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Parametric analysis of quasi-periodic metastructures with damped local resonators

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Abstract. Metastructures or (quasi-)periodic structures have several applications such as vibration, acoustic and vibroacoustic control since they may exhibit bandgaps, that are frequency ranges where elastic or acoustic waves cannot propagate and can be obtained either due to periodicity that may lead to the Bragg scattering phenomena or by adding local resonators distributed along the host structure. For typical engineering structures, the addition of local resonators to the host structure also allows to increase its damping properties leading to another potentially effective way to reduce or mitigate undesired vibrations. The main novelty of the concept of metastructures with local resonances is the fact that the periodicity of the host structure and/or of the local resonators distribution may add to the overall performance. The main objective of the proposed work is to present recent results and analyses on the addition of periodic and/or quasi-periodic local resonators to beam- and plate-type structures, including an assessment of the effects of the parameters of the local resonators and of their distributions on the host structures on the vibration mitigation performance in the lower frequency ranges. Results indicate that added mass constraints tend to restrict satisfactory performances to narrow frequency ranges and, for that reason, the effective damping potentially added by the local resonators are essential to provide satisfactory performances over a wider frequency range. For the same reason, it is also shown to be essential to analyze distributions of different resonator types that are tuned to different target resonance frequencies in order to provide a multimodal or broadband vibration mitigation solution.
Keywords: metastructures, (quasi-)periodic structures, vibration control

INTRODUCTION

The use of distributed small vibration absorbers or local resonators for vibration mitigation in low-weight flexible structures, such as beam- and plate-type structures, has attracted several research groups in recent years. This is due to the fact that not only this allows the implementation of vibration absorption without much structural modification, if compared with the use of a single large vibration absorber, but also makes it possible to have different resonators tuned to different frequencies providing a multimodal vibration mitigation (Igusa and Xu, 1994). The idea of having multiple distributed vibration absorbers, that could potentially be distributed in a periodic fashion, also allow to explore the effect of periodicity or quasi-periodicity that has also attracted much attention in recent years due to the development of metastructures.

Metastructures or (quasi-)periodic structures have several applications such as vibration, acoustic and vibroacoustic attenuation, acoustic absorption and insulation, vibration and shock mitigation, wave propagation control and focusing, among others. They may exhibit bandgaps, that are frequency ranges where elastic or acoustic waves cannot propagate and can be obtained either due to periodicity that may lead to the Bragg scattering phenomena or by adding local resonators distributed along the host structure (Hussein et al., 2014; Liu et al., 2000). These two types of bandgaps may also be combined in some cases. Metastructures have many interesting applications for wave (Frank Pai, 2010), vibration (Reichl and Inman, 2017) and vibroacoustic (Claeys et al., 2016) control applications. For typical engineering structures, the addition of local resonators to the host structure also allows to increase its damping properties leading to another potentially effective way to reduce or mitigate undesired vibrations. Therefore, this research effort is related to the search for solutions considering distributed low-size mechanical vibration absorbers added to low-weight flexible structures.

Peng and Frank Pai (2014) discussed the design of spring-mass-damper subsystems distributed on a vibrating plate, concluding that the amount of damping in internal resonators is crucial to the vibration control performance. A latter work included dissipation in the internal vibration absorbers to reduce vibration amplitudes of a sandwich beam (Chen et al., 2017). Also focused on attenuating impact loads on sandwich-type structures, the inclusion of discrete mass-spring vibration absorbers inside the core of a sandwich beam was proposed in (Sharma and Sun, 2016). Later, the same concept was considered in (Liu et al., 2018) to improve acoustic insulation properties, while maintaining the lightweight nature of sandwich plates. The use of continuous internal resonators as vibration absorbers inside a honeycomb core of sandwich plates motivated by vibration control treatments that do not rely on significant strains on the host structure, as do viscoelastic constrained layer damping treatments, was studied in (Yu and Lesieutre, 2017). It is also clear from these studies that an extension to multiple types of resonators properly distributed on the host structure could potentially lead to better and/or multimodal performances.

Thus, there is growing interest in the use of distributed local resonators in flexible structures to improve its structural vibration control properties. On the other hand, there is still room for improvement in existing techniques and solutions, particularly in terms of the design and optimization of local resonators parameters and their distribution. This work presents some simple but yet useful analyses on the effect of tuning and damping parameters of distributed local resonators and, also, on strategies to determine potentially good distributions for beam- and plate-type host structures.

FINITE ELEMENT MODELING OF A METASTRUCTURE WITH LOCAL RESONATORS

It is chosen here to first model the host structure using the finite element method and, afterwards, provide the coupling with local resonators. Thus, the equations of motion of the host structure are written as

$$\mathbf{M}_S\ddot{\mathbf{u}}_S + \mathbf{C}_S\dot{\mathbf{u}}_S + \mathbf{K}_S\mathbf{u}_S = \mathbf{b}_S f, \;\; y = \mathbf{c}_S\dot{\mathbf{u}}_S, \tag{1}$$

in which $\mathbf{M}_S$, $\mathbf{C}_S$ and $\mathbf{K}_S$ state for the mass, damping and stiffness matrices of the host structure. $\mathbf{u}_S$ is a vector containing the n generalized nodal displacements (degrees-of-freedom, dof). The excitation input is defined using a distribution vector $\mathbf{b}_S$ and an applied force f. Observation output y is defined in terms of a distribution vector $\mathbf{c}_S$. A uniform modal damping is assumed for the host structure but, to facilitate the coupling with local resonators, it is rewritten in physical coordinates such that

$$\mathbf{C}_S = \mathbf{M}_S\boldsymbol{\phi}\boldsymbol{\Lambda}\boldsymbol{\phi}^T\mathbf{M}_S, \;\; \boldsymbol{\Lambda} = 2\xi\boldsymbol{\Omega}, \tag{2}$$

where the matrices containing the natural frequencies $\boldsymbol{\Omega} = \mathrm{diag}(\omega_1, \omega_2, \ldots, \omega_n)$ and vibration modes $\boldsymbol{\phi} = [\phi_1 \ldots \phi_n]$ are obtained from the undamped eigenproblem of the host structure

$$(-\omega_j^2\mathbf{M}_S + \mathbf{K}_S)\phi_j = \mathbf{0}. \tag{3}$$

The mass-spring-damper local resonators corresponding matrices may be written as

$$\mathbf{M}_R = \mathrm{diag}(m_{r1}, m_{r2}, \ldots, m_{rn_r}), \; \mathbf{C}_R = \mathrm{diag}(c_{r1}, c_{r2}, \ldots, c_{rn_r}), \; \mathbf{K}_R = \mathrm{diag}(k_{r1}, k_{r2}, \ldots, k_{rn_r}), \tag{4}$$

where m_{ri}, c_{ri} and k_{ri} are mass, damping and stiffness coefficients of the i-th resonator. n_r is the number of resonators attached to the host structure and also the number of additional dof in the coupled system. The individual mass and stiffness coefficients of each resonator will be defined later and will be used to provide various distributions of different resonator types over the host structure. The sum of the resonators' masses will also be used to limit the total added mass by the resonators. For the sake of simplicity and to allow some generalization of the results, the damping coefficients are defined based on an equivalent modal damping coefficient, such that

$$c_{ri} = 2\xi_r\sqrt{m_{ri}k_{ri}}. \tag{5}$$

By defining the finite element nodes where the local resonators are attached and assuming that only the transversal displacements of the host structure couple with the resonators' sub-system, the resulting coupled equations of motion may be written in the following form

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_R \end{bmatrix}, \; \mathbf{C} = \begin{bmatrix} \mathbf{C}_S & -\mathbf{L}^T\mathbf{C}_R \\ -\mathbf{C}_R\mathbf{L} & \mathbf{C}_R \end{bmatrix}, \; \mathbf{K} = \begin{bmatrix} \mathbf{K}_S & -\mathbf{L}^T\mathbf{K}_R \\ -\mathbf{K}_R\mathbf{L} & \mathbf{K}_R \end{bmatrix}, \tag{6}$$

in which $\mathbf{L}$ is a boolean matrix of dimension $n_r \times n$ that allows to associate any given local resonator with the corresponding nodal displacement of the host structure. In the particular case studied here of beam- and plate-type structures, the nodal displacements of interest are the transversal (out-of-plane) deflections of the host structure. The matrix $\mathbf{L}$ may be generally written in the form

$$\mathbf{L} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & & & & \cdots & & 0 \\ 0 & & \cdots & & & 0 & 1 & 0 & & \cdots & 0 \\ \vdots & & & & & & & \ddots & & & \vdots \\ 0 & & & \cdots & & & & 0 & 1 & 0 \; \cdots & 0 \end{bmatrix}. \tag{7}$$

Then, the equations of motion for the coupled structure-resonators system read

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{b}f, \; y = \mathbf{c}\dot{\mathbf{u}}, \tag{8}$$

in which $\mathbf{u} = [\mathbf{u}_S^T \;\; \mathbf{u}_R^T]^T$ contains all dof of the coupled system, composed of the nodal displacements of the host structure and of the displacements of resonators' masses. The input and output distribution vectors are defined as $\mathbf{b} = [\mathbf{b}_S^T \;\; \mathbf{0}^T]^T$ and $\mathbf{c} = [\mathbf{c}_S \;\; \mathbf{0}]$.

The frequency response of the system will be used as metric to evaluate the vibration attenuation performance and assess the effect of different parameters. This may be obtained directly from the second-order system (8) or using an equivalent state-space system, such that, for instance

$$H(\omega) = \tilde{y}/\tilde{f} = \mathrm{i}\omega\mathbf{c}\left(-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K}\right)^{-1}\mathbf{b}. \tag{9}$$

APPLICATION TO A BEAM-TYPE HOST STRUCTURE

As a first example, a beam-type structure is considered. First, specific details of the finite element model considered for the beam and its parameters are given. Then, the model is used to perform parametric analyses to assess the effect of mass and damping of the resonators on the vibration attenuation performance. Finally, tuning and distribution strategies of the resonators along the host beam are discussed.

Beam-type host structure description and modeling

A uniform and homogeneous cantilever beam is considered as host structure. It is assumed that point local resonators may be attached to the upper surface of the host beam as shown in Fig. 1. The geometric and material properties of the host beam are defined as: mass density $2700\,\mathrm{kg\,m^{-3}}$, Young's modulus $70\,\mathrm{GPa}$, length $600\,\mathrm{mm}$, width $25\,\mathrm{mm}$, thickness $2\,\mathrm{mm}$, modal damping ratio 0.5%. A point transversal force f is applied at position $15\,\mathrm{mm}$ away from the clamp and the transversal velocity y is observed at the free tip of the beam.

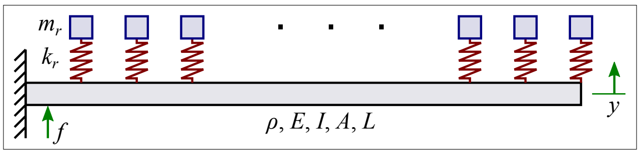


Figure 1: Cantilever beam with uniformly distributed resonators.

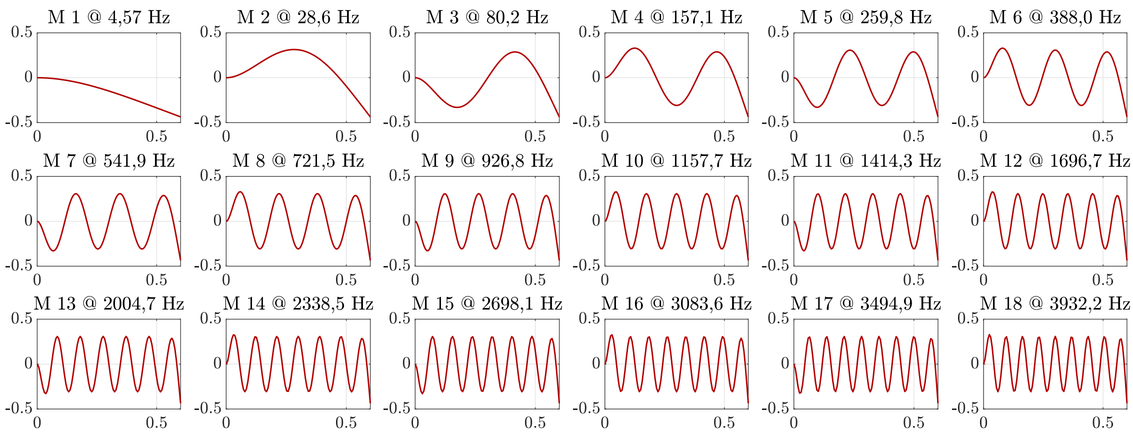


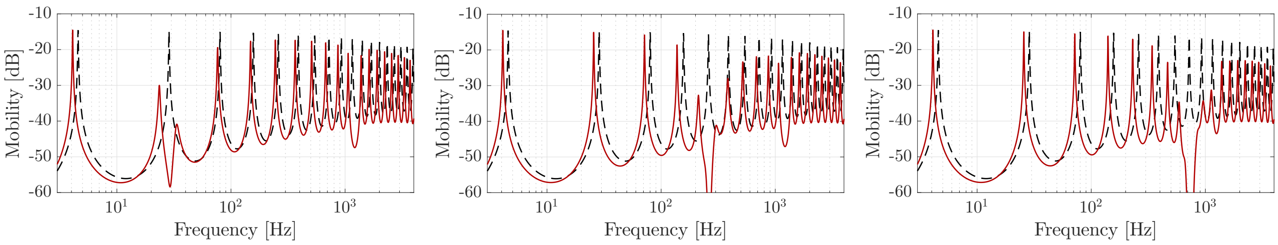
Figure 2: First 18 vibration modes of the cantilever host beam.

The host beam is modeled using Bernoulli-Euler hypotheses and discretized using Hermite cubic interpolation. Following a convergence analysis, 40 finite elements were kept for the subsequent analyses. The first 18 natural frequencies and vibration modes of the host beam are shown in Fig. 2.

Parametric analyses of added mass and damping ratio of local resonators

Ten resonators uniformly distributed along the host beam length and with tunable natural frequency were considered. The resonators masses were defined such that the total relative added mass (resonators mass divided by that of the host beam) may assume values $\mu = [5, 10, 15]\%$. Then, the resonators' stiffnesses were used to tune their natural frequencies accordingly. In all cases, all resonators are assumed to have the same damping ratio which may assume the values $\xi_r = [2, 5, 10]\%$. For the sake of generality, the parametric analyses for resonators' mass and damping ratio were performed for three tuning frequencies at 29, 260 and 735 Hz, arbitrarily chosen to be near some of the first ten natural frequencies of the host beam, namely the second, fifth and eighth (Fig. 2).

The frequency responses of the beam with the ten identical resonators tuned at each selected tuning frequency are shown in Fig. 3, for $\mu = 10\%$ and $\xi_r = 5\%$. It is clear that the resonators affect mainly a narrow frequency range around the tuning frequency. When the tuning frequency is in a low modal density region, as it is the case for 29 Hz, the vibration attenuation is almost only observable very close to the second resonance (Fig. 3a). On the other hand, when tuning at 735 Hz, the vibration amplitude is visibly attenuated in a frequency range that includes the 7th, 8th, 9th and 10th original resonance peaks.



(a) ω_r = 29 Hz (b) ω_r = 260 Hz (c) ω_r = 735 Hz

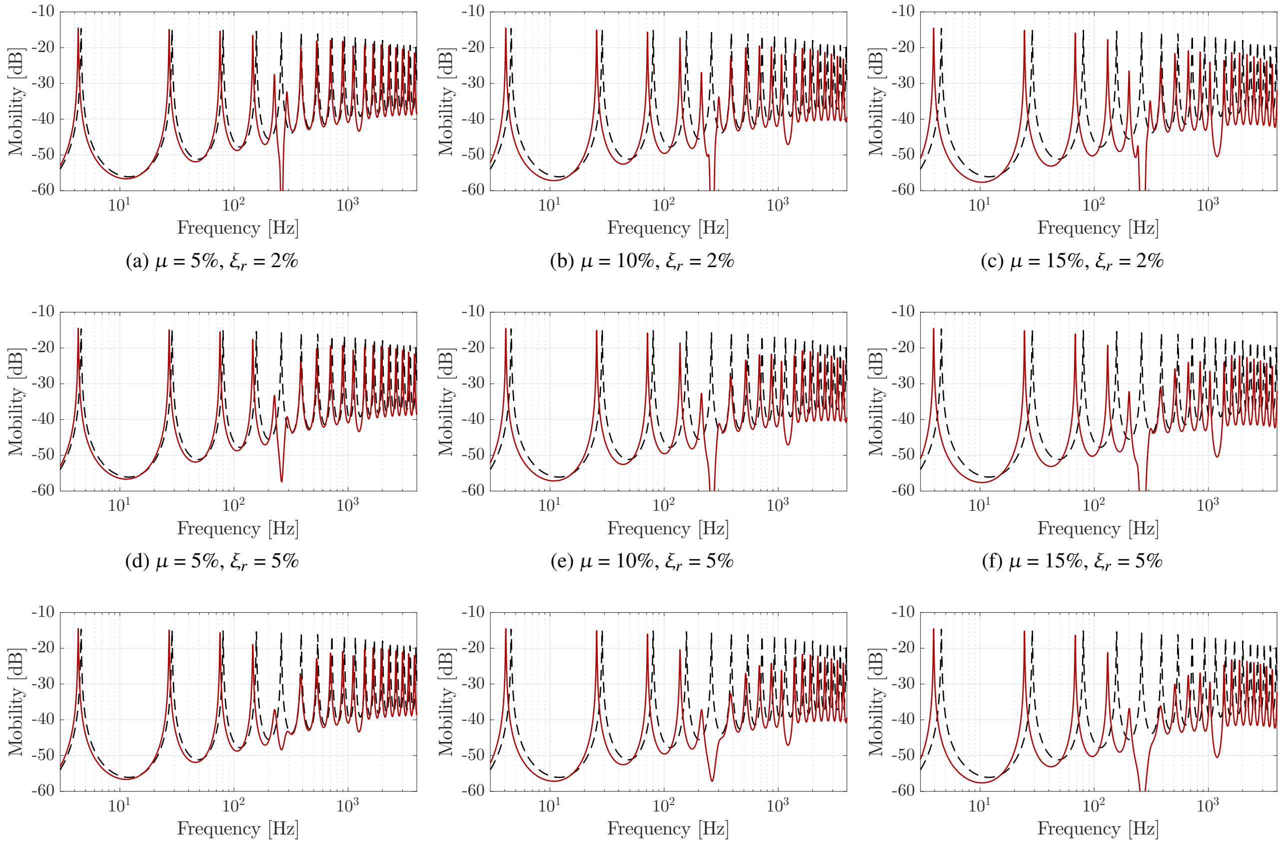
Figure 3: Frequency response of the cantilever beam for different tuning of the uniformly distributed resonators (μ = 10%, $\xi_r = 5\%$).

It is also worth noting in Fig. 3, that an additional bandgap not related to the local resonators tuning appears near 1000 Hz. In fact, this is due to the Bragg scattering phenomena caused by the periodicity in the resonators positioning. This bandgap frequency depends on the number of resonators since this determines the corresponding wavelength. The combination (or proximity) of local resonance and Bragg bandgaps also leads a bridge phenomenon that could be useful, leading to an attenuation of the resonance peaks between these bandgaps.

Next, the effect of resonators' added mass and damping ratio on the vibration attenuation performance was assessed and is shown for the intermediate tuning frequency of 260 Hz in Fig. 4. It is worthwhile to notice that increasing the added mass generally leads to a widening of the frequency range with clear amplitude reduction (bandgap). On the other hand, an increase in the damping ratio yields greater amplitude reduction at the peaks around the bandgap, but decreases the amplitude reduction inside the bandgap. This particularly allows to attenuate the resonance peaks outside the bandgap. Thus, it seems that both parameters should be considered for a satisfactory vibration mitigation within a given frequency range.

Local resonators tuning and distribution strategies for multimodal control

Since various independent local resonators are attached to the beam, it is worthwhile to assess whether it would be interesting to tune them differently aiming at a multimodal vibration attenuation (or multi-bandgap) strategy. This was done here considering that resonators are tuned at either 260 Hz (near fifth original resonance) or 735 Hz (near eighth original resonance). Their distribution can be carried out following various strategies. Here, four distribution strategies are presented. The first three follow a predefined geometric pattern by interchanging resonator types. In the first one, the first five resonators (closer to the clamped end) are tuned to the lower frequency while the remaining ones are tuned to the higher frequency. The second distribution inverts this disposition. In the third one, a periodic higher/lower frequency tuning is repeated along the beam. The fourth distribution, on the other hand, is defined based on the mode shapes of the host beam. The displacement of the two mode shapes (normalized to a unitary maximum displacement) are compared at a given resonator's location and the resonator is tuned to the frequency corresponding to the mode shape that presents the highest displacement at this location. Whenever the displacements are close enough such that definition is harder, the



(a) μ = 5%, ξ_r = 2% (b) μ = 10%, ξ_r = 2% (c) μ = 15%, ξ_r = 2%

(d) μ = 5%, ξ_r = 5% (e) μ = 10%, ξ_r = 5% (f) μ = 15%, ξ_r = 5%

(g) μ = 5%, ξ_r = 10% (h) μ = 10%, ξ_r = 10% (i) μ = 15%, ξ_r = 10%

Figure 4: Effect of relative added mass and resonators' damping ratio on the frequency response of the cantilever beam ($\omega_r = 260\,\text{Hz}$).

choice is made to lead to a better uniformity between the number of each resonator type. That is why, in all four cases, there are five of each resonator type.

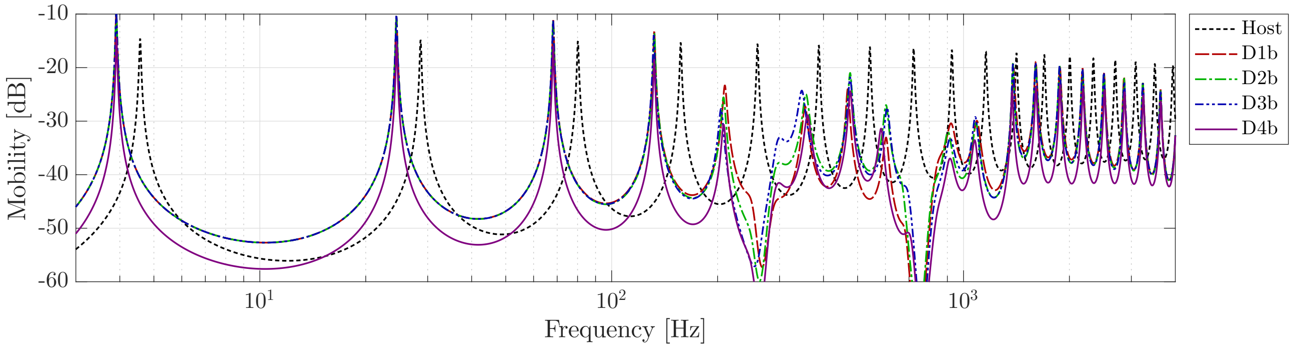


Figure 5: Frequency responses for four different distributions of resonators tuned to modes 5 and 8. D1b: 5-5-5-5-5-8-8-8-8-8; D2b: 8-8-8-8-8-5-5-5-5-5; D3b: 8-5-8-5-8-5-8-5-8-5; D4b: 8-5-5-8-8-5-5-8-8-5.

Figure 5 shows the obtained frequency responses for the four resonators distribution strategies compared to the host beam (without resonators). It is noticeable that all distributions lead to satisfactory reduction in vibration amplitude near the fifth and eighth original resonance peaks, although the peak amplitudes in this frequency range varies depending on the chosen distribution. It is worth noting also that another bridge phenomenon is formed between the 5th and 8th resonance peaks, besides the one already formed between the 8th and 10th resonance peaks due to the Bragg bandgap, leading to a reasonable vibration amplitude reduction is the entire frequency range between the 5th and 10th resonance peaks (approximately 200 to 1200 Hz). Overall, the distribution based on the mode shapes leads to the best amplitude reduction at remaining peaks inside this range.

APPLICATION TO A PLATE-TYPE HOST STRUCTURE

A similar analysis was performed for a plate-type host structure with the main difference that, in this case, the distributions follow bi-dimensional patterns. Also, the damping modeling of the local resonators was changed from viscous to hysteretic by using a complex resonator stiffness.

Plate-type host structure description and modeling

A uniform and homogeneous metallic plate clamped in all edges is considered as the host structure, as shown in Fig. 6. Material properties are the same considered for the beam. Geometrical properties are: length $L_y = 400$ mm, width $L_x = 300$ mm, thickness $h = 3$ mm. The plate is modeled using Reissner-Mindlin hypotheses and discretized using Hermite bi-cubic interpolation for transversal displacements (non-conforming). A mixed interpolation scheme (MITC) is also used to implement the finite element model to prevent shear locking. A regular finite element mesh was predefined such that it is also useful for the regular distribution of resonators that are attached to selected nodes (Fig. 6). For the evaluation of the frequency responses, an input transversal force is applied near the lower-left corner while the output transversal velocity is evaluated at every finite element node, which frequency response amplitudes are averaged and defined as the observed output. Mode shapes and natural frequencies of the host plate are shown in Fig. 7.

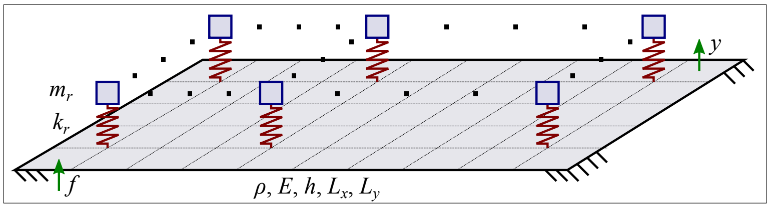


Figure 6: Clamped plate with uniformly distributed resonators.

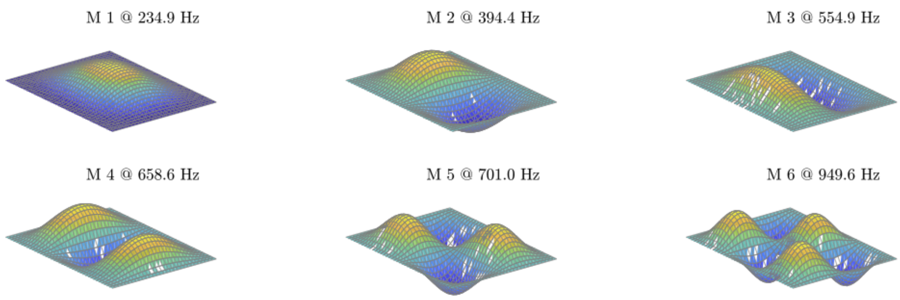


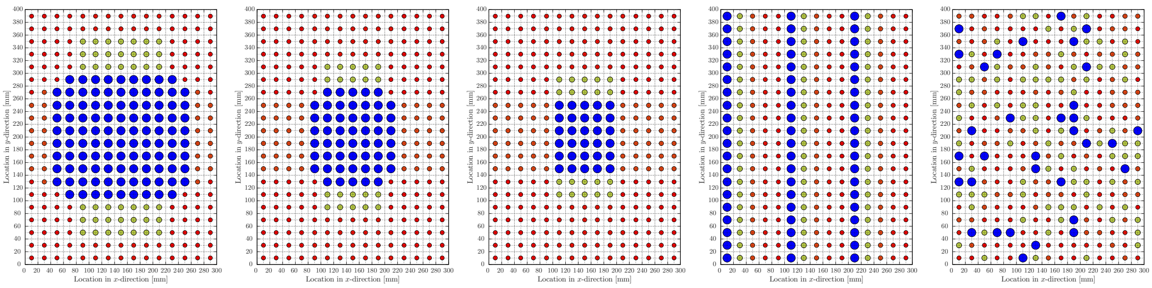
Figure 7: First 6 vibration modes of the host clamped plate.

Different from the beam case, in the plate case, a complex modulus approach is considered for the resonators so that their stiffness are made complex in the form $k_r^* = k_r(1 + \mathrm{i}\eta_r)$, in which η_r is the effective loss factor. Thus, c_r is not used in this case. The approach to obtain the original damping matrices of the host structure are, however, kept unchanged. The loss factors of the resonators are all assumed to be equal and assume the value $\eta_r = 10\%$. The relative mass added by the resonators is defined as $\mu = 10\%$.

Local resonators tuning and distribution strategies for multimodal control

Aiming at a multimodal vibration mitigation, five resonator types are defined, each tuned to one of the first five original resonance frequencies of the host beam (around 235, 394, 555, 659 and 701 Hz). This is done by changing their masses and adjusting the resonators' stiffness to limit the total added mass by the resonators so that $\mu = 10\%$.

Then, the question is how to distribute these resonator types over the available plate nodes. Here, five distribution strategies are presented and compared. The first three are based on the mode shapes of the host structure, differing by the way the mode shapes are normalized before determining the mode shape with maximum displacement at any given resonator's location. In the first one (Modal #1), the mode shapes are normalized by the inverse of the corresponding squared natural frequency. This leads to a higher prioritization of the lower frequencies. The second one (Modal #2) considers a normalization by the inverse of the natural frequencies (non-squared). The third one (Modal #3) normalizes the mode shapes for unitary maximum displacement. As shown in the schematic representation of these distributions (Fig. 8abc), the first distribution indeed presents a higher number of larger masses (not to scale), in the center of the plate, that are tuned to the first original resonance frequency. By changing the normalization, as in the second and third distributions, the number of resonators tuned to the higher frequencies tends to increase. Thus, the third distribution ends up prioritizing the last two of the five target frequencies.



(a) Modal #1, Dm1 (b) Modal #2, Dm2 (c) Modal #3, Dm3 (d) Lines, Dl (e) Random, Dr

Figure 8: Distribution of resonator types over the plate according to different criteria.

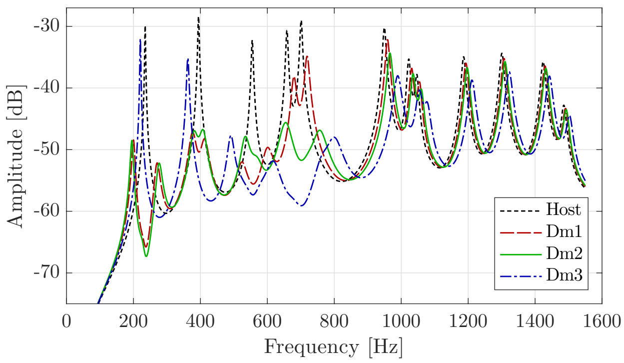


Figure 9: Averaged frequency response of the plate for different resonator types distribution strategies based on mode shapes ($\mu = 10\%$, $\eta_r = 10\%$).

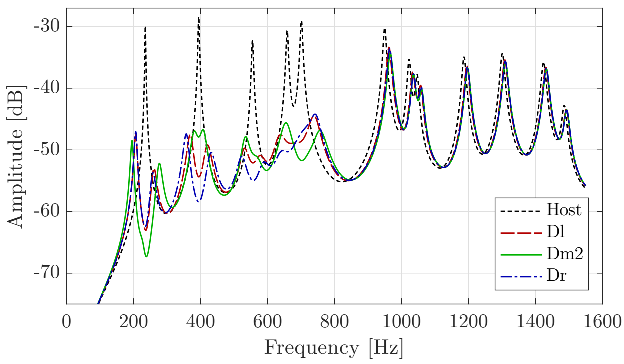


Figure 10: Averaged frequency response of the plate for different resonator types distribution strategies: modal, in lines and random ($\mu = 10\%$, $\eta_r = 10\%$).

The frequency responses for these three distributions are shown in Fig. 9 and, as expected, the first distribution (Dm1) is mainly effective to reduce the first three resonance peaks, while the third one (Dm3) leads to satisfactory reduction only for the last three resonance peaks. The second distribution (Dm2), on the other hand, is able to satisfactorily balance the vibration mitigation over the entire frequency range. Thus, it is suggested here to keep the second approach for the mode shape normalization.

For comparison purposes, two other arbitrary distribution strategies were considered. In the first one, shown in Fig. 8d (Lines, Dl), the resonator types are distributed along vertical lines ordered from higher to smaller masses. Although arbitrary, this distribution leads to the same number of each resonator type which, at least, guarantees a numeric uniformity in targeting the five resonance peaks. Another arbitrary distribution is set by randomly selecting the resonator type at each node, shown in Fig. 8e (Random, Dr). This distribution also leads to highly uniform placement of resonator types. Indeed, as shown in Fig. 10, both arbitrary distributions yield very satisfactory results, even when compared to the modal strategy

(Modal #2, Dm2). It is possible to affirm that there is some advantage in the modal distribution, mainly for the first resonance peak region, but the arbitrary distributions, particularly the random one, perform better at other resonance peaks.

CONCLUSIONS

This work presented some recent results and analyses on the addition of periodic and/or quasi-periodic local resonators to beam- and plate-type structures, including an assessment of the effects of the parameters of the local resonators and of their distributions on the host structures on the vibration mitigation performance in the lower frequency ranges. Results indicate that added mass constraints tend to restrict satisfactory performances to narrow frequency ranges and, for that reason, the effective damping potentially added by the local resonators are essential to provide satisfactory performances over a wider frequency range. For the same reason, it is also shown to be essential to analyze distributions of different resonator types that are tuned to different target resonance frequencies in order to provide a multimodal or broadband vibration mitigation solution.

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