TRANSIENT RESPONSE OF LINEAR BAR STRUCTURES INTERACTING WITH THREE-DIMENSIONAL SOIL PROFILES: COUPLING THE BEM WITH MODAL ANALYSIS

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Abstract: This work reports an investigation on the in plane transient response of one-dimensional bar structures interacting with the dynamic response of three-dimensional soil profiles in the z direction. The analyzed system is comprised of a bar structure laying on a foundation block, which in turn is supported by a soil profile. In this coupled system the excitation may be external, that is, forces applied directly to the bar structure or on the foundation block, or internal, that is an excitation due to an incoming wave field that impinges upon the foundation block. From the methodological point of view, the transient response of the coupled soil-foundation-structure system is obtained by a modal superposition procedure in which the considered modal parameters, eigenfrequencies and eigenmodes, were synthesized by a procedure that already encompasses the influence of the soil response on the structural modal quantities. The classical structural modal analysis is performed in terms of relative displacements, that is the displacements of the structure relative to the block foundation displacements. The dynamic response of the structure in relative coordinates is coupled to the soil-foundation frequency response, which were synthesized by a 3D direct version of the Boundary Element Method. The coupling of the structural frequency response in modal coordinates with the soil frequency response allows the determination of frequency response functions (FRFs) for the coupled system, in which the soil response is already considered. Modified modal quantities are extracted from the FRFs of the coupled system rendering a set of orthogonal equations that can be integrated to obtain the system transient response. This approach also allows to include in the analysis an arbitrary number of structural modes. It is well known that the transient response of linear structures, depending on the kind of excitation mechanism, does not need to consider all the structural modes. With the approach described above, this article aims to make a modal analysis of a structure over a homogeneous half-space with a rigid layer under it. Besides that, it is intended to investigate the effects of the rigid layer on the structure modal quantities.

Keywords: Dynamic Soil-Structure Interaction, Transient Response, Boundary Element Method, Modal Analysis.

INTRODUCTION

The transient response of structures interacting with the soil is a topic of growing interest and significant importance. Understanding how structures respond to external excitations in the time domain is crucial for project optimization and safety, and the interaction of the structure with the soil must be considered for better representation of reality. Different methodologies have been proposed to calculate the response of structures interacting with an elastic half-space in the time domain, as in Tovo *et al* (2019), in which the proposed methodology consists of coupling the vertical responses of the structure and the soil in the time domain iteratively, where the structure is modeled by the Finite Element Method (FEM) and the soil is modeled by the Indirect Boundary Element Method (IBEM).

Also, in Ferraz (2021) a methodology was proposed that performs the coupling of the structure and soil responses in the frequency domain, in which the frequency response functions (FRFs) of the structure obtained already include the influence of the soil. After using classical methods to extract modal parameters in these FRFs, the transient responses are determined from the numerical integration of the equations of motion based on the modal quantities of the structure with influence from the soil. This methodology has the advantage of being possible to determine transient responses of soil-structure systems in which only FRFs and external forces are known.

This paper aims to obtain transient responses of soil-structure systems through the numerical integration of the uncoupled equations of motion of the structure in terms of the modal parameters extracted with soil influence. Furthermore, it is intended to investigate the influence of the inclusion of a rigid layer under a homogeneous half-space, which is a complex soil arrangement and may present difficulties in the determination of an accurate transient responses.

MATHEMATICAL FORMULATION

The systems analyzed in this work are represented in Fig. 1, where system I consists of a structure with 4 degrees of freedom (dofs) coupled to a square rigid massless foundation on top of stratum-over-bedrock. System II consists of the same system, but with the replacement of the structure with 4 dofs for a single foundation. It is intended to extract the modal parameters from the FRFs of systems I and II and to determine the transient responses of these systems using the extracted modal basis. Only responses in the z direction are analyzed.

The excitation force F is applied in m_4 in system I, and on the foundation in system II. The height from the surface of the half-space to the bedrock, defined by h_s, is the same for both systems.
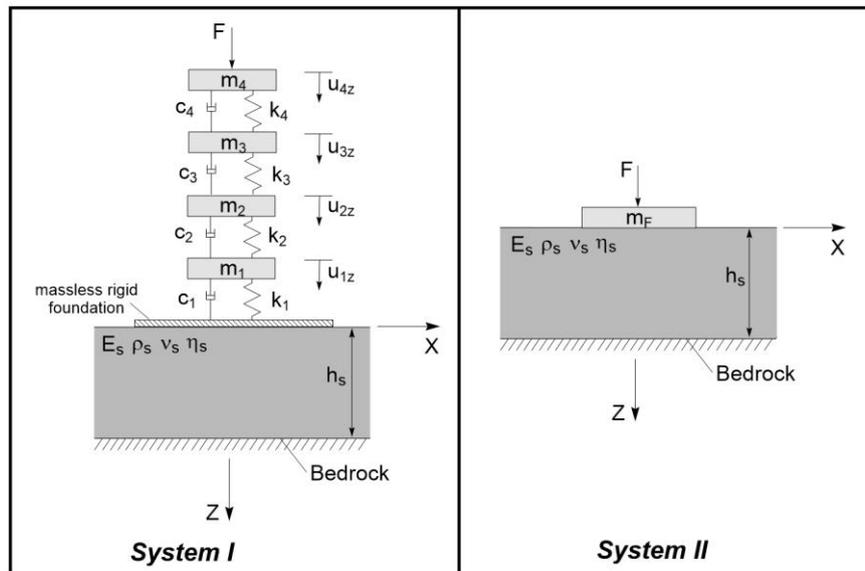

Figure 1 – Soil-structure systems.

The formulation used in this work and the analyzes performed from the systems in Figure 1 are presented below.

Soil-foundation formulation

The soil response is obtained using the classical three-dimensional Direct Boundary Element Method (DBEM) with constant quadrilateral elements. The coupling of the soil with the foundation is modelled using a strategy similar to the one presented by Carrion (2007).

Consider a rigid foundation interacting with the soil, which in turn is modelled as a stratum-over-bedrock, as shown in the figure below.
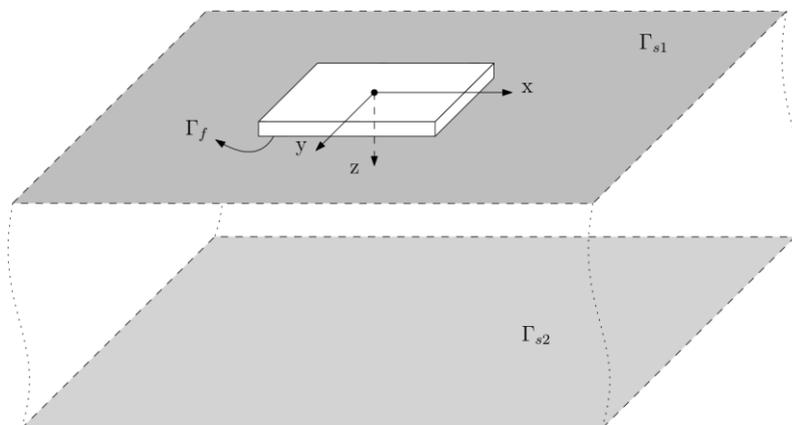

Figure 2 – Foundation on top of stratum-over-bedrock.

In this case, Γ_f is the boundary of the soil structure interface, Γ_{s1} is the boundary of the free surface of the soil, and Γ_{s2} is the boundary of the bedrock. Considering $\Gamma_s = \Gamma_{s1} \cup \Gamma_{s2}$ the discretized boundary of the soil that is not in contact with the rigid foundation.

The classical BEM equation can be written in terms of Γ_f and Γ_s:

$$\begin{bmatrix} H_{ff} & H_{fs} \\ H_{sf} & H_{ss} \end{bmatrix} \begin{Bmatrix} u_f \\ u_s \end{Bmatrix} = \begin{bmatrix} G_{ff} & G_{fs} \\ G_{sf} & G_{ss} \end{bmatrix} \begin{Bmatrix} t_f \\ t_s \end{Bmatrix} \tag{1}$$

The displacements on the soil-foundation interface $\{u_f\}$ can be written in terms of the displacements of the rigid foundation, $\{u_0\} = \{u_x, u_y, u_z, \phi_x, \phi_y, \phi_z\}^T$, using a kinematic compatibility matrix $[C]$, as shown below.

$$\{u_s\} = [C]\{u_0\} \tag{2}$$

Also, the tractions of the soil-foundation interface $\{t_f\}$ are related to the external forces f applied to the rigid foundation through a matrix of equilibrium equations $[D]$ (Carrion, 2007), as shown below.

$$\{f\} = [D]\{t_f\} \tag{3}$$

The problem can be rewritten in terms of the displacements of the rigid foundation u_0 and the external forces f applied to it.

$$\begin{bmatrix} H_{ff}C & H_{fs} & -G_{ff} & -G_{fs} \\ H_{sf}C & H_{ss} & -G_{sf} & -G_{ss} \\ 0 & 0 & D & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ u_s \\ t_f \\ t_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ f \end{Bmatrix} \tag{4}$$

Now, assuming the tractions t_{s1} on Γ_{s1}, and the displacements u_{s2} on Γ_{s2} are known, Eq. (4) can be written and rearranged as

$$\begin{bmatrix} H_{ff}C & -G_{ff} & H_{fs1} & -G_{fs2} \\ H_{sf}C & -G_{sf} & H_{ss1} & -G_{ss2} \\ 0 & D & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ t_f \\ u_{s1} \\ t_{s2} \end{Bmatrix} = \begin{bmatrix} -H_{fs2} & G_{fs1} \\ -H_{ss2} & G_{ss1} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_{s2} \\ t_{s1} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ f \end{Bmatrix} \tag{5}$$

Now assuming that the tractions on the free surface of the soil are zero $t_{s1} = 0$, and the bedrock is fixed $u_{s2} = 0$, the equation becomes

$$\begin{bmatrix} H_{ff}C & -G_{ff} & H_{fs1} & -G_{fs2} \\ H_{sf}C & -G_{sf} & H_{ss1} & -G_{ss2} \\ 0 & D & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ t_f \\ u_{s1} \\ t_{s2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ f \end{Bmatrix} \tag{6}$$

This system of equations can be solved to find the values of u_0 for different forces f applied to the rigid foundation. With these results it is possible to generate a compliance matrix that relates u_0 to f in the following equation.

$$\{u_0\} = \frac{1}{aG}[N]\{f\} \tag{7}$$

where a is half the length of the foundation's side and G is the shear modulus of the soil.

In Fig. 3, it can be seen the real and imaginary parts of the flexibility $N_{uzFz}(\omega)$ obtained for a homogeneous half-space and the flexibility adding a rigid layer under the half-space. The height from the surface of the half-space to the

bedrock is $h_s = 5$m, in this case. The presence of a rigid layer alters the flexibility behavior, resulting in the appearance of numerous resonances, which can influence the responses of structures on this type of soil profile.
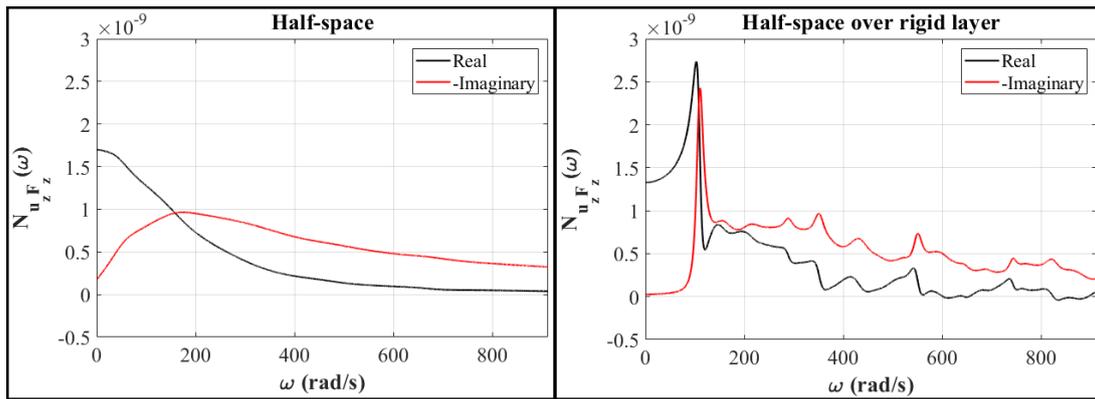

Figure 3 – Comparison between the flexibilities N$_{uzFz}$(ω) for the half-space with and without bedrock.

Determination of FRFs with soil influence

The methodology used in this work allows obtaining FRFs of structures with the influence of the soil. This is exemplified in the case of Fig. 4, where the structure (Subsystem I) is represented by a system of 3 dofs and the soil (Subsystem II) is represented by a system of 2 dofs. Furthermore, the FRFs obtained for each subsystem separately and for the structure after coupling are observed. It is possible to notice 5 resonances in the FRF of the structure after coupling, even having only 3 dofs. It is worth mentioning that the natural frequencies of the coupled system are different from the natural frequencies of the subsystems. It should also be emphasized that the fixed base system with 2 dofs in subsystem II is not used in the following formulations and analyses, but a real soil arrangement model.
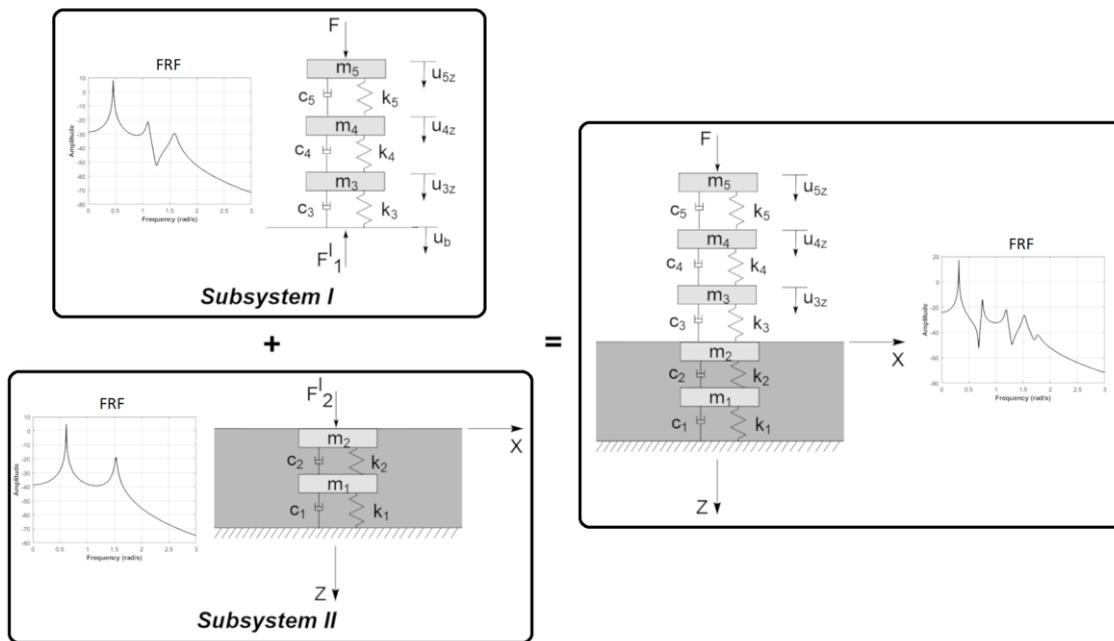

Figure 4 – FRF of coupled soil-structure system.

For system I, the methodology proposed by Louzada (2019) was adopted, in which the response of the structure, in the frequency domain in terms of the relative displacements of each dof, is coupled to the response of the soil by the equilibrium of forces and the kinematic compatibility between the two subsystems (structure and soil), as illustrated in Fig. 4. The dynamic response of the structure, $\{U_z(\omega)\}$, after coupling is represented in Eq. (8).

$$\{U_z(\omega)\} = \underbrace{\left([\Phi][H(\omega)]\left([\Phi]^T + \omega^2\{\Gamma\}[S_{bz}(\omega)]\right) + [S_{bz}(\omega)]\right)}_{[\alpha(\omega)]}\{F_{ext}(\omega)\} \tag{8}$$

where, $[\Phi]$ is the modal forms matrix, $[H(\omega)]$ is the transfer function matrix, $[S_{bz}(\omega)]$ is the compliance matrix of the soil response after coupling, $\{\Gamma\}$ is the generalized modal load coefficient and $\{F_{ext}(\omega)\}$ is the external forces vector.

From Eq. (8), one can obtain the FRFs with soil influence, which correspond to the components of the compliance matrix $[\alpha(\omega)]$.

For system II, the balance of forces is performed at the soil-foundation interface and Eq. (9) is obtained, which defines the dynamic response of the foundation with the influence of the soil. The term that relates the displacement and the excitation force corresponds to the FRF of the system.

$$U_{zF}(\omega) = \underbrace{\frac{N_{uzFz}(\omega)}{1 - \omega^2 m_F N_{uzFz}(\omega)}}_{FRF(\omega)} F_{ext}(\omega) \tag{9}$$

Modal parameters extraction

The Rational Fraction Polynomial method (RFPM) was chosen to extract the modal parameters (natural frequency, damping factor and modal form) from the FRFs with soil effects, which is a classic extraction method presented in Ewins (2000), and a more detailed formulation can be seen in Richardson and Formenti (1982). This method is based on the approximation of the FRF curve by means of complex orthogonal polynomials, represented in Eq. (10). From this approximation, the relationships between the poles and residuals of the partial fractions and the modal quantities are determined.

$$H(\omega) = \frac{\sum_{k=0}^{2N-1} a_k (i\omega)^k}{\sum_{k=0}^{2N} b_k (i\omega)^k} \tag{10}$$

Transient response by modal superposition

To calculate the transient responses, as stated in Ferraz (2021), the equations of motion of the structure in the soil-structure system are transformed into a set of decoupled equations defined in terms of the modal parameters in the time domain, as noted in Eq. (11). To carry out the transformation to decoupled equations, the orthogonality conditions defined in Fu and He (2001) were used.

$$[I]\{\ddot{q}_z(t)\} + [2\xi_n \omega_n]\{\dot{q}_z(t)\} + [\omega_n^2]\{q_z(t)\} = [\Phi]^T \{F_{ext}(t)\} \tag{11}$$

where, $[I]$ is an identity matrix, q_z is the modal coordinate, ξ_n and ω_n are the damping factor and the natural frequency of the nth mode, respectively. Then, the Newmark method was chosen to perform the numerical integration of the equations of motion, due to its unconditional stability for any time step using $\gamma = 0.25$ and $\beta = 0.5$, according to Chopra (2012). Finally, the relationship $\{u_z\} = [\Phi]\{q_z\}$ is applied to transform the transient response of the structure back to the physical coordinate.

NUMERICAL RESULTS

For the analysis, the systems seen in Fig. 1 were used as reference, and the properties defined for the soil are seen in Tab. 1. The flexibility $N_{uzFz}(\omega)$ obtained for the case of soil on a rigid layer can be seen in the graph on the right in Fig. 3, where $h_s = 5m$ and $a = 1m$.

Table 1 – Soil properties

Density	Young Modulus	Shear Modulus	Shear velocity	Poisson's ratio
$\rho_s = 2700$ kg/m^3	E = 234 MPa	G = 90 MPa	$v_s = 341{,}6$ m/s	$v = 0{,}3$

Regarding the mass of the systems, a portion equivalent to the mass of the soil under the structures was used., which is defined by Eq. (12), where B is a percentage of soil mass equivalent to the structure mass.

$$m_{str} = B(\rho_s a^2 h_s) \tag{12}$$

To evaluate the influence of soil resonances on the composition of the structure FRFs with soil effects, $B = 0.2$ was used. The mass defined for each dof of system I is $m_i = m_{str}/4$, while for system II the mass is $m_F = m_{str}$. Then, the properties defined for the structure of system I are seen in Tab. 2. According to Caughey (1960), the definition chosen for the damping was proportional to the mass and stiffness ($C = \mu M + \beta K$), and for that, the damping factors $\xi_1 = \xi_4 = 0.02$, resulting in $\mu = 5.11$ and $\beta = 4.12e^{-5}$. For system II, $m_F = 8.48e^3 \, kg$.

Table 2 – Structure properties

Mass	Stiffness	Damping
$m_1 = 2.12e^3$ kg	$k_1 = 4.03e^8$ N/m	$c_1 = 2.74e^4$ kg/s
$m_2 = 2.12e^3$ kg	$k_2 = 4.03e^8$ N/m	$c_2 = 2.74e^4$ kg/s
$m_3 = 2.12e^3$ kg	$k_3 = 4.03e^8$ N/m	$c_3 = 2.74e^4$ kg/s
$m_4 = 2.12e^3$ kg	$k_4 = 4.03e^8$ N/m	$c_4 = 2.74e^4$ kg/s

As it is possible to observe resonances in the flexibility of the soil over a rigid layer and only the axial displacement is being analyzed, an analogy can be made with a fixed-free bar model, in which, according to Rao (2010), the natural frequencies are determined from Eq. (13).

$$\omega_n = \frac{n\pi v_s}{2h_s} \qquad n = 1,3,5...$$

(13)

The natural frequencies for n equal to 1, 3, 5 and 7 are obtained using Eq. (13) and the soil properties in Tab. 1. The results obtained using $h_s = 5m$, are seen in Tab. 3.

Table 3 – Analytical natural frequencies of the soil

1st Mode	2nd Mode	3rd Mode	4th Mode
$\omega_1 = 107.3$ rad/s	$\omega_2 = 321.9$ rad/s	$\omega_3 = 536.5$ rad/s	$\omega_4 = 751.1$ rad/s

The excitation force used to determine the transient responses is defined in Eq. (14), which is applied in the z direction in m_4.

$$F(t) = \begin{cases} 0, & se \quad t < 0.01s \quad ou \quad t > 0.05s \\ 5000N, & se \quad 0.01s \leq t \leq 0.05s \end{cases}$$

(14)

System I

First, the analysis is performed for system I from Fig. 1. The FRFs of the structure with soil effects are determined from Eq. (6), as seen in Fig. 5, using the flexibility of the half-space over a rigid layer (right graph in Fig. 3). The RFPM is applied to extract the modal parameters, and for the analyzed case, 7 natural frequencies were extracted, represented in the graphs by the red lines.
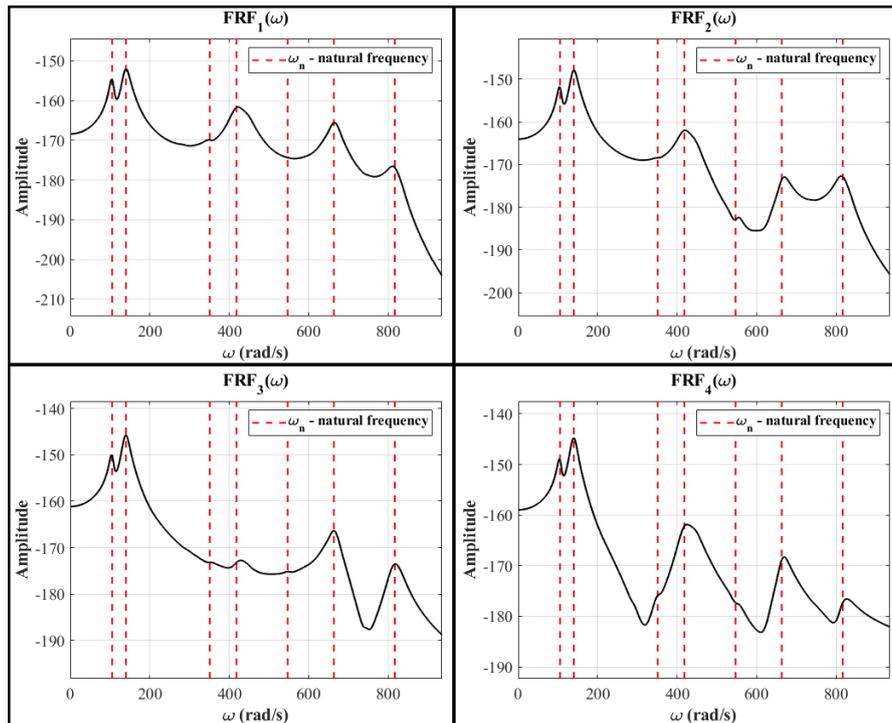
Figure 5 – FRF of system I (bedrock).

All modal quantities extracted from the 4 FRFs are seen in Tab. 4. As can be seen from Fig. 5 and Tab.4, after the inclusion of the soil effects, the FRFs of the structure have new 3 resonances, i.e., the structure with 4 dofs coupled to the soil arrangement can be represented by an "expanded" modal base with 7 dofs.

Table 4 – Extracted modal parameters for system I

Natural Frequency	Damping Factor	Modal form matrix						
$\omega_1 = 105.50$ rad/s	$\xi_1 = 0.0594$							
$\omega_1 = 140.02$ rad/s	$\xi_1 = 0.0723$	0.004	0.006	-0.002	-0.008	0.004	0.018	-0.006
$\omega_3 = 351.17$ rad/s	$\xi_3 = 0.0399$	0.004	0.009	-0.001	-0.009	-0.003	-0.004	0.005
$\omega_2 = 418.59$ rad/s	$\xi_2 = 0.0411$	0.005	0.011	0.001	0.001	-0.005	-0.012	-0.005
$\omega_5 = 547.42$ rad/s	$\xi_5 = 0.0252$	0.005	0.012	0.002	0.009	0.001	0.006	0.004
$\omega_3 = 663.37$ rad/s	$\xi_3 = 0.0519$							
$\omega_7 = 817.03$ rad/s	$\xi_7 = 0.0082$							

By integrating Eq. (11) with the extracted modal parameters, the transient responses related to the 4 dofs of the structure in the physical coordinate are obtained, as can be seen in Fig. 6. For comparison purposes, the transient response was also determined by applying the IFFT and using the convolution with the excitation force, as implemented by Louzada (2019). The parameters used in this method can be seen in Tab. 5. The excitation force used is represented in Eq. (14).

Table 5 – Parameters of IFFT and convolution method

Number of points	N = 60038
Maximum frequency	$\omega_{max} = 2738.5$ rad/s
Maximum time	$t_{max} = 137.8$ s
Frequency increment	$\Delta\omega = 0.0456$ rad/s
Time increment	$\Delta t = 0.0023$ s
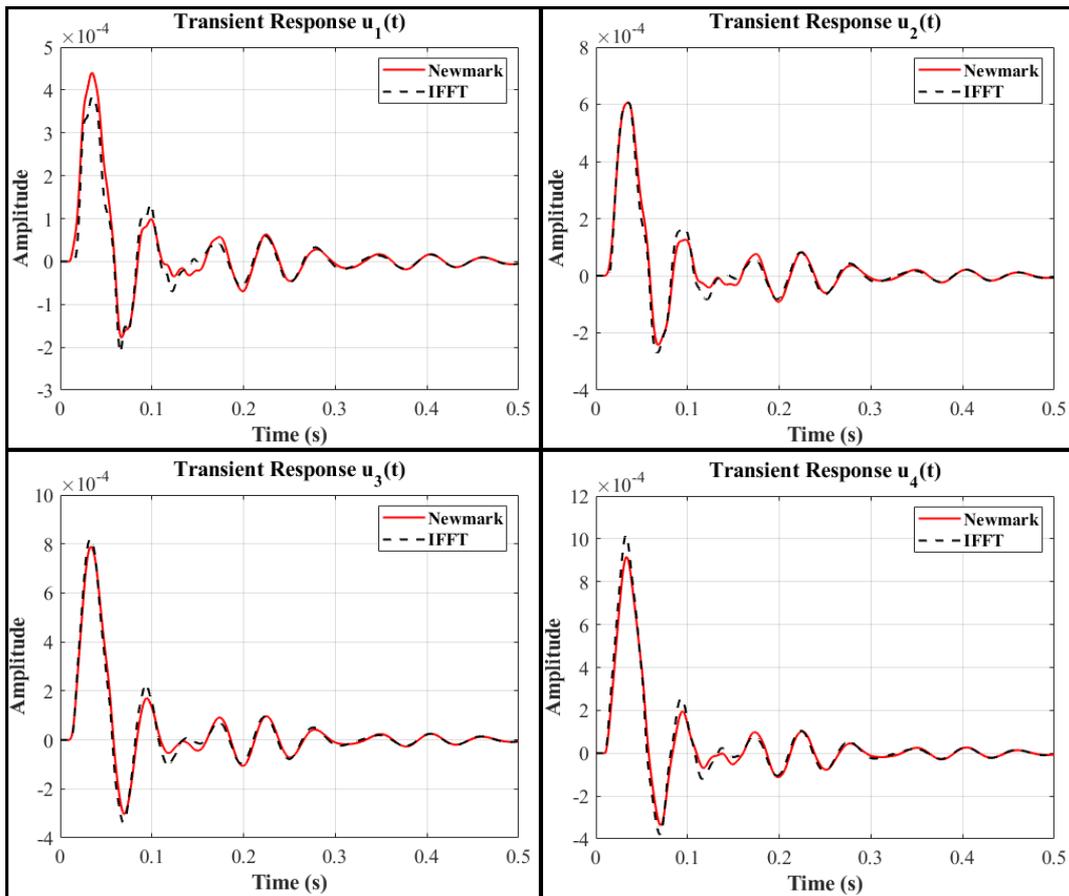
Figure 6 – Transient responses of system I (bedrock).

It is noticed that the biggest differences in the amplitudes between the transient responses obtained by the Newmark numerical integration method and by the IFFT method occur in the peaks of up to 0.2s, approximately. In order to measure this difference, the errors in the L_2 norm between the transient responses were calculated, as noted in Tab. 6.

Table 6 – L₂ norm errors

1st dof	2nd dof	3rd dof	4th dof
0.34e⁻⁸	0.30e⁻⁸	0.33e⁻⁸	0.33e⁻⁸

System II

Now, the analysis is performed for system II in Fig. 1, where the block of mass is directly over the soil surface. The analytical natural frequency of the system can be estimated, since it is possible to determine the static stiffness of the soil on the bedrock by Eq. (15), as defined in Gazetas (1983). Then, the estimate for the natural frequency of system II is $\omega_F = 275.96 \ rad/s$.

$$K_s = \frac{4Ga}{1-\nu}\left(1+1.28\frac{a}{h_s}\right) \tag{15}$$

From Eq. (9), the FRF for system II is determined, as can be seen in Fig. 7. As the mass of the foundation represents only 20% of the mass of the soil, defined by parameter B, it can be seen that the FRF of the foundation has a great influence of the many resonances of the flexibility of the soil profile used (bedrock), as seen in the right graph in Fig. 3. After that, the RFPM was also used to extract the modal parameters and 14 natural frequencies were obtained, represented by the vertical lines in red on the graph in the left on Fig. 7.
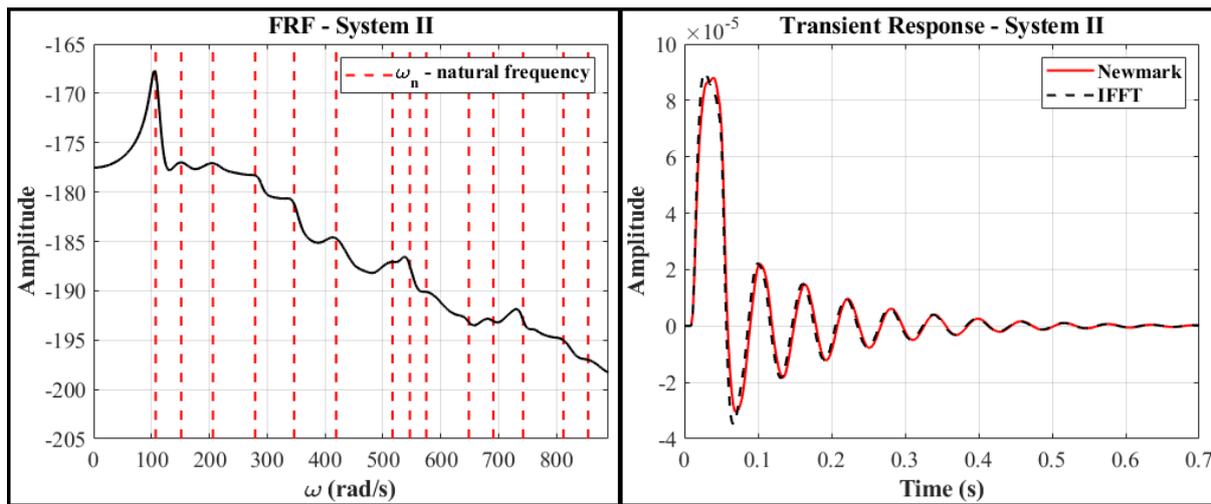
Figure 7 – FRF and transient response of system II (bedrock).

As for system I, the integration of Eq. (11) with the extracted modal parameters is performed and the transient response of the foundation is obtained in the physical coordinate, as seen in the graph on the right in Fig. 5. The transient responses were also determined by applying the IFFT and convolution with the parameters of Tab. 5. The parameters used in the analysis are seen in Tab. 5 and the excitation force used is also from Eq. (14). It is noticed that there is a greater difference between the peaks of the transient responses up to approximately 0.1s, but with a better approximation than that seen in the response of system I. The error between the responses in the L2 norm was also calculated, in which it was obtained 4.3e⁻¹⁰.

Transient response for different soil profiles

In order to analyze the influence of soil profiles on the responses of a system, the structure of system I was used as a reference to calculate the responses and obtain the modal parameters using a fixed base, a homogeneous half-space (left graph in Fig. 3) and the half-space over a rigid layer (right graph in Fig. 3). In Fig. 8, the 2nd dof FRF for the three types of structure bases can be seen, in which significant differences can be seen between them.
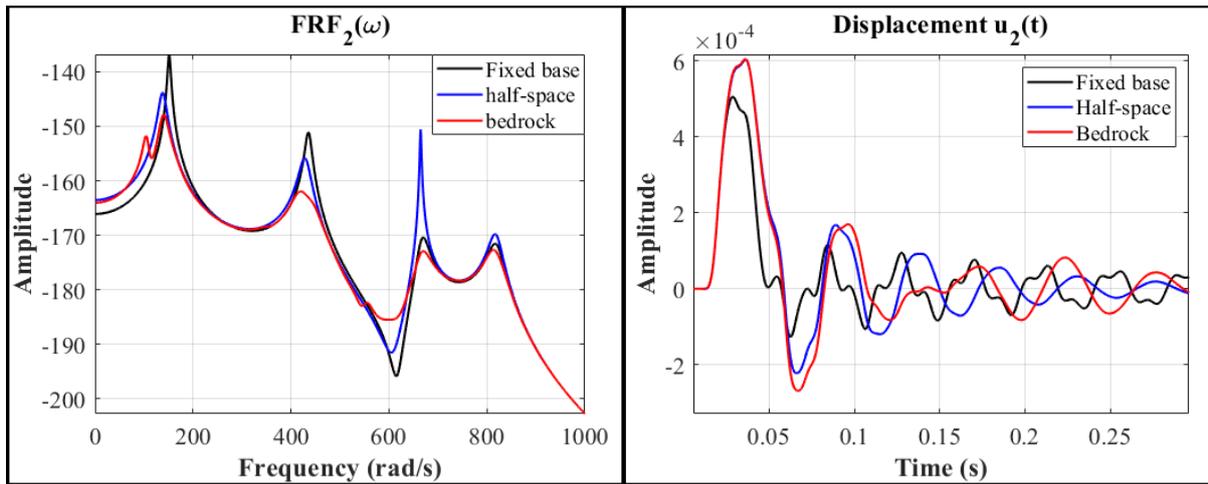
Figure 8 – FRF and transient response of system I (2nd dof) for different soil profiles.

The modal parameters extracted for the systems with a fixed base and for the homogeneous half-space are seen in Tab. 7, while for the system with a half-space on a rigid layer, the modal quantities are the same as those in Tab. 4. It is noticed that the systems with half-space have higher damping factors in relation to the fixed base system. Furthermore, the resonances for the case with half-space over rigid layer can be clearly seen, especially in the first two modes.

Table 7 – Extracted modal parameters for fixed base and homogeneous half-space

Natural Frequency		Damping Factor	
Fixed base	Half-space	Fixed base	Half-space
$\omega_1 = 151.38$ rad/s	$\omega_1 = 137.19$ rad/s	$\xi_1 = 0.0200$	$\xi_1 = 0.0896$
$\omega_2 = 435.89$ rad/s	$\omega_2 = 427.70$ rad/s	$\xi_2 = 0.0148$	$\xi_2 = 0.0540$
$\omega_3 = 667.82$ rad/s	$\omega_3 = 664.15$ rad/s	$\xi_3 = 0.0176$	$\xi_3 = 0.0325$
$\omega_4 = 819.21$ rad/s	$\omega_4 = 818.18$ rad/s	$\xi_4 = 0.0200$	$\xi_4 = 0.0234$

In Figure 7, in the graph on the right, it is possible to observe the transient responses calculated for the 3 cases by numerical integration by the Newmark method using their respective extracted modal parameters. The initial amplitudes of the responses of the systems with half-space are greater than those with a fixed base, since they have lower stiffness. On the other hand, the fixed base system presents a smaller decay of the amplitudes with the time, since it has smaller damping factors.

CONCLUDING REMARKS

In this paper it was possible to evaluate the coupling between the stationary responses of the structure subsystem with the soil subsystem for the determination of transient responses directly in the time domain using extracted modal parameters, since the values obtained converged with those of the method using the algorithm of the IFFT and linear convolution. Furthermore, it was shown that the use of the expanded modal base, with the modal parameters extracted after the coupling between the subsystems is valid, since good results were obtained for the two analyzed systems, confirmed by the L_2 norm.

Furthermore, it is important to have a good definition of the model of a system to be studied, since the dynamic behavior of the system changes completely for the different bases considered, in this case, for the structure of the system I. Besides that, from the modal quantities extracted, it was possible to notice that the half-space considerably increases the damping of the structure, as well as reduces the natural frequencies. And for the case where it is on a rigid layer, new modes appear on the composition of the FRF due to this "layer" of soil.

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