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A topology optimization of a compliant mechanism using piezoelectric material

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Abstract: The present study aimed to apply piezoelectric materials in the context of compliant mechanisms. To synthesize these mechanisms, the topology optimization approach was used. Topology optimization is a mathematical method to assign material in a prescribed domain, usually a Finite Element Method approach is taken and the goal is to minimize compliance while reducing mass. Unlike the usual topology problem, the approach employed in this study for a compliant mechanism was to maximize the output displacement. This formulation for the optimization problem produces kinemactly efficient mechanism designs. One of the characteristics of the compliant mechanisms is the lack of joints and hinges, which facilitates the use of them in micro and nano scale, since no assembly is necessary and there are no gaps between parts. The topology optimization method used was an adapted SIMP method for non-isotropic materials and the input parameter of the optimization problem uses the piezoelectric material characteristic of generating strain from electric fields. This approach makes feasible the project of micro mechanisms powered by electric fields and, therefore, can be used in a number of different applications, among them embedded systems, with the ever-shrinking nature of computational devices; and medical use, since most of the piezoelectric materials are ceramics and crystals, non-reactive with human tissues.

Keywords: Compliant Mechanism, Topology optimization, Piezoelectric material

INTRODUCTION

Piezoelectric materials are able to generate an electric charge in response to applied mechanical stress and the invert process, generating mechanical energy through an applied electric field. The direct piezoelectric effect was first studied by Pierre e Jacques Curie (1880 apud Moheimani and Fleming, 2006), the so called inverse piezoelectric effect was first theorized and proved by Lippmann (1881 apud Moheimani and Fleming, 2006) and later the Curie brothers confirmed this reversibility. The piezoelectric effect is most commonly used for electromechanical sensors and accelerometers microprocessors. The inverse effect is mostly used for acoustic wave generation and electromechanical actuators.

The use of piezoelectric materials as actuators is interesting for its precision of movement and backlash-free motion, provided it is in a microscale capacity (Mamiya, 2006). To remediate the lack of strain produced by a piezoceramic (PZT), usually around 0.1% (Canfield and Frecker, 2000), a compliant mechanism (CM) is usually attached to the actuator, for amplifying the mechanism kinematics.

A compliant mechanism uses elastic deformation of its material to transmit force and motion, instead of using hinges and gears. Highly scalable, CMs are appropriate for micro and nano scale applications, since the lack of multiple pieces requires little to no assembly, are easy to manufacture and are frictionless, therefore lubricant free mechanisms. Compliant mechanisms are not failed free, mainly the use of part of the input energy is utilized for the deformation of the mechanism itself, the fatigue susceptibility and the difficulty to make a design methodology for real life applications (De Leon et al., 2015). Among the techniques used to design CMs, topology optimization has been one of the most adaptable and systematic.

Topology optimization (TO) is the optimization of shape and size of a domain to achieve the maximum performance for the system. Usually in engineering design this means minimizing the amount of material applied and the strain energy of structures while maintaining their mechanical strength (Bendsøe et al., 2003 apud Yuksel, 2019), in other words, minimizing the body compliance. In CMs the opposite is true, the goal is to maximize the compliance of the system. There are several distinct methods for the topology optimization, among them: Homogenization, Density, Discrete and Boundary Variation Methods.

The Density method for topology optimization proposed by Bendsøe (1988 apud Yuksel, 2019) is the most popular TO method. The Solid Isotropic Material with Penalization method (SIMP) is computationally efficient, robust, adjustable, easy to understand and does not require homogenization of the microstructure (Rozvany, 2000).

The study of compliant mechanisms with piezoelectric material can have real life applications in the (Carbonari and Silva, 2003) high precision mechanics, such as mechanisms of photographic machines, computers hard disks, electromechanical surgical instruments and in the new field of microelectromechanical systems (MEMS). This paper aims to highlight one methodology for the topology optimization of such mechanisms.

PIEZOELECTRIC CONSTITUTIVE EQUATIONS

The piezoelectric material always has the mechanical and electrical fields coupled, if assumed that both are linear, in accordance with the IEEE standard (Gonçalves, 2015), the material can be modeled in a matrix format, as:

$$\sigma = c^E S - e^T E \tag{1}$$

$$D = eS + \varepsilon^S E \tag{2}$$

where σ and S are the stress and strain tensors, D and E are the electric displacement and electric field, c^E is the elastic tensor, ε^S the dielectric tensor, and e is the piezoelectric strain tensor. The elastic tensor c^E has the superscript E to represent the constant electric field E, of which it's under the influence of. The same can be said about ε^S and the strain tensor. The superscript T means transposed matrix, and distinct authors assume inverse representations of piezoelectric strain (Zheng et al., 2008 and Reinhardlerch, 2000), in this paper the non-transposed bidimensional matrix e is 3 by 2 in size. The electric field is defined as the gradient of the electric potential ϕ, as expressed in Eq. (3). The strain tensor S is the Lagrangian strain with the second-order term neglected, represented by Eq. (4)

$$E = -\nabla\phi \tag{3}$$

$$S = \frac{1}{2}(\nabla \mathrm{u} + (\nabla \mathrm{u})^{\mathrm{T}}) \tag{4}$$

In Tab. 1 the properties that were used in this paper are shown. The material PZT-5A is a piezoceramic widely used for actuators, with high temperature range and thermal stability and with a high strain output (Caetano, 2017). The piezoceramic is made out of Lead Zirconate Titanate and has hexagonal symmetry. It behaves in a transversely isotropic capacity, having four distinct constants on the stiffness matrix.

Table 1 – Properties of the PZT-5A material

	PZT-5A		
c^E	C_{11}^E	12.1	$10^{10} N/m^2$
	C_{13}^E	7.52	
	C_{33}^E	11.1	
	C_{44}^E	2.11	
e	e_{31}	-5.4	C/m^2
	e_{33}	15.8	
	e_{51}	12.3	
ε^S	ε_{11}^S	916	dimensionless
	ε_{33}^S	830	

BENCHMARK NUMERICAL IMPLEMENTATION

To solve the numerical problem, a Python code was created, utilizing the FEniCS computing platform (Alnaes et al., 2015), as well as the dolphin adjoint package and the method of moving asymptotes (MMA) created by Svanberg (1987). Utilizing Ubuntu, a free and open-source Linux distribution based on Debian with Spyder, a free and open-source scientific environment written in Python, the author created a program that resolves the topology optimization problem for a displacement inverter.

The inverter mechanism is usually defined as a two-dimensional CM that has an input on one side and an output on the opposite side, in the opposite direction. The Fig. 1 shows the domain and boundary conditions for the classic inverter problem. The square area is divided in its symmetric line with the appropriate degrees of freedom, the lower left corner is restricted and the input, represented by a continuous arrow, is applied in the center of the left side (upper left in the simplified mirrored problem). The output is represented by a dashed arrow.

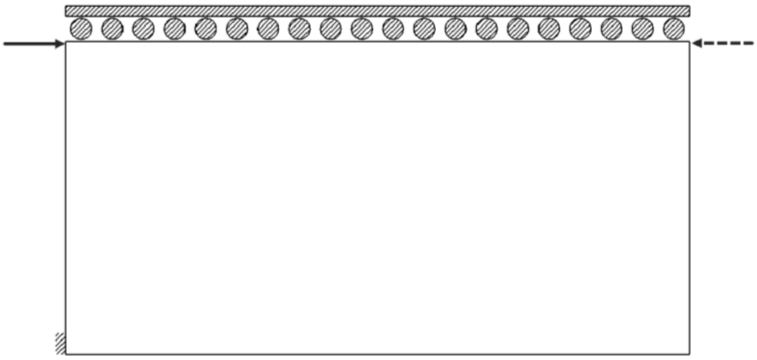


Figure 1 – Domain and boundary conditions for an inverter mechanism

In the FEniCS environment, the author created the mesh, and then the boundary conditions. The utilized mesh was a 150 by 75 units grid, since the elements themselves are triangles, the total amount of elements in the mesh is 22500. Equation (5) is the weak formulation for the static equilibrium of an isotropic material with no body forces and no effects of piezoelectricity and it was used for solving the benchmark problem to test the program.

$$2\mu \int_{\Omega} Sym(\nabla u) : Sym(\nabla \delta u) d\Omega + \lambda \int_{\Omega} (\nabla \cdot u)(\nabla \cdot \delta u) d\Omega + \int_{\Gamma_{in}} k_{in}(l_{in} \cdot u)(l_{in} \cdot \delta u) d\Gamma$$

$$+ \int_{\Gamma_{out}} k_{out}(l_{out} \cdot u)(l_{out} \cdot \delta u) d\Gamma = \int_{\Gamma_{in}} t \cdot \delta u \, d\Gamma \tag{5}$$

with μ and λ being the Lamé parameters and Sym being the symmetric of a matrix, defined as Eq. (6), Eq. (7) and Eq. (8) respectively. The u being the displacement trial function, δu the displacement test function, t the pressure in the input, l_{in} and l_{out} being the vectors representing the direction of input and output, respectively. The ν represents the Poisson's ratio and E the Young's modulus. Table 2 shows the properties of the isotropic material used in this paper.

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \tag{6}$$

$$\mu = \frac{E}{2(1+2\nu)} \tag{7}$$

$$Sym(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{x} + \boldsymbol{x}^T) \tag{8}$$

Table 2 – Properties of the aluminum material

	Aluminum	
ν	0.33	dimensionless
E	7.1	$10^{10} N/m^2$

The topology optimization using SIMP utilizes a pseudo density (ρ) in the range of 0 to 1 to regulate the influence of the material properties in each iteration, the penalization is in the form of an exponent in the pseudo density, that usually is set to 3 (De Leon et al., 2020). Equation (9) interpolates the Young's modulus using the pseudo density.

$$E = E_{min} + (E_0 - E_{min})(\rho^{\eta}) \tag{9}$$

Where E_0 is the initial modulus of elasticity, η is the penalization and E_{min} is a small positive number, it can't be zero to avoid ill-conditioning. A problem in the use of SIMP methodology is the possible appearance of checkerboard patterns (Sigmund and Petersson 1998). To overcome this problem, a Helmholtz-type density filter was implemented (Lazarov and Sigmund, 2010). Helmholtz-type density filter defines the variable as a solution of a Helmholtz-type differential equation with homogeneous Neumann boundary conditions, in an implicit manner. The termination's criteria for the program was set as a maximum number of iterations (3000) or to a convergence tolerance of a magnitude of order four less than the objective function.

The objective function, as presented by Souza (2020), appears in Eq. (10). The usual optimization problem minimizes an objective function (J), therefore, for maximizing the displacement the function was inverted. The Eq. (10) shows that J is a function of u, that, itself is a function of all the problem conditions and ρ, so $J(u)$ can be write as $J(\rho)$. Rewriting Eq. (11) using the inverter problem, Eq. (12) was obtained.

$$J = -\int_{\Gamma_{out}} l_{out} \cdot u d\Gamma \tag{10}$$

$$\begin{array}{lll} minimize & f_0(x) & \\ (x \in R) subject\ to & f_i(x) \leq \hat{f}_i, & for\ i = 1, \dots, m \\ & \underline{x}_j \leq x_j \leq \underline{x}_j & for\ j = 1, \dots, n \end{array} \tag{11}$$

where $f_0(x)$ being $J(\rho)$, $f_i(x)$ being the restriction, in this case the volume restriction. The chosen volume was 30% of the topology domain. With ρ being a vector that has its components within a lower and upper bound of null to unity (0 to 1). To aid in the discretization of ρ to a binary output, a smooth Heaviside function was introduced, in the Eq. (13) the β determinates the slope and η the threshold of the function. The β and η values chosen were 16 and 0.5 respectively.

$$\begin{array}{lll} minimize & J(\rho) & (\rho \in R) \\ subject\ to & V(\rho) - V_c \leq 0, & \\ & -\rho_j \leq 0 & for\ j = 1, \dots, n \\ & \rho_j - 1 \leq 0 & for\ j = 1, \dots, n \end{array} \tag{12}$$

$$\tilde{\rho}_i = \frac{\tanh(\beta\eta) + \tanh\big(\beta(\rho_i - \eta)\big)}{\tanh(\beta\eta) + \tanh\big(\beta(1 - \eta)\big)} \tag{13}$$

BENCHMARK RESULTS

The results for the inverter problem took 39880 seconds (11.1 hours) of computer time, performing 2584 iterations, Fig. 2 shows the evolution of the objective function in function of iterations. It's possible to see some usual behavior for optimization problems, such as a few setbacks; around 400 iterations, some plateaus before the inversion of direction (around 50 iterations) and at the end of the program.

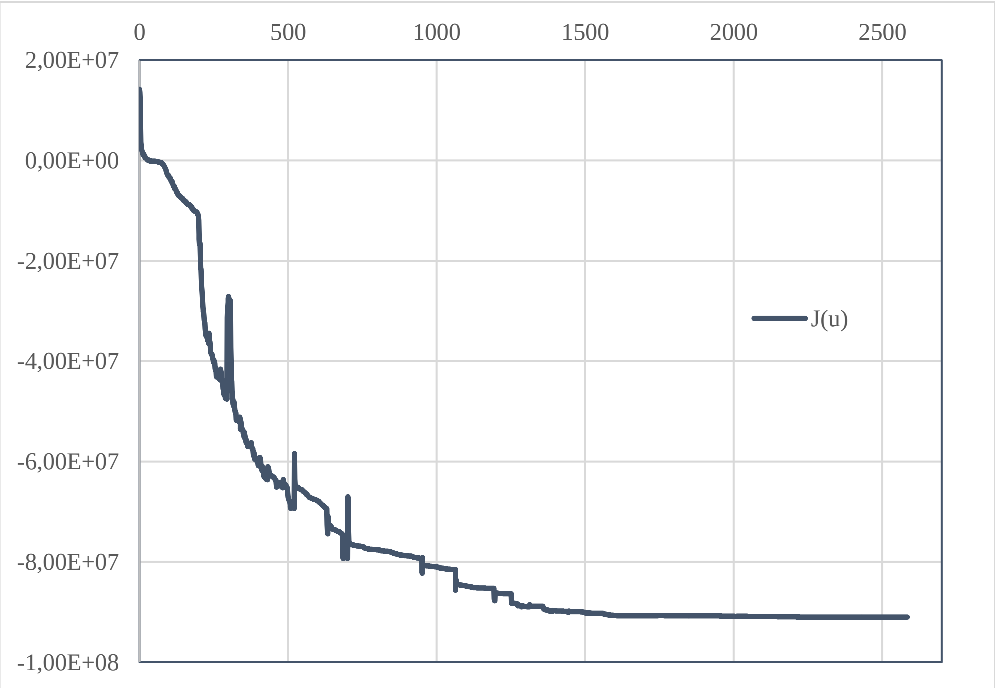


Figure 2 – Objective function for different iterations.

The solution for the optimization problem is representative of a matrix of pseudo densities that signify where should and where should not have material in the domain. Figure 3 is a plot of those densities, the penalization and the filter working, the expected gray area and checkerboard patterns are minimum.

Figure 3 – Pseudo density in the domain.

The discreteness of the solution can be calculated with Eq. (14), first introduced by Sigmund (2007). The $\tilde{\rho}$ represents the final density, after the optimization and the filtering. N is the total number of elements and M_{nd} is the measure of non-discreteness, ranging from 100% for all elements being 0.5 to 0%, with elements being either 1 or 0. The result M_{nd} for the inverter problem was 1.2%.

$$M_{nd} = \frac{\sum_{i=1}^{N} 4\widetilde{\rho_i}(1-\widetilde{\rho_i})}{N} \times 100\% \tag{14}$$

For benchmark purposes the obtained distribution of material aligns itself with the known literature (De Leon, 2015), as shown in the Fig 4 bellow.

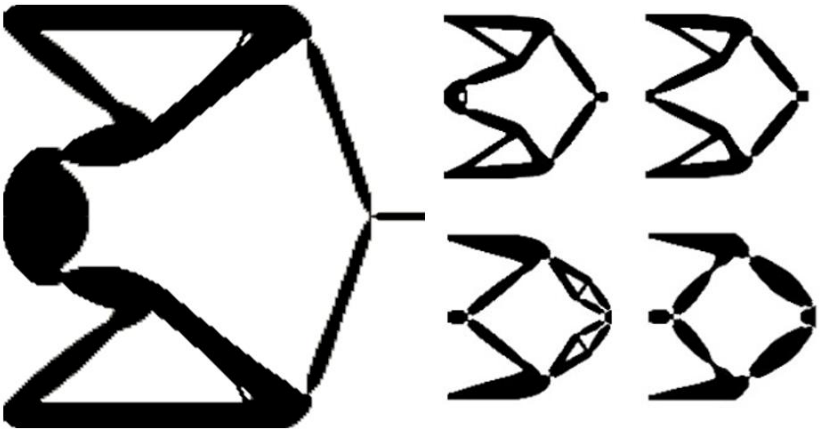


Figure 4 – Inverter obtained by the authors (left) and ones obtained in the literature (De Leon, 2015) (right).

PIEZOELECTRIC IMPLEMENTATION AND RESULTS

Flexotensional transducer is a piezoceramic connected to a CM to amplify and modify the output displacement. The one defined in this paper can be seen in Fig. 5. With two symmetric axes, the two-dimensional mechanism has a piezoelectric material in its core, surrounded by aluminum. The objective of this particular flexotensional transducer is represented by the arrows located in the corner of Fig. 5, that is, have a positive displacement in the x axis and a negative one in the y axis. The boundary condition of electric potential is represented by the ground symbol and Q.

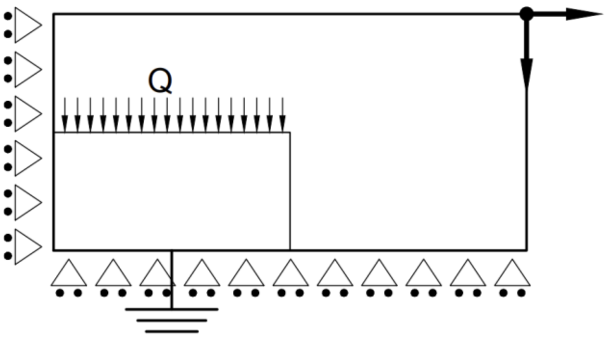


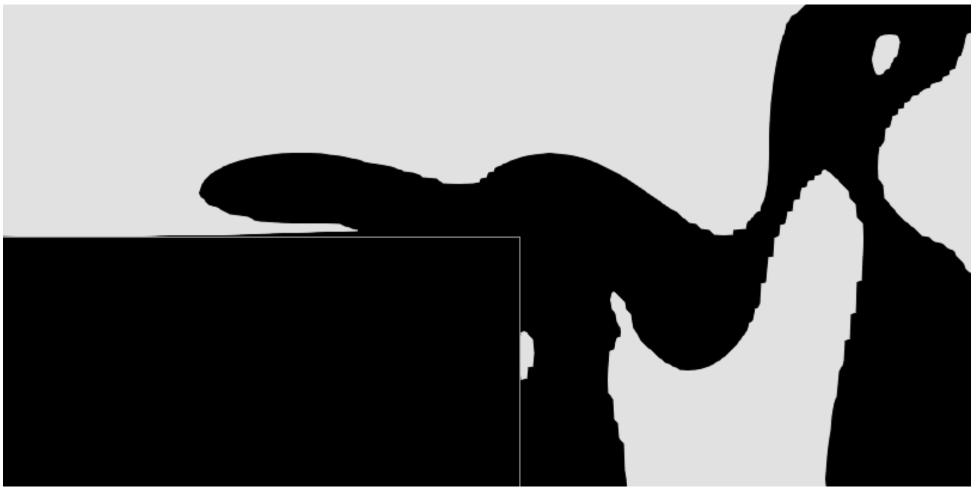
Figure 5 – Domain and boundary conditions for the Flexotensional transducer

The energy equation for a static problem with piezoelectric material and no body forces, surface forces or surface charges is Eq. (15). The dual material was created by changing the properties of the material according to the mesh, places where the PZT-5A was needed were not changed and the domain where the aluminum governs was applied a material with null piezoelectric properties and isotropic elastic properties.

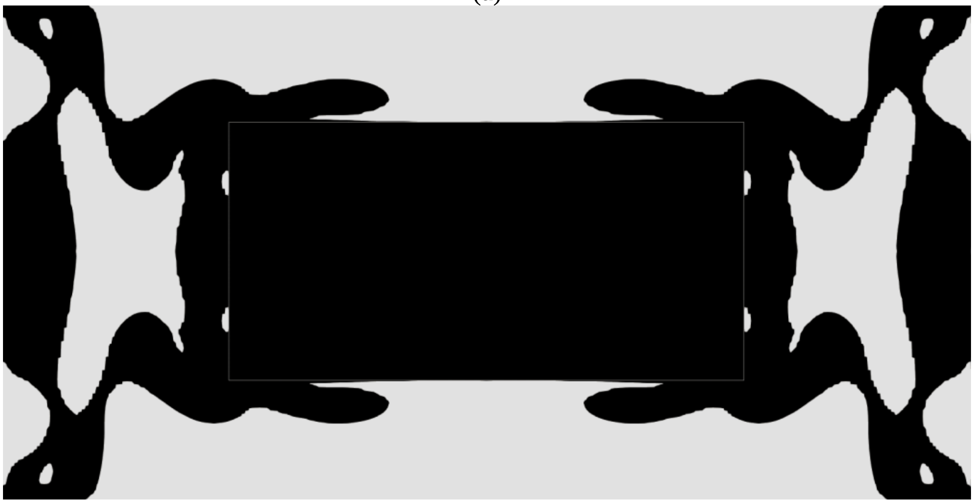
$$\int_{\Omega} \sigma : S d\Omega + \int_{\Omega} D \cdot E d\Omega \tag{15}$$

When calculating the stress tensors (σ) for the isotropic part of the mesh, the SIMP method was applied, for the rest of the mesh the pseudo density was implied as one and not countable for the volume restriction of 30% of the domain. With this methodology, some work done by the MMA is put to waste, since the method tries to reduce the material in the piezo domain and it is latter render worthless by the impose ρ.

The continuation of the implementation bears no distinction with the previous one. The optimized topology generated after a thousand iterations can be seen in Fig. 6, where the demarcated rectangle is the PZT-5A, and the outer material is aluminum (black) or void (gray).With the electric charge applied, the piezo material contracts in the y (vertical) direction while expanding in the x(horizontal), amplifying the outer point movement. The result topology has yet improvements to be made, more noticeably the suspend mass above the PZT-5A.



(a)



(b)

Figure 6 – Topology obtained for the flexotensional transducer (a) and the full CM, without the symmetry (b)

CONCLUSION

Analyzing the output of just the piezoceramic in the corners, we have a displacement of 1.2 (micrometers) in the x direction and -0.6 in the y. With the final mechanism, 1.9 micrometers on x and -0.7 on y in the corners of the CM. Representing an increase of 50% in the overall kinematics of the mechanism, without using more piezoceramic material and increasing the total design space by 3.5 times the original, the methodology proposed in this paper works well for the design of a compliant mechanism with the intend to amplify the output of a piezoelectric material. Further tests occur and the author expects to analyze the stress in the mechanism to aid in the designs.

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