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Ordered SIMP multi-material topology optimization considering volume constraints per material

Otavio Augusto Alves da Silveira[1], Lucas Farias Palma[2]

[1] Center for Optimization and Reliability in Engineering (CORE), Department of Civil Engineering, Federal University of Santa Catarina, Rua João Pio Duarte Silva, s/n, Florianópolis, 88040-900, SC, Brazil.
[2] Center for Research in Geophysics and Geology (CPGG), Institute of Geosciences (IGEO), Federal University of Bahia, Rua Barão de Jeremoabo, Salvador, 40170-290, BA, Brazil.

Abstract. The ordered SIMP interpolation approach is used in this paper to formulate a multi-material compliance minimization problem with volume constraints per material. Originally proposed for a mass and cost constrained design, the ordered SIMP can be modified to account for this type of constraints. One of the motivations for such a modified problem is the possibility of comparison with other multi-material approaches where volume constraints are typically used. The physical meaning of the design variables as normalized mass densities is lost in the approach proposed in this work. The entire design variable interval, [0,1], can be arbitrarily divided between the different material phases. Density filtering with a threshold projection is used to prevent the formation of checkerboard patterns and ensure mesh independence. The effectiveness of the proposed formulation is evaluated through numerical examples and compared with other literature results.
Keywords: topology optimization, multi-material, ordered SIMP, volume constraint

INTRODUCTION

The use of multi-material models in structural optimization studies, specifically topology optimization, opens up interesting possibilities by taking into account the specifics of each material. Among the various studies in multi-material topology optimization, Tavakoli and Mohseni (2014) propose a general and simple-to-implement algorithm to solve this type of problem. The multi-phase problem is divided into a set of two-phase topology optimization sub-problems. An external iteration based on the block coordinate descend method couples the solutions of the sub-problems. Minimum structural and thermal compliance problems are analyzed, and volume constraints per material are taken into account.

To solve multi-material topology optimization problems, Zuo and Saitou (2017) propose an ordered SIMP method. The idea is to define a multi-material model as a function of normalized densities using a series of SIMP interpolations. The technique has the advantage that the number of design variables is independent of the number of candidate materials. The approach is applied to solve structural optimization problems under mass and cost constraints, and it employs the sensitivity filter to avoid the formation of checkerboard patterns.

Sanders et al. (2018) applied the ZPR design variables update scheme, which was originally proposed for ground structures, to a continuum multi-material topology optimization. The method was used to solve problems of compliance minimization with an arbitrary number of candidate materials. Volume and mass constraints can be applied to all materials, a specific material, the entire domain, or sub-regions of it.

Using the ordered SIMP method, Xu et al. (2021) presented a solution to the stress-constrained multi-material topology optimization problem. A new ordered SIMP-like interpolation function is proposed for the interpolation of the relaxed stress measure. Simultaneously, the stress measurement is properly scaled to reflect the various yield strengths. The global stress constraint is defined using the p-norm function in connection with the stability transformation method.

Zhang et al. (2018) proposed a new concurrent multi-scale topology optimization method for cellular structures with various types of microstructures. At the macroscale, different microstructures are treated as different materials. An ordered SIMP-based topology optimization is used to determine the overall structure layout. Each macroscale element is viewed as a separate microstructure on the microscale.

Silveira and Palma (2021) presented modifications to the mathematical equations that define the interpolation of the material model in the ordered SIMP method. Those changes aim to improve the penalization of densities as well as the representation of non-monotonous material properties. Furthermore, due to discontinuities in the sensitivities of the material model, the authors propose the use of density filtering. Furthermore, owing to discontinuities in the sensitivities of the material model, the authors propose the use of density filtering. Finally, the paper employs the threshold projection

of the filtered densities to reduce the number of intermediate densities.

Within the context of multi-material topology optimization applications, this work proposes to formulate a compliance minimization problem with volume constraints per material using the ordered SIMP method. Therefore, this method of solving multi-material problems can be compared to previous studies that uses volume constraints, such as Sanders et al. (2018) and Tavakoli and Mohseni (2014). In addition to this introduction, the paper is organized as follows: the multi-material model proposed by Zuo and Saitou (2017) is briefly described in the following section. Next, we review the modification of the ordered SIMP multi-material interpolation proposed in Silveira and Palma, 2021. The following section defines a minimum compliance multi-material problem with volume constraints per material. A results section provides numerical examples of how the ideas proposed throughout the text can be applied. Finally, the conclusion is presented.

BRIEF INTRODUCTION TO AN ORDERED SIMP

To deal with multi-material topology optimization, Zuo and Saitou (2017) proposed an ordered SIMP formulation. When considering M candidate materials for a multi-material design, the normalized mass densities can be sorted in ascending order as:

$$\bar{\rho}_1 < \cdots < \bar{\rho}_m < \cdots < \bar{\rho}_M \quad \text{where} \quad \bar{\rho}_m = \frac{\rho_m}{\rho_{max}} \quad m = 1, \ldots, M \tag{1}$$

and $\rho_{max} \equiv \rho_M$ denotes the highest density of all candidate materials.

The primary purpose of an ordered SIMP is to describe other material properties based on normalized densities, such as Young's Modulus or cost. The normalized modulus of elasticity of a candidate material can be stated as $\bar{E}_m = E_m/E_M$, and it is crucial to note that in a general formulation, E_M may differ from E_{max}, and this should be true for other material properties as well. Although a flexible heavier material is not a viable candidate for mass constrained compliance minimization, aiming for lighter and more expensive materials is frequent in terms of cost.

Using well-known characteristics of a two-phase penalization based topology optimization (cf. Zhou and Rozvany, 1991 or Bendsoe, 1989), the authors of the original study apply them to an ordered SIMP interpolation. In a two-phase SIMP-based optimization, the elastic modulus decreases at a slow rate when the density variable goes to zero. In addition, elastic modulus increases rapidly as density approaches 1. This behavior tries to favor a stiffer structure. Intermediate densities are unfavorable because the obtained stiffness is small relative to the mass of the material.

The authors provided mathematical formulas that attempted to provide these features to an ordered SIMP. In other words, when the normalized density of a finite element $\bar{\rho}_e$ approaches that of a flexible candidate material $\bar{\rho}_m$, the rate of change of the elastic modulus should be low; and when the normalized density of a finite element approaches that of a stiffer candidate material, the rate of change of the elastic modulus should be large.

Silveira and Palma ordered SIMP

Silveira and Palma (2021) proposed changes to the material model that interpolate the mechanical properties of candidate materials based on a critique of the mathematical expressions proposed by Zuo and Saitou (2017). Despite agreeing on some points expressed in the original work (Zuo and Saitou, 2017), the mathematical formula for the interpolations might be written in a more elaborated manner to address the characteristics addressed. The modified interpolation proposed in Silveira and Palma (2021) will be briefly presented here.

If the modulus of elasticity of a material $m+1$ is greater than the modulus of elasticity of a material m, as is generally expected, the equation for the interpolated normalized modulus of elasticity, $\bar{E}_e$, valid for $\bar{\rho}_e \in [\bar{\rho}_m, \bar{\rho}_{m+1}]$, could be written as

$$\bar{E}_e(\bar{\rho}_e) = a(b(\bar{\rho}_e - c))^p + d \tag{2}$$

where a and b are vertical and horizontal scaling coefficients, respectively, given by

$$a = \bar{E}_{m+1} - \bar{E}_m \quad \text{and} \quad b = \frac{1}{\bar{\rho}_{m+1} - \bar{\rho}_m} \tag{3}$$

with c and d being horizontal and vertical shift coefficients, respectively, given by

$$c = \bar{\rho}_m \quad \text{and} \quad d = \bar{E}_m \tag{4}$$

On the contrary, if candidate material $m+1$ is more flexible than material m, the normalized modulus of elasticity should be interpolated by

$$\bar{E}_e(\bar{\rho}_e) = a(b(c-\bar{\rho}_e))^p + d \tag{5}$$

where the vertical scaling coefficient a is given by

$$a = \bar{E}_m - \bar{E}_{m+1} \tag{6}$$

and the horizontal and vertical shift coefficients, c and d, are given by

$$c = \bar{\rho}_{m+1} \quad \text{and} \quad d = \bar{E}_{m+1} \tag{7}$$

Figures displaying plots of the ordered interpolation of the modulus of elasticity will be presented in the numerical results section to keep the work concise.

MULTI-MATERIAL COMPLIANCE MINIMIZATION WITH VOLUME CONSTRAINTS PER MATERIAL

As previously stated, the ordered SIMP multi-material model was originally proposed to solve a compliance minimization problem subject to mass and cost constraints. The ordered normalized densities, which serve as the design variables in this case, are calculated from the actual mass densities. This paper, on the other hand, considers a multi-material structural problem in which the goal is to minimize compliance subject to volume constraints per material. This mathematical problem can be defined in the following terms for finite elements:

$$\begin{aligned} \min_{\mathbf{x}}: \quad & c(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} \bar{E}_e(x_e)\mathbf{u}_e^{\mathrm{T}}\mathbf{k}_0\mathbf{u}_e \\ \text{s.t.}: \quad & \mathbf{K}\mathbf{U} = \mathbf{F} \\ & g_j = \sum V_j^e \leq V_j^{max} \\ & \mathbf{0} \leq \mathbf{x} \leq \mathbf{1} \end{aligned} \tag{8}$$

where c is the compliance given from the stiffness global matrix $\mathbf{K}$ and nodal displacement vector $\mathbf{U}$, but it is usually calculated from the contributions of each finite element.; $\bar{E}_e$ is the modulus of elasticity, a function of the normalized density, $\bar{\rho}_e$, associated with the finite element e; $\mathbf{u}_e$ is the elemental nodal displacement vector and $\mathbf{k}_0$ is the stiffness matrix of an element with unitary modulus of elasticity. The normalized density, $\bar{\rho}_e$, is a function of the design variable x_e, as will be explained in the next section. The structure is maintained in an equilibrium between the internal forces and the vector of external nodal forces $\mathbf{F}$. Finally, g constraints define the maximum volume allowed for each candidate material, which can be calculated as

$$V_j^{max} = f_j V_T \tag{9}$$

where f_j is the volume fraction for candidate material j. The total volume of the structure, V_T, can be easily calculated by adding the volumes of all finite elements.

$$V_T = \sum V^e \tag{10}$$

It is important to note that the physical meaning of the design variables as normalized mass densities is lost in the approach proposed in this work. As a result, the normalized densities, $\bar{\rho}_m$, that define each candidate material can be chosen arbitrarily and not derived from the actual mass densities. There is no interest in the mass of the structure in terms of constraint in this study, and it is not a dynamic problem.

The contribution of a finite element e with density $\bar{\rho}_e$, contained in the interval $[\bar{\rho}_m, \bar{\rho}_{m+1}]$, to the volume of material m is given by

$$V_m^e = \frac{(\bar{\rho}_{m+1} - \bar{\rho}_e)}{(\bar{\rho}_{m+1} - \bar{\rho}_m)} V^e \quad , \tag{11}$$

and the contribution to the volume of material $m+1$ is given by

$$V_{m+1}^e = \frac{(\bar{\rho}_e - \bar{\rho}_m)}{(\bar{\rho}_{m+1} - \bar{\rho}_m)} V^e \quad . \tag{12}$$

Finally, it is important to note that

$$V_m^e + V_{m+1}^e \equiv V^e \quad . \tag{13}$$

Density filtering and threshold projection

When using density-based procedures, the appearance of checkerboard patterns and mesh dependence are common issues in topology optimization (Sigmund and Petersson, 1998). The use of filters, either on sensitivities or densities, is the most well-known and simplest way to impose constraints on the design and avoid these problems (Bendsoe and Sigmund, 2004). Although receiving little attention in the literature, multi-material topology optimization based on an ordered SIMP also presents these issues.

A density filter is used in the results presented in this paper (Bruns and Tortorelli, 2001). The density filter converts the original densities x_e, i.e. the design variable, into a filtered density $\tilde{x}_e$ by taking a weighted average of the finite element densities that lie within a circle of radius r_{min} centered on the element being filtered. The equations that define this type of filter are well-known in the topology optimization literature (Bruns and Tortorelli, 2001; Sigmund, 2007).

Filters such as densities or sensitivities are known to cause gray transition zones between solid and void components in a two-phase topology optimization. This is also a problem when using an ordered SIMP to optimize multi-material structures. As a result, the final topologies contain a significant number of intermediate-density elements, not just between the lighter solid material and void, but also between multiple solid candidate materials. This paper employs the threshold projection proposed by Xu et al. (2010) to deal with intermediate densities. As in Silveira and Palma (2021), this work uses an adapted version for ordered SIMP based multi-material optimization of the expression rewritten by Wang et al. (2011).

As a result, a specific filtered density $\tilde{x}_e$ will be projected to the limits of an interval $[\bar{\rho}_m, \bar{\rho}_{m+1}]$ depending on whether it is above or below a threshold η that determines a relative location within this interval. The projected density $\bar{\tilde{x}}_e$, which in this study corresponds to the normalized density $\bar{\rho}_e$ and is also known as physical density, is calculated as follows:

$$\bar{\tilde{x}}_e = \bar{\rho}_m + (\bar{\rho}_{m+1} - \bar{\rho}_m) \frac{\left[\tanh(\beta\eta) + \tanh\left(\beta\left(\frac{\tilde{x}_e - \bar{\rho}_m}{\bar{\rho}_{m+1} - \bar{\rho}_m} - \eta\right)\right)\right]}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))} \tag{14}$$

where β is a projection parameter which has its value gradually increased over the iterations of the optimization procedure.

Provided that the normalized density $\bar{\rho}_e$ is replaced by the filtered and projected density $\bar{\tilde{x}}_e$, the sensibilities of the objective function and constraints with respect to the physical densities are given by simple derivatives of the material properties. Its important to note that the projected density $\bar{\tilde{x}}_e$ is a function of the filtered density $\tilde{x}_e$, which is a function of the original design variable x_e, i.e. $\bar{\tilde{x}}_e(\tilde{x}_e(x_e))$.

NUMERICAL RESULTS

This section will present some numerical results for multi-material topology optimization considering the minimum compliance with volume constraints per material problem formulated in this paper. Figure 1 shows the design domain and boundary conditions to an MBB beam. Results will consider two or three candidate solid materials, in addition to void. In all cases, the initial density value is 0.2 (for all finite elements), and the radius of the density filter is 6.

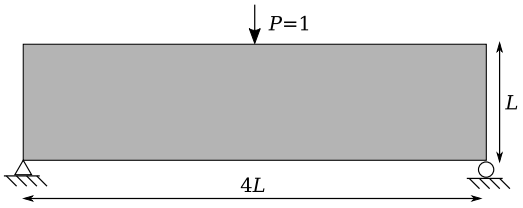

Figure 1 – Domain and boundary conditions to MBB beam.

All problems were optimized with and without projection of the filtered densities. For the solutions where densities were not projected, 100 iterations were performed with the penalty parameter p held constant and set to 3. For the approach with threshold projection of filtered densities, p is initially set to 1 and increased by 0.5 every 20 iterations until the final value equals 5, resulting in a total of 180 iterations. In this case, the β parameter was increased geometrically at each iteration until it reached 10.

MBB Beam

The parameters $L = 240$ and $P = 1$ will be considered for the MBB beam problem. Half of the design domain is discretized using, 240 by 120, 4-node plane isoparametric finite elements, taking into account the symmetry of the structure about the vertical midline. The elastic moduli of the candidate materials are listed in Tab. 1. In the same table, the volume fractions for each material in the cases of two and three solid materials are listed. In the case of only

Table 1 – Material properties and volume fractions

		Void	MAT3	MAT2	MAT1
	$\bar{E}_m$	E_{min}[1]	0.2	0.5	1.0
2 solid materials	f_j	2/4	-	1/4	1/4
3 solid materials	f_j	3/6	1/6	1/6	1/6

two candidate solid materials, MAT1 and MAT2, the total interval of design variables was arbitrarily divided into equal parts, obtaining $\bar{\rho} = [0, 0.5, 1.0]$. Figure 2 shows the ordered multi-material SIMP interpolation of elastic modulus for the designs with 2 and 3 solid materials listed in Table 1.

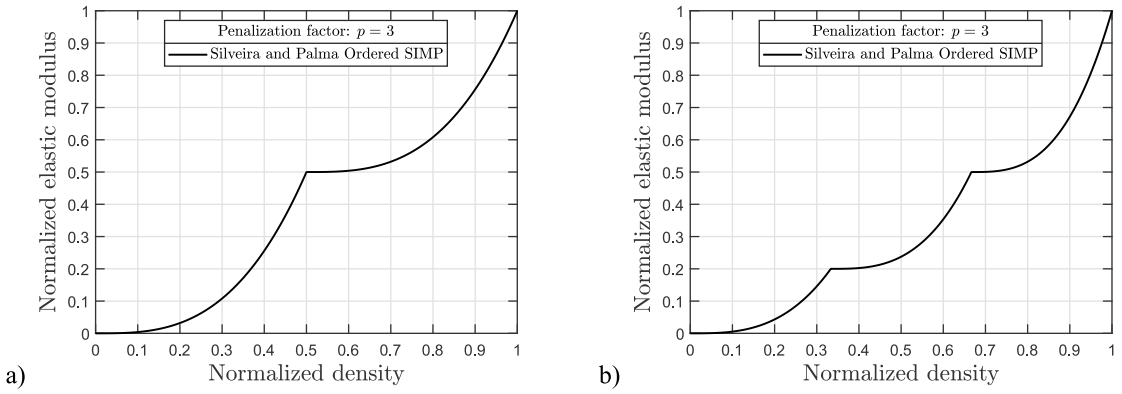

Figure 2 – Ordered multi-material SIMP interpolation of elastic modulus. a) 2 solid materials, and b) 3 solid materials

Figure 3 depicts the optimized topologies of the MBB beam for the analyses that use only density filtering and the solutions that use threshold projection after filtering. This figure also shows the results from Sanders et al. (2018) for a problem with the same geometry, boundary conditions, candidate materials, and volume fractions but different parameters. The topologies obtained in this work are similar to those obtained by Sanders et al. (2018). The projection approach results in a significant decrease in intermediate densities when compared to the approach that only uses density filtering. In terms of the final value of the objective function, the solution without projection obtained a value of 26.83, while the solution with projection obtained a value of 25.91. Sanders et al. obtained a final value of 27.03 for compliance in this case with two solid materials.

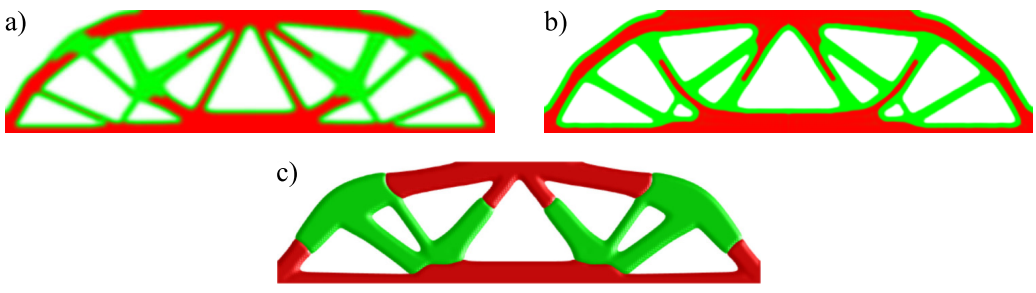

Figure 3 – Optimized topologies of MBB beam considering 2 candidate solid materials. a) Ordered SIMP filtered, b) Ordered SIMP filtered and projected, and c) Sanders et al. (2018) results.

The objective function and volume fractions histories for the analysis without projection of filtered densities and with a constant penalty in the material model are depicted in Figure 4. When evaluating the volume fractions plot, it is important to note that the Method of Moving Asymptotes (MMA) has been set up in such a way that meeting the constraints is more flexible at the beginning of the iterative process. In other words, while the volume constraints must be satisfied at the final topology, they may not be met in the early iterations.

Due to the difficulties in establishing initial values for the design variables, this method of configuring the MMA was chosen.

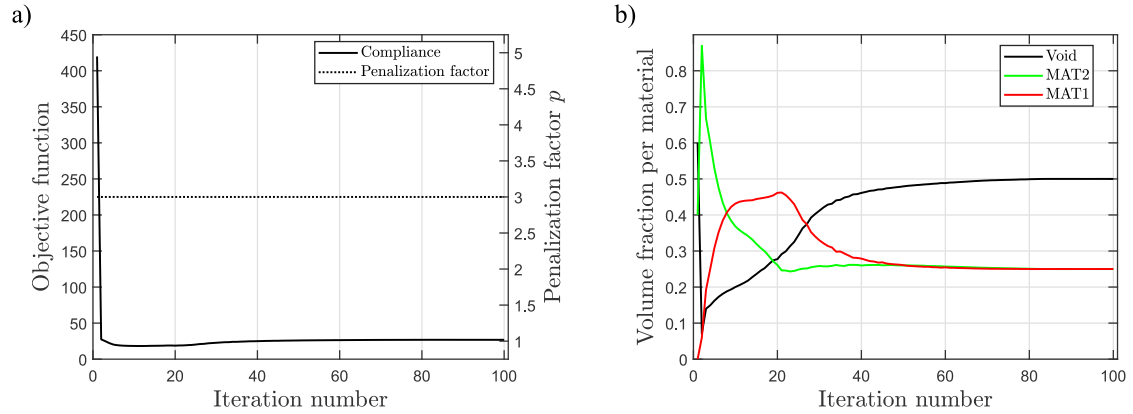

Figure 4 – Results over iterations, using just filtering approach, of MBB beam considering 2 candidate solid materials. a) Objective function, and b) Volume fraction.

Figure 5 depicts the plots of the objective function and constraints (volume fractions) for the analysis that takes into account the approach with threshold projection of densities. After a large initial drop, the objective function shows an increase in value up to about iteration 150, followed by a decrease in later iterations. This non-monotonic behavior is caused by the variation of the p and β parameters throughout the iterative process. The variation of the projection parameter (β) in the form of a geometric progression can be seen in the same image. It is worth noting in the plot of volume constraints that there are abrupt changes in the behavior of the volume fractions when the penalty value is increased (every 20 iterations).

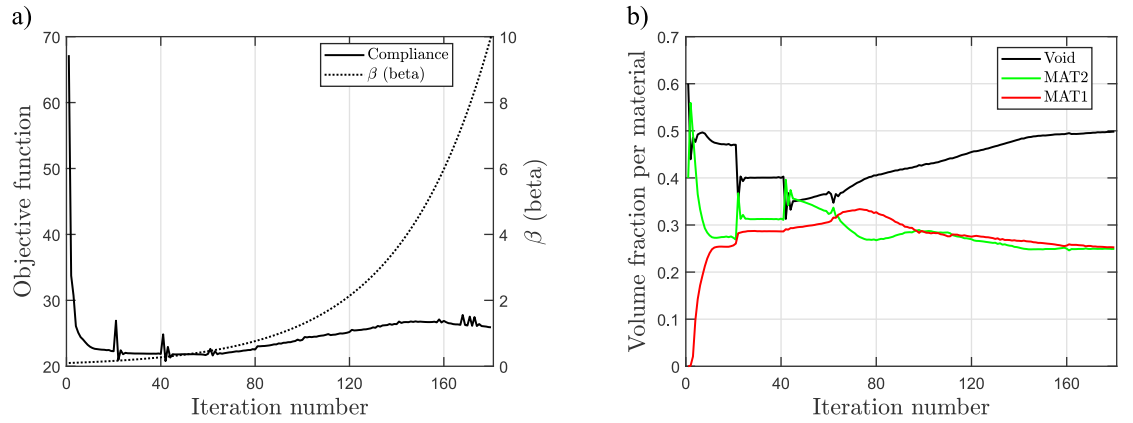

Figure 5 – Results over iterations, using filtering and projection approach, of MBB beam considering 2 candidate solid materials. a) Objective function and β parameter, and b) Volume fraction.

Figure 6 depicts the optimized topologies of the MBB beam when three solid materials, in addition to the void, are considered. The volume fractions and material properties are given in Tab. 1. The normalized densities are set to $\bar{\rho} = [0, 1/3, 2/3, 1.0]$, reaffirming their arbitrary choice. When comparing the topologies to the results of Sanders et al. (2018), it is observed that the approach with projection is more similar than the approach with only density filtering. In terms of length scales, the r_{min} for density filtering is related to the thickness of a material candidate rather than the feature length scale of solid or void phases. As discussed in Silveira and Palma (2021), it is observed here that using

filters of average nature (such as sensitivity or density) with ordered SIMP imposes an additional restriction. The filtering demands the presence of a transition material between a heavier and a lighter material. For example, MAT3 (blue) can be seen between MAT2 (green) and void, or MAT2 (green) between MAT1 (red) and MAT3 (blue). The optimal compliance values for the filtered and projected topologies were 33.54 and 33.28, respectively. The topology obtained by Sanders et al. (2018) for three solid materials has a compliance of 35.18.

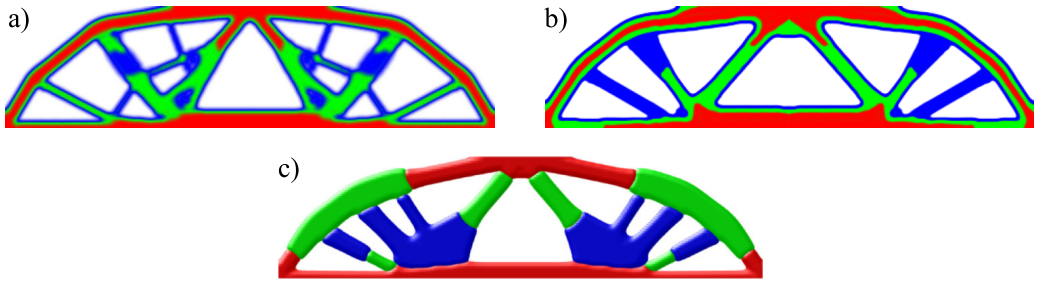

Figure 6 – Optimized topologies for MBB beam considering 3 candidate solid materials. a) Ordered SIMP filtered, b) Ordered SIMP filtered and projected, c) Sanders et al. (2018) results.

Finally, Fig. 7 shows the behavior of volume fractions for the two approaches, with and without projection. Again, despite the volume constraints not being met throughout all the optimization iterations, the final topology satisfies the volume constraints imposed on each candidate material.

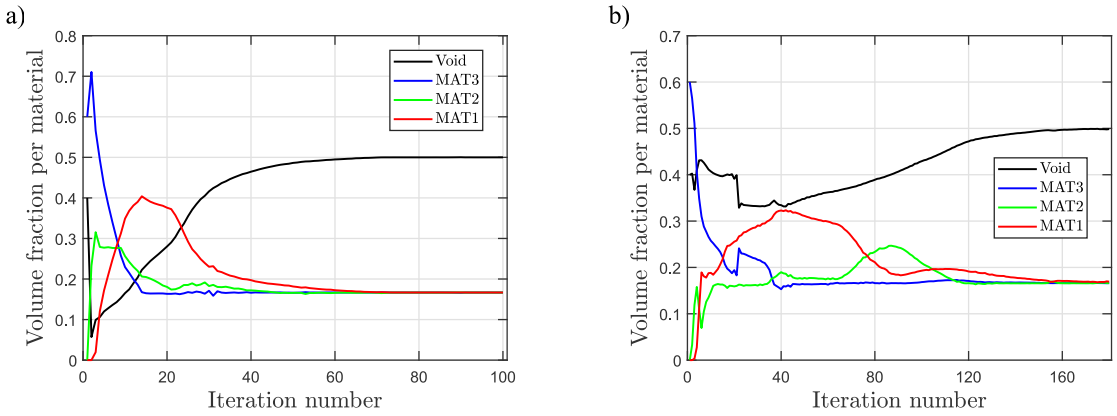

Figure 7 – Volume fractions over iterations. a) Ordered SIMP filtered, b) Ordered SIMP filtered and projected.

CONCLUSIONS

This paper presents the ordered SIMP method for multi-material topology optimization of structures considering volume constraints per material. Results for topologies as well as the histories of the objective function and constraints were shown. Two approaches were considered: by filtering the densities and by projecting the densities after filtering. The optimizations considered the use of two or three solid materials, in addition to void. When using an ordered SIMP method, the average nature of the density filter demands a transition material. When a material volume fraction is small, the need for a transition material can make optimization difficult. The optimal topologies obtained and the values for the objective function are comparable to the results from the literature.

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