MECSOL 2022 - Proceedings of the 8th International Symposium on Solid Mechanics
M.L. Bittencourt, J. Labaki, L.C.M. Vieira Jr. and E. Mesquita (Editors), Campinas SP, Brazil, October 17th to 19th, 2022

Optimization of Pendulum Type Tuned Mass Dampers for Vibration Control of Slender Structures Under Seismic Loads

Omar Gutierrez Campana [1], Reyolando M. L. R. F. Brasil [2], Marcelo Araújo da Silva [2] and Tatiane Cristine Pereira de Carvalho [1]

[1] Structural and Geotechnical Engineering Department, Polytechnic School University of São Paulo

Av. Professor Mello Moraes, 2373, 05508-900, São Paulo, Brazil

ing.omargutierrez@usp.br, tatiane.cristine.carvalho@gmail.com

[2] CECS, Federal University of ABC, Alameda da Universidade, S/N, 09606-045, São Bernardo do Campo, SP, Brazil F, reyolando.brasil@ufabc.edu.br, marcelo.araujo@ufabc.edu.br

Abstract: Seismic motion is one of the most dangerous events for civil structures. To mitigate the effect of vibrations, induced by the base excitation created by seismic waves, one of the devices used is a vibration absorber called TMD (Tuned Mass Damper). This paper presents the optimization process of a TMD placed in a slender structure. The work begins with the finite element modeling of a slender high rise structure, doing a modal analysis, is obtained the predominant natural frequency and by applying a known force at the top of the structure, is obtained the stiffness equivalent to a 1 degree of freedom (DOF) system, with these values is calculated the representative mass for an equivalent model of 1 DOF. Then, a mass-spring-damper system is modeled equivalent to the original structure, to which a pendulum-type mass is attached, initially the characteristics of mass, pendulum length and damping of the TMD are random, within permissible limits. The structure of many degrees of freedom is thus transformed into a system of 2 DOFs (1 DOF of the structure and 1DOF of the pendulum). These systems were forced at their base by means of a seismic record of accelerations, obtaining the maximum displacement and displacements history of the principal structure during seismic action and the comparison of the variation of these values in the 2 models was made. Through of methods of the mathematical optimization theories, the optimal parameters of TMD were obtained, in this case, the maximum displacement was reduced in 33.6% and the vibrations were minimized in 39% and concluding that the placement of a TMD with optimal characteristics is a good option for vibration control in a tall and slender structures subjected to seismic movements.

Keywords: Passive control, Tuned mass damper, Vibration Control, Optimization

INTRODUCTION

A structure is called slender when one of its dimensions is much larger in relation to the others, in this study this dimension is the height of the structure in relation to the dimensions of its base, due to their slenderness, these structures need vibration control systems, which can be passive or active. Among the passive vibration control systems of structures there are the TMD's. The concept of TMD dates from the 1940s (Hartog, 1985), which consists of a smaller additional mass, which adds 1 degree of freedom to the structure, and has a natural period similar to the predominant period of the structure. According to Lara, Caidedo and Valencia (2021), TMD is one of the most used and proven devices in recent years due to its simple mechanical behavior and high force control capability.

Tuned mass dampers with optimal properties are able to control vibrations and are also applied in skyscrapers to control vibrations produced by wind and on other occasions it has been used for seismic retrofitting. (Bekdas, Nigdeli, 2013). According to Elias, Matsagar and Datta (2016), TMDs are the most popular and widely used passive vibration control devices. Their effectiveness in controlling earthquake-induced vibrations was demonstrated by Pinkaew et al. (2003) and Parulekar and Reddy (2009).

TMD is placed at a specific point on the structure in such a way that when dynamic excitation occurs, the period of the additional mass is tuned to the period of the load and out of phase with the period of the structure, the energy produced by the vibration of the main structure is transferred to the TMD, progressively minimizing the vibrations of the main structure. With an adequate mass ratio, TMD becomes very effective in minimizing the response of the main structure. The use of a TMD is suitable for reducing translations and rotations of tall structures under seismic loads for any angle of incidence of earthquakes. (Ueng, Lin, Huang, 2008). According to Bastian (2016), some examples of tall buildings built with TMD systems are: Shanghai Tower (China), 632m high, built in 2015; Citigroup Center in New York (USA), 279m high, built in 1977; Petronas Towers, Kuala Lumpur (Malaysia), with 452m. tall built in 1998; Taipei 101 (Taiwan), 508m high, completed in 2003.

A study by the *Council On Tall Buildings and Urban Habitat* (CTBUH), shows the increase in the construction of tall buildings (over 200m) from the year 1980 to 2021, where it can be seen that in the year 2000 there were 25 buildings larger than 200m and for the year 2018 it increased up to 166 (CTBUH, 2021).

Slender structures, as they are more flexible, suffer greater displacements, which can cause structural damage due to the forces that can be caused by earthquakes or winds. In addition, vibrations increase and cause discomfort and insecurity to occupants, in this way it is necessary to implement a structural protection system. In 2018, the CTBUH prepared a list of the 50 tallest buildings built with damping systems, through which Figure 1 was prepared, in which a trend towards preference in the use of TMD's is observed with 48% compared to other types of damping systems.

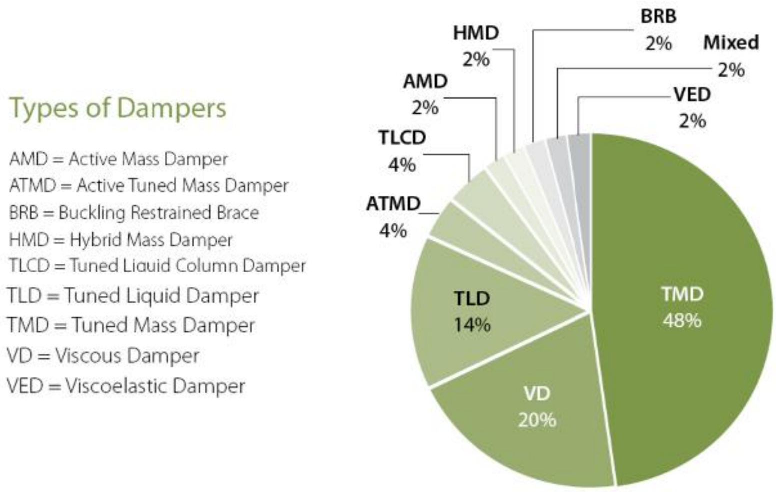

Figure 1 - Types of damping used in the 50 tallest buildings in the world (CTBUH, 2018).

Therefore, the increase in the construction of tall buildings and the preference for the use of TMDs, encourage the study of the optimization of the parameters of a TMD.

PENDULUM TYPE TUNED MASS DAMPERS FORMULATION

Model description

For the present work, the equations of motion were derived from the idealization of the system shown in Figure 2, where the parameters *M*, *K*, and *C* correspond to equivalent parameters of the mass, stiffness and inherent damping of a slender main structure for a structure with 1 degree of freedom (DOF); m_0, L and c_0 correspond to the mass, length and damping of the pendulum-type TMD, which are the variables to be optimized; q_1 and q_2 , are the generalized coordinates adopted, where q_1 is the displacement of the highest part of the structure with respect to its base and q_2 is the rotation of the pendulum with respect to the vertical and finally u_s is the displacement imposed by the earthquake.

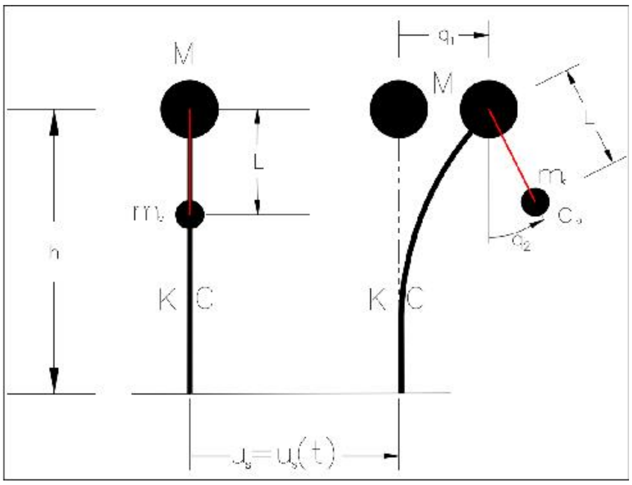

Figure 2 - Idealized structural system pendulum type of 2 DOF.

Equation of motion

The equations of motion were derived by means of Lagrange, adopting the generalized coordinates $q_1(t)$ and $q_2(t)$, as indicated in the Figure 2, being:

$$(M+m_0)\,\ddot{q}_1(t)+m_0L\big(\ddot{q}_2(t)\cos q_2\,(t)-\dot{q}_2{}^2(t)\sin q_2(t)\big)+Kq_1(t)+C\dot{q}_1(t)=-(M+m_0)\ddot{u}_S(t) \tag{1}$$

$$m_0L^2\ddot{q}_2(t)+m_0L\ddot{q}_1(t)\cos q_2\,(t)+c_0\dot{q}_2(t)=-m_0\;L\,(g\sin q_2\,(t)+\ddot{u}_S(t)\cos q_2(t)) \tag{2}$$

Assuming small displacements and rotations, the equations are linearized as follows:

$$(M+m_0)\ddot{q}_1(t)+m_0L\ddot{q}_2(t)+Kq_1(t)+C\dot{q}_1(t)=-(M+m_0)\ddot{u}_S(t) \tag{3}$$

$$m_0L^2\ddot{q}_2(t)+m_0\,L\ddot{q}_1(t)+c_0\dot{q}_2(t)+m_0\;L\,gq_2(t)=-m_0\,L\ddot{u}_S(t) \tag{4}$$

where g is gravity, in matrix form we have:

$$\begin{bmatrix} M+m_0 & m_0L \\ 1/L & 1 \end{bmatrix}\begin{Bmatrix}\ddot{q}_1(t)\\ \ddot{q}_2(t)\end{Bmatrix}+\begin{bmatrix} K & 0 \\ 0 & g/L \end{bmatrix}\begin{Bmatrix} q_1(t)\\ q_2(t)\end{Bmatrix}+\begin{bmatrix} C & 0 \\ 0 & \frac{c_0}{m_0L^2} \end{bmatrix}\begin{Bmatrix}\dot{q}_1(t)\\ \dot{q}_2(t)\end{Bmatrix}=\begin{Bmatrix}-(M+m_0)\\ -1/L\end{Bmatrix}\ddot{u}_S(t) \tag{5}$$

DETERMINATION OF MODEL PARAMETERS

For the determination of the properties of stiffness and equivalent mass of the 1 DOF model, a slender building model was created through the use of commercial software in finite elements (*Extended Three Dimensional Analysis of Building Systems -Etabs 2019*), as shown in Figure 3.

Figure 3 – Finite element model.

Once the structure was built, using the same software, the modal analysis of the model was carried out in order to obtain the periods, frequencies and eigenvalues of the vibration modes, how is showed in the Table 1.

Table 1 – Features of the first 10 vibrate modes

Mode	Period T s	Frequency ω rad /s	Eigen value λ rad²/s²	Dir
1	**6.949**	**0.9042**	**0.8176**	**Y**
2	6.87	0.9146	0.8366	X
3	1.795	3.5006	12.2543	Y
4	1.765	3.56	12.6734	X
5	1.684	3.7308	13.191	Rot
6	0.8	7.8571	61.7338	Y
7	0.787	7.9835	63.7366	X
8	0.612	10.2637	105.344	Rot
9	0.463	13.5613	183.91	Y
10	0.459	13.6823	187.2052	X

Table 1 shows that the first 4 modes are translational in both X and Y axes, and the fifth mode is rotational. Can also be seen that the period difference between the first translational mode in each direction (T_Y=6.949s and T_x=6.870s) is almost 4 times the period value for the second mode (T_Y=1.795s and T_x=1.765s). Whereby, for this work, only the first mode of vibration was considered, and that the earthquake has the same direction as the direction Y of the structure.

Parameters of the principal structure and earthquake selection

After applying a static force *F* of known value, on top of the finite element model, the displacement *u* produced was obtained. So, by applying the expression *K=F/u*, it was possible to obtain the stiffness *K* for an equivalent system of 1 DOF.

Applying a static load of 1.00E+06N the displacement produced was 0.057918m, therefore using the expression *K=F/u* the equivalent stiffness will be 1.72658E+07N/m.

To calculate the equivalent mass *M*, the Eq. (6) and (7) were used, where ω_n is the natural frequency of the structure in function of λ that is the eigenvalue corresponding to the natural frequency (Eq. (6)). The other hand, is possible to obtain ω_n in function of equivalent stiffness *K* and equivalent mass *M* (Eq. (7)).

$$\omega_n{}^2 = \lambda \tag{6}$$

$$\omega_n = \sqrt{\frac{K}{M}} \tag{7}$$

Equating Eq. (6) and (7) we get:

$$M = \frac{K}{\lambda} \tag{8}$$

Substituting the value of the eigenvalue corresponding to the first mode of vibration (λ=0.8176 rad^2/s^2) and equivalent stiffness (*K*=1.72658E+07N/m), it is possible to obtain the equivalent mass *M*= 2.11176E+07 kg.

In the case of the inherent damping, Smith et al. (2010) reviewed and updated damping data measured by several authors for buildings with different heights and construction material. concluding for buildings larger than 200m of reinforced concrete, the damping rate ξ is not greater than 2%, which is the assumed value. Hence, by applying the Eq. (9) it is possible to find the damping coefficient *C* as critical damping rate, where C_c is the critical damping of principal structure, ω_n is the first natural frequency of the principal structure.

$$C = \xi C_c \tag{9}$$

Where:
$$C_c = \sqrt{4\,K\quad} = 2M\omega_n \tag{10}$$

Hence:
$$C = 2\xi \quad \omega_n \tag{11}$$

Thereupon, the equivalent model of the 1 DOF is showed in the Figure 4.

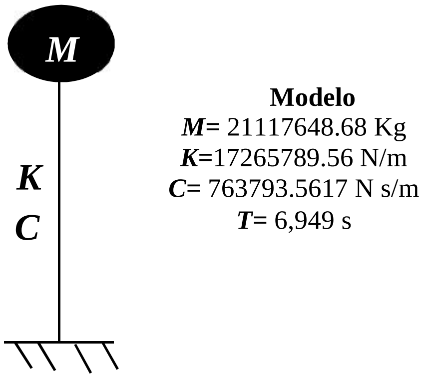

Figure 4 – Equivalent model of the 1 DOF.

The accelerogram of the Imperial Valley (UUEE) earthquake of magnitude 7.5 Mw that occurred in 1940 was selected as a source of seismic excitation (Figure 5). The Fourier analysis of the accelerations was carried out and it was found that the predominant period is T=0.851s. In addition, a response spectrum analysis was carried out, where it was found that for a damping rate of 2%, the maximum displacements are produced in structures with predominant periods between 6.5s and 8s, being 0.48m the maximum displacement, corresponding to a period of 7.1s.

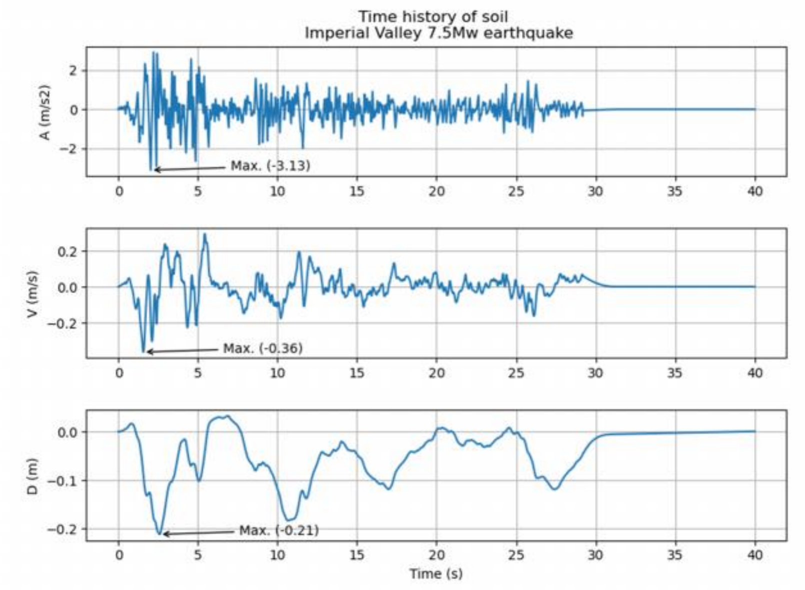

Figure 4 – Time history of soil of the Imperial Valley earthquake.

OPTIMIZATION OF TMD PROPERTIES

Optimization is a mathematical process, which aims to obtain extreme values of a function (objective function), which depends on one or more variables. This function may or may not have equality or inequality constraints (Brasil e Silva, 2019). An extreme value of a function is called a maximum or minimum, which is an optimal point of the objective function.

To optimize the characteristics of TMD, the optimization package called Gekko in the Python programming language was used. For the optimization process, it is necessary to define the objective function, the design variables, the constraints, and the initial conditions of the state variables.

Objective function

In this work, the objective function minimizes the maximum displacement q_1 corresponding to the displacement of the main mass M (Figure 4).

Design Variables

TMD Longitude	:	L_{TMD};
TMD Mass	:	m_0 (% mass of M);
TMD Damping ratio	:	ξ_0.

Restraints

During the optimization process and at each time step, certain conditions and limits have to be met for the decision variables, as they are:

TMD moment of inertia :

$$\mathcal{J} = mL^2 \tag{12}$$

TMD damping coefficient :

$$c_0 = 2\xi_0 \mathcal{J} \sqrt{\frac{g}{L}} \tag{13}$$

Equations of state at time horizon *t* :

$$\dot{q}_1(t) = \frac{dq_1}{d} \tag{14}$$

$$\ddot{q}_1(t) = \frac{-Kq_1(t) - C\dot{q}_1(t) + m \quad q_2(t) + \frac{c_0 \dot{q}_2(t)}{L}}{M} - \ddot{u}_S(t) \tag{15}$$

$$\dot{q}_2(t) = \frac{dq_2}{d} \tag{16}$$

$$\ddot{q}_2(t) = \frac{Kq_1(t)+C\dot{q}_1(t)-(M+m)gq_2(t)}{M} - \frac{(M+m)c_0\dot{q}_2(t)}{M \quad L^2} \tag{17}$$

Limits for L_{TMD}, m_0 and ξ_0 :

$1 \leq L \leq 30;$

$0{,}005M \leq m \leq 0{,}05M;$

$0{,}005 \leq \xi_0 \leq 0.20.$

The initial conditions of position and velocity of q_1 and q_2 for the model at time $t=0$, were equal to zero, because the response of our structure starts from rest.

RESULTS AND DISCUSSION

The displacement response of the idealized structure of 1 DOF was obtained from Eq. (18) which describes the system response under seismic loads. The solution was obtained by the 4th order *Runge Kutta* method, using the stiffness characteristics *K* and equivalent mass *M*, and *C*, calculated for the model.

$$M\ddot{q}_1(t) + C\dot{q}_1(t) + Kq_1(t) = -M\ddot{u}_S(t) \tag{18}$$

The solution of the 2 DOF model in initial conditions (TMD without optimizing) was also carried out by 4th order *Runge Kutta* resolving Eq. (5).

The results obtained from the TMD optimization process are presented in the Tab. 2, considering 10 different sets of start values for the design variables. The optimization was performed considering only the forced vibration regimen, registered in the accelerogram, without considering the free vibration regimen.

In this table, it is possible to observe that the 10 different starting points of decision variables lead to the same local optimum point (L_{OPT}=13.683m, m_{opt}=5.00% and ξ_{opt}=0.10), these optimal values obtained do not guarantee to be a global solution.

As can be seen in this table, the optimal value for the TMD mass is 5%, the same as the upper limit of the restraint, being reached in the optimization process, making the mass relation a variable with active restraint.

Table 2 – Optimal points obtained considering the objective function maximum displacement.

Optimization	Initial condition			Objective	Optimal values of variables		
	L	%M	taxa ξ_{TMD}	Function Value	L_{OPT}	$\%M_{OPT}$	taxa ξ_{OPT}
1	28.00	4.00	0.05	1313378.72	13.683	5.00	0.10
2	25.00	4.00	0.04	1313378.72	13.683	5.00	0.10
3	10.00	0.50	0.01	1313378.72	13.683	5.00	0.10
4	8.00	2.00	0.10	1313378.72	13.683	5.00	0.10
5	10.00	4.00	0.05	1313378.72	13.683	5.00	0.10
6	22.00	2.00	0.01	1313378.72	13.683	5.00	0.10
7	22.00	0.50	0.08	1313378.72	13.683	5.00	0.10
8	9.00	2.00	0.02	1313378.72	13.683	5.00	0.10
9	5.00	2.00	0.01	1313378.72	13.683	5.00	0.10
10	18.00	3.50	0.08	1313378.72	13.683	5.00	0.10

Figure 5 shows the responses of the main structure subjected to the Imperial Valley earthquake, where it is observed that the maximum displacement reached for the structure without TMD was 0.4794m, which occurred in the third cycle of vibration, in the case of the structure with TMD in initial conditions, the maximum displacement reached for the structure was 0.3557m, which occurred in the second cycle of vibration.

The random selection of the characteristics of the TMD may or may not lead to a decrease in the maximum displacement of the structure, in the case of this initial point (L_0=10.0m, m_o=4.00% and ξ_o=0.05) a maximum displacement of 0.3557m was reached, which means a reduction of 25.8% of the maximum displacement of the structure.

The maximum displacement reached with the optimal TMD conditions was 0.3209m, which means a displacement reduction of 33.06%. In addition, it is possible to observe that the optimal TMD conditions not only reduce the maximum displacement, but also reduce the vibration as a whole in both regimes.

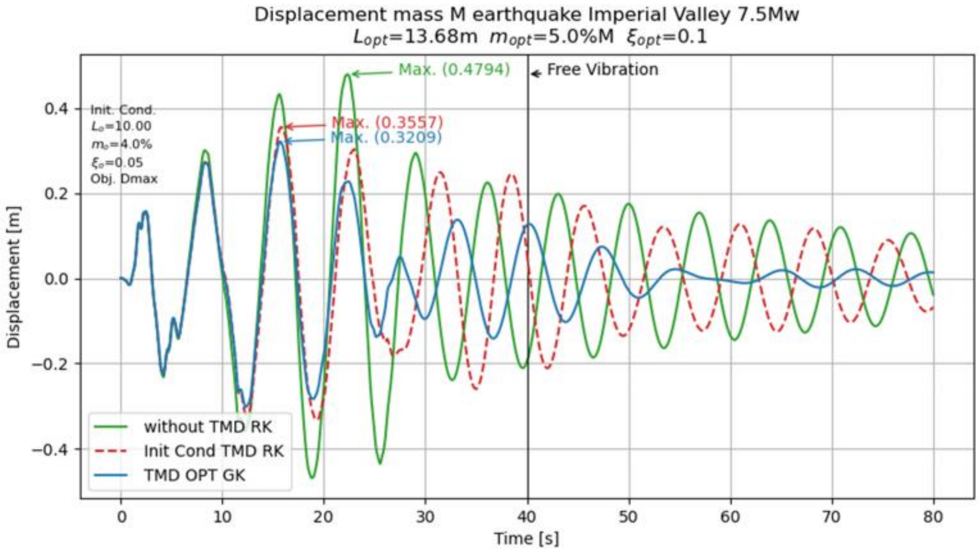

Figure 5 – Results of the optimization of TMD.

CONCLUSSION

The use of optimization theories guarantees for this work, a decrease in the maximum displacement of the structure and this leads to a decrease in its vibration, in the forced regime, and even in the free vibration regime. concluding that the placement of a TMD with optimal characteristics is a good option for vibration control in a tall and slender structures subjected to seismic movements.

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