impose situations where the system cannot overcome these energy barriers, presenting a poor performance (Daqaq *et al.*, 2014; Harne & Wang, 2013). The improvement in the performance of energy harvesting systems is associated with a decrease of the energy barriers (Yang & Towfighian, 2017; Lan & Qin, 2017). Other advanced strategies that have been exploited includes the synergistic use of smart materials is another promising development (Adeodato *et al.*, 2021; Silva *et al.*, 2015) and control (Barbosa *et al.*, 2015).

This work aims to investigate mechanical energy harvesting systems using smart materials and adaptability capacity. The main motivation is to propose a smart and adaptive energy harvesting system capable of changing energy barriers, allowing better performance for different loading sources. The idea is to change the magnetic interaction through the movement of magnets, which is defined from the dynamical response that is based on the input energy, allowing energy barriers to be overcome. The analysis of the energy barrier needs an appropriate description of the magnetic interaction. The literature presents different approaches to this, either using a polynomial associated with Duffing equation (Zhou *et al.*, 2013; de Paula *et al.*, 2015) or using another polynomial calibrated from experimental data (Cao *et al.*, 2015a, 2015b). Nevertheless, the most coherent approach treats magnetic interactions from the forces between magnets. Some works consider the magnetic forces from a dipole represented by a point (Aboulfotoh *et al.*, 2013; Leng *et al.*, 2017; Stanton *et al.*, 2010). Wang *et al.* (2020) show that these approaches do not properly represent the experiments and, therefore, proposed an approach considering two points. This work presents a novel magnetic interaction model that uses multiple points to represent the magnetic interaction, based on the single point dipole methodology. The adaptability of the system is conferred from the position of the magnets, which can be altered to change the magnetic interaction and the energetic barriers. A dynamical analysis of the multistable energy harvesting system allows exploring the main aspects of the energy harvesting, predicting the most suitable operating conditions according to the excitation source.

2. ENERGY HARVESTING SYSTEM

The proposed energy harvesting device is a smart and adaptive system composed by a magneto-elastic beam interacting with a system of three magnets that can change its position, and subjected to a base excitation, Figure 1. The magnets are represented by three points in the schematic figure since magnetic interaction is described using a multipoint model, as discussed in next section.

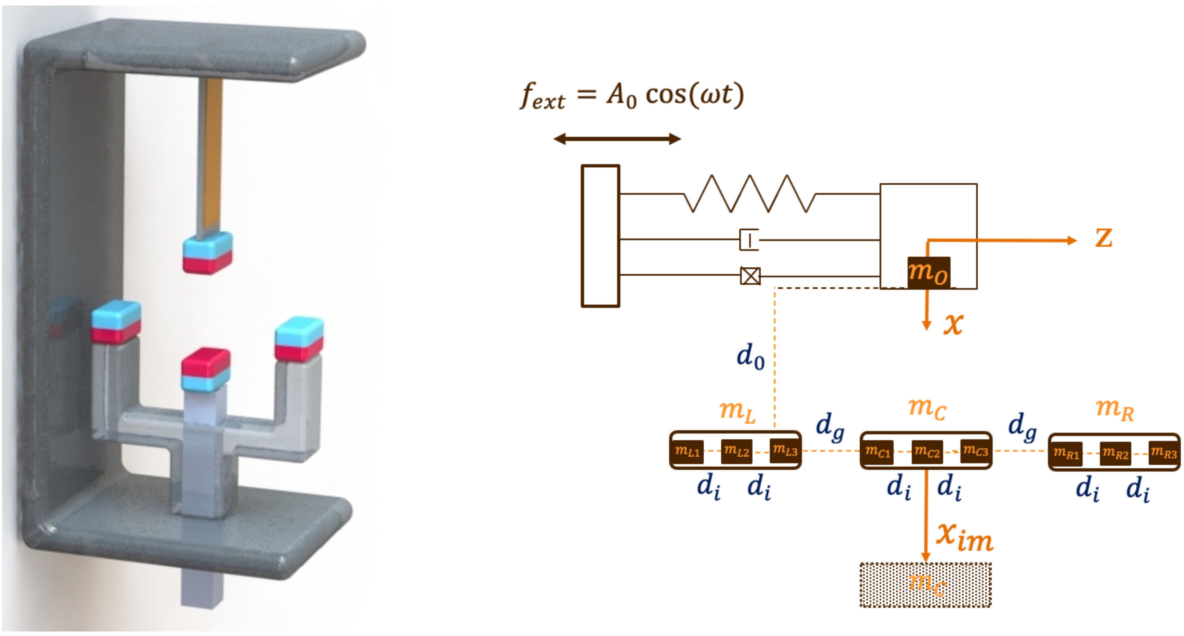

Figure 1. **Smart, adaptive energy harvesting system showing the three-dimensional representation and the equivalent nonlinear oscillator.**

Based on the first vibration mode, it is possible to treat this system as a one degree of freedom oscillator. By assuming that the magneto-elastic beam is connected to an electric circuit by a megnetostrictive material (Figure 1). The equations of motion are written assuming that *z* is the relative displacement and *i* is the electric current (Erturk & Inman, 2011a; Engdahl *et al.*, 2000),

$$\begin{gathered} \ddot{z} + 2\xi\omega_n\,\dot{z} + \omega_n^2\,z - \theta i = f_{ext} + f_{mag} \\ \frac{1}{L}(R\,i - \theta\,\dot{z}) = 0 \end{gathered} \tag{1}$$

where ξ is the dissipation coefficient, ω_n is the natural frequency associated with the linear system, θ is the magnet-mechanical coupling coefficient, $f_{ext} = A_0\cos(\omega t)$ is the external force, R is the electrical resistance, L is the inductance. Magnetic interactions are modeled by considering four magnets: one is the tip mass O, and the others are represented by L (left), C (center) and R (right). Moreover, the position of the central magnet C can alter its position, defined by x_{im}. The horizontal distance of the magnets C-L and C-R is d_g (z-direction). Besides, the distance between the tip mass O and the center magnet C is given by $d = d_0 + x_{im}$; $f_{mag} = F_{mag}/m$ is related to the magnetic force, described in the sequel.

2.1. Magnetic Interaction Model

There are different alternatives to model magnetic interactions. A polynomial fit is an alternative that is usually represented by the Duffing equation (Zhou *et al.*, 2013). Other alternatives are proposed by different polynomials (Cao *et al.*, 2015a, 2015b). A more consistent approach uses the magnetic field interactions. Aboulfotoh *et al.* (2013), Leng *et al.* (2017), Stanton *et al.* (2010), Tan *et al.* (2015) exploited the equivalent point dipole method (Figure 2). Wang *et al.* (2019) and Wang *et al.* (2020) proposed a more sophisticated approach representing the magnet by two points, which presents a better fit with experimental data.

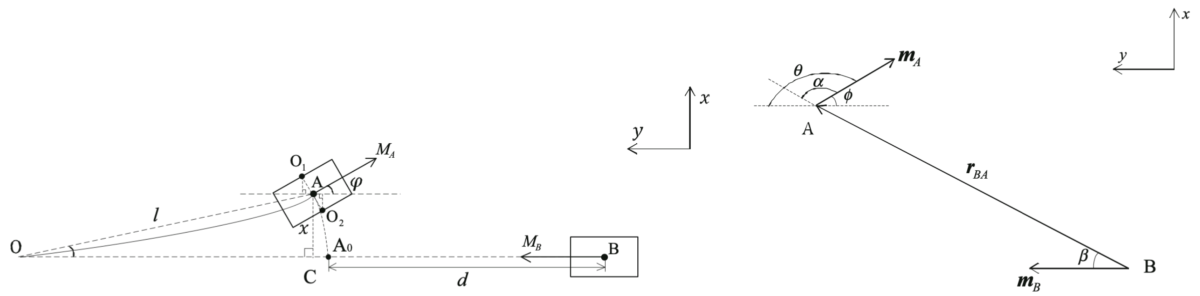

Figure 2. **Magnetic interactions modeled from the equivalent point dipole method (Tan *et al.*, 2017)**

Here, a novel alternative is proposed by considering the single point dipole method but using multiple points to represent the dipole. On this basis, each dipole is represented by a sequence of points allowing a better representation that presents a smoother force fields. In this regard, consider the magnetic interaction between two magnets, A and B,

$$\boldsymbol{B}_{BA} = -\frac{\mu_0}{4\pi}\nabla\frac{\boldsymbol{m}_B\cdot\boldsymbol{r}_{BA}}{r_{BA}^3} \tag{2}$$

where μ_0 is the permissibility, $\boldsymbol{m}_i$ is the dipole magnetic moment that is associated with the magnet $i = O, L, R, C$. Vector $\boldsymbol{r}_{AB}$ is the source of the magnetic moment from A to B. The potential energy of this energy is given by,

$$U_m = -\boldsymbol{B}_{BA}\cdot\boldsymbol{m}_A = -\frac{\mu_0}{4\pi}\nabla\frac{\boldsymbol{m}_B\cdot\boldsymbol{r}_{BA}}{r_{BA}^3}\cdot\boldsymbol{m}_A \tag{3}$$

The magnetic force on the dipole A imposed by dipole B is the following,

$$\boldsymbol{F} = -\nabla U_m = -\frac{\mu_0}{4\pi}\nabla\left[\left(\nabla\frac{\boldsymbol{m}_B\cdot\boldsymbol{r}_{BA}}{r_{BA}^3}\right)\cdot\boldsymbol{m}_A\right] \tag{4}$$

Tan *et al.* (2015) showed that this equation can be simplified as follows,

$$\begin{aligned} \boldsymbol{F} = \frac{3\mu_0 m_A m_B}{4\pi r_{BA}^4}[&\hat{\boldsymbol{r}}_{BA}(\hat{\boldsymbol{m}}_A\cdot\hat{\boldsymbol{m}}_B) + \hat{\boldsymbol{m}}_B(\hat{\boldsymbol{m}}_A\cdot\hat{\boldsymbol{r}}_{BA}) + \hat{\boldsymbol{m}}_A(\hat{\boldsymbol{m}}_B\cdot\hat{\boldsymbol{r}}_{BA}) \\ &- 5\hat{\boldsymbol{r}}_{BA}(\hat{\boldsymbol{m}}_A\cdot\hat{\boldsymbol{r}}_{BA})(\hat{\boldsymbol{m}}_B\cdot\hat{\boldsymbol{r}}_{BA})] \end{aligned} \tag{5}$$

where the vectors $\hat{\boldsymbol{m}}_A, \hat{\boldsymbol{m}}_B, \hat{\boldsymbol{r}}_{BA}$ represent the unit vectors on the direction of $\boldsymbol{m}_A, \boldsymbol{m}_B, \boldsymbol{r}_{BA}$, respectively. In general, the force can be expressed by $\boldsymbol{F} = F_x\,\boldsymbol{i} + F_z\,\boldsymbol{k}$ and x-direction term can be neglected (Tan *et al.*, 2015). By performing a discretization of each dipole using *n* points, it is possible to express each force by the equation,

$$F_{BAz} = \frac{1}{n}\sum_{k=1}^{n} F_{BAk} \tag{6}$$

The potential energy of the energy harvesting device, U, is defined by combining the beam potential energy, $U_p = \frac{1}{2}kz^2$, and the magnet interactions, $U_m = \int_{z_1}^{z_2} F_z\, dz$: $U = U_p + U_m$. Therefore, based on the same multipoint approach, the magnetic energy can be expressed by,

$$U_m = \frac{1}{n}\sum_{k=1}^{n} U_{BAk} \tag{7}$$

Under these assumptions, it is possible to define the forces at each magnet. By considering $n = 3$, magnetic forces are written as follows,

$$\begin{aligned} F_{jOk} = \frac{3\mu_0 m_O m_j}{4\pi r_{jO}^4}\Bigg[&-\frac{\left(z+\varsigma_j d_g+\psi_k d_i\right)}{r_{jO}}\cos\theta + \frac{d_0}{r_{jO}}\sin\theta \\ &- 5\, d_0 \frac{\left(z+\varsigma_j d_g+\psi_k d_i\right)}{r_{jO}^2}\left(-\frac{d_0}{r_{jO}}\cos\theta \right. \\ &\left. + \frac{z+\varsigma_j\, d_g+\psi_k d_i}{r_{jO}}\sin\theta\right)\Bigg] \qquad (j = L, R) \qquad (k = 1,2,\dots,n) \\ F_{COk} = \frac{3\mu_0 m_O m_C}{4\pi r_{CO}^4}\Bigg[&-\frac{(z+\psi_k d_i)}{r_{CO}}\cos\theta + \frac{d}{r_{CO}}\sin\theta \\ &- 5d\,\frac{(z+\psi_k d_i)}{r_{CO}^2}\left(-\frac{d}{r_{CO}}\cos\theta + \frac{z+\psi_k d_i}{r_{CO}}\sin\theta\right)\Bigg] \end{aligned} \tag{8}$$

where

$$\varsigma_j = \begin{cases} -1\,; & \text{if } \ j = L \\ +1\,; & \text{if } \ j = R \end{cases} \qquad\qquad \psi_k^{(n=3)} = \begin{cases} -1\,; & \text{if } \ k = 1 \\ 0\,; & \text{if } \ k = 2 \\ +1\,; & \text{if } \ k = 3 \end{cases} \tag{9}$$

Note that the definition of ψ_k depends on the number of points employed to represent the dipole, n. These expressions consider the definition of the following parameters,

$$\begin{aligned} r_{LO} &= \sqrt{\left(z-d_g\right)^2 + d_0^2} \\ r_{RO} &= \sqrt{\left(z+d_g\right)^2 + d_0^2} \\ r_{CO} &= \sqrt{z^2+d^2} \\ \cos\theta &= \frac{\sqrt{l^2-z^2}}{l} \\ \sin\theta &= \frac{z}{l} \end{aligned} \tag{10}$$

Based on that, the force on magnet O, the tip mass, is given by

$$F_{mag} = F_{LOz} + F_{COz} + F_{ROz} \tag{11}$$

Therefore, remembering that it is assumed that $n = 3$, each of the forces is written by

$$F_{jO} = \tfrac{1}{3}\left(F_{jO1} + F_{jO2} + F_{jO3}\right) \qquad (j = L, C, R) \tag{12}$$

And the energy is given by, $U_m = U_{LO} + U_{CO} + U_{RO}$, associated with the following expressions,

$$U_{jOk} = \frac{\mu_0 m_O m_j}{4\pi r_{jO}^3}\left[\frac{3d_0^2}{r_{jO}^2}\cos\theta - \frac{3\left(z+\varsigma_j d_g + \psi_k d_i\right)d_0}{r_{jO}^2}\sin\theta - \cos\theta\right]$$
$$(j = L, R) \qquad (k = 1,2,\ldots n) \tag{13}$$
$$U_{COk} = \frac{\mu_0 m_O m_C}{4\pi r_{CO}^3}\left[\frac{3d^2}{r_{CO}^2}\cos\theta - \frac{3(z+\psi_k d_i)d}{r_{CO}^2}\sin\theta - \cos\theta\right]$$

3. NUMERICAL SIMULATIONS

This section presents numerical simulations that are carried out employing the fourth order Runge-Kutta method. Parameters presented in Table 1 are employed for all simulations.

Table 1. **Energy harvesting parameters (Tan *et al.*, 2017).**

d_0 [mm]	2.5
d_g [mm]	6.3
d_i [mm]	0.8
x_{im} [mm]	[−0.3, 0.0, 0.3, 1.5]
ω_n [rad/s]	161.91
θ [N/V]	0.1203
ξ	0.02
A_0 [m/s^2]	10
ω [rad/s]	161.91
m_O [A m^2]	0.015
$m_{L,R}$ [A m^2]	0.015
m_C [A m^2]	0.006
l	102.85

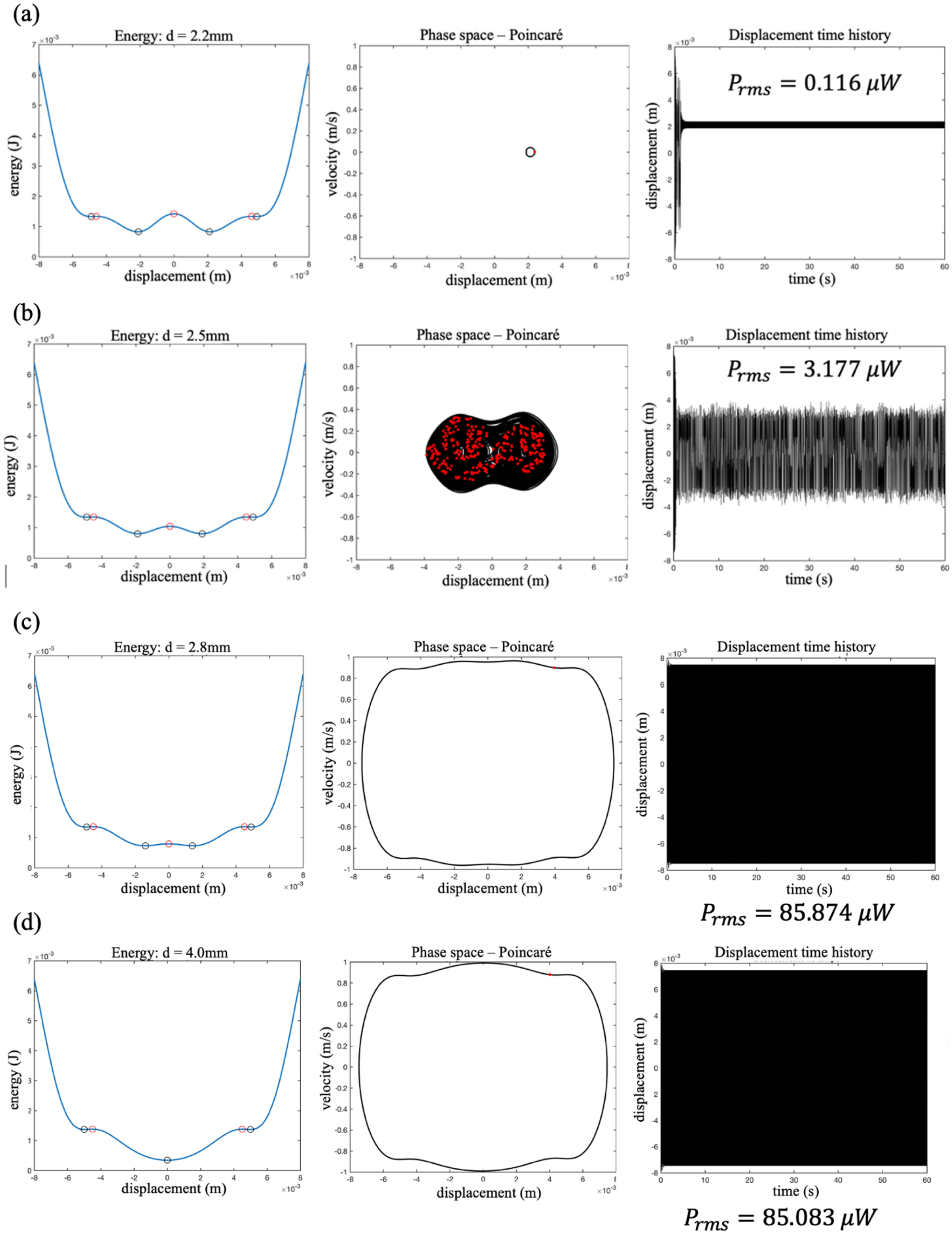

Figure 3. **Dynamical characterization of the energy harvester for different magnetic interactions defined by distance *d* promoted by changing the position of the C magnet, x_{im}: (a) *d* = 2.2; (b) *d* = 2.5; (c) *d* = 2.8; (d) *d* = 4.0.**

The analysis of the energy harvesting device starts with a dynamical characterization for different magnetic interactions promoted by distinct positions of the magnet C, defined by distance x_{im}, which alters the distance d. The idea is to exploit different kinds of responses that are used to define the patterns for adaptive behavior. Costa *et al.* (2020) showed that nonlinear dynamics perspective is an essential point that should be employed to evaluate the energy harvesting device design. Figure 3 presents results for distinct positions of the magnets. Energy curves are presented together with phase space, Poincaré maps and power time history. RMS power is also indicated, being calculated for steady state behavior, discharging the transient period. Initially, an intra-well period-1 response is achieved, showing that the system is not able to overcome the energy barrier. Afterward, a chaotic response is achieved visiting all wells. The other two cases present a high energy level, and inter-well period-1 behavior is observed. It should be pointed out that the energy is dramatically increased from the first to the third cases, and the last case is compatible with the third one. The last case is a situation where the magnet C does not have influence on system dynamics.

The major motivation of the adaptive system is to change system configuration based on dynamical aspects. Therefore, consider different scenarios by assuming an initial condition, $z_0 = -5$ mm. The first case starts with $x_{im} = 0.3$, a low energy situation associated with case (a). By promoting a system configuration change following the sequence of dynamical patterns (a)-(b)-(c)-(d), the system can generate a power of $1.581\ \mu W$. On the other hand, a different sequence (b)-(a)-(d)-(c) increases the generated power to $2.285\ \mu W$. These kinds of behaviors show the possibilities of the adaptive system for energy harvesting purposes.

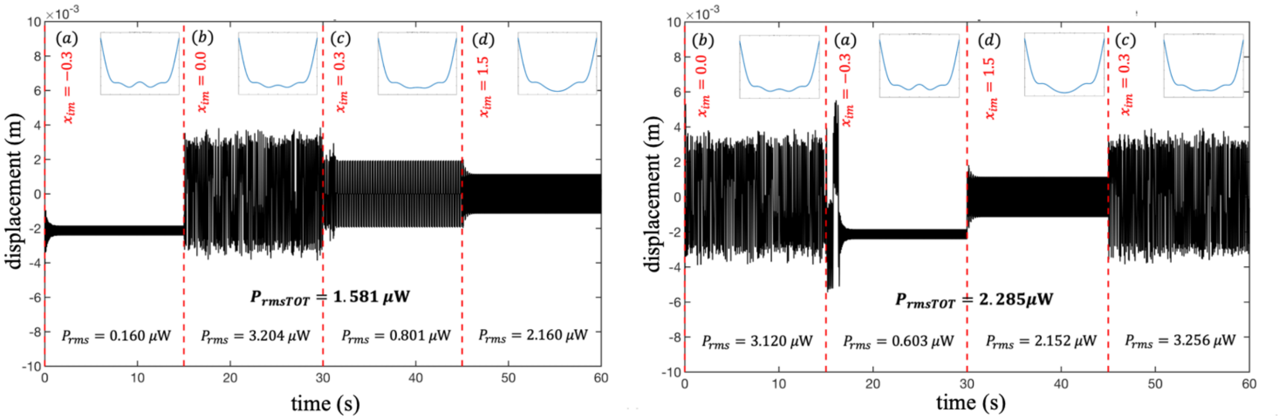

Figure 3. **Different sequences of adaptive behavior guided by the dynamical patterns.**

4. CONCLUSIONS

This paper shows the analysis of adaptive, smart energy harvesting systems with magnetic interactions. The idea is to use the adaptive behavior from the movement of magnets to enhance energy harvesting capacity. A novel model to describe magnetic interactions is proposed based on equivalent point dipole method but considering several points to have a smoother curve. Energy harvester analysis establishes the dynamical characteristics of the system that are employed to define patterns employed to guide adaptive behavior. Results show that energy barriers can be properly overcome by considering appropriate dynamical changes based on the dynamical patterns. The main conclusion is that nonlinear dynamics is the essential point to guide adaptive actuations, promoting better performance to energy harvesters.

5. ACKNOWLEDGEMENTS

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