MECSOL 2022 - Proceedings of the 8th International Symposium on Solid Mechanics
M.L. Bittencourt, J. Labaki, L.C.M. Vieira Jr. and E. Mesquita (Editors), Campinas SP, Brazil, October 17th to 19th, 2022

Performance analysis of the RILEM constitutive model for Steel Fiber Reinforced Concrete

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Abstract: In a context of inexorably growing engineering challenges, the search for the concomitant achievement of structural safety, economic efficiency and environmental sustainability leads to the development of materials of increasing efficiency. Several studies show Steel Fiber Reinforcement Concrete (SFRC) as a promising material due to its post-cracking behavior related to the enhanced material toughness. Despite its good performance and broad applicability, the literature still lacks a comprehensive constitutive model to precisely describe its tensile behavior. In this work, a cross-sectional analysis approach is employed to determine the post-cracking behavior of SFRC beams, considering the multilinear Fiber Reinforcement Concrete (FRC) constitutive model RILEM TC 162-TDF. The results compared with literature data and statistical analysis demonstrated that the model generally presents overestimated predictions compared to the loading values observed in the experimental campaigns, resulting, in the worst-case scenario (force proportionality limit - Flop) a mean absolute error of about 8.5. Although the residual forces also presented overestimated results, these were on average 3 to 4 times smaller than those found for the Flop, evidencing a higher accuracy of the RILEM model in predicting residual loads.

Keywords: steel fiber reinforced concrete, constitutive model, RILEM recommendation, cross-section analysis

INTRODUCTION

Technologies associated with civil construction are in constant development and evolution. According to Di Prisco, Plizzari, and Vandewalle (2009), the FRC is one of the most relevant innovations in the field of special concrete. Such material may be defined as the association of the concrete with short and discrete fibers randomly distributed. Its application has proven to be economically and productively efficient, given the simplifications provided in the executive processes, allied to cracking and durability improvements (CPH, 2008).

To achieve a more accurate structural design, which would allow disseminate the use of this technique for a larger number of applications, it is essential to have constitutive models that faithfully reflect the behavior of the FRC. However, differently from compression, the tensile behavior of FRC is not alike the response of conventional concrete. As a result, one of the main obstacles is, precisely, the development of a model of constitutive equations that allow adequately characterize the tensile behavior of the material (Álvarez et al., 2010).

The emergence of specific standards for the use of FRC has been a very important tool. These standards provide greater security for the solutions adopted for this type of structure. During the past 20 years, normative guidelines and recommendations that define the basis of FRC have been developed to facilitate its design as well as expand its use. However, among the different existing constitutive equations, there is currently not a single one that adequately characterizes the behavior of the FRC (Álvarez, 2013).

Therefore, this paper aims to compare the results of the multilinear constitutive model proposed by RILEM TC162-TDF with experimental data from 3-point bending tests (3PBT), standardized by EN 14651 and available in the literature. The analysis was performed using a database consisting of 81 series for a total amount of 528 beams subjected to three-point-bending tests performed at the University of Brescia. In addition to this database, two other papers (Salvador (2012), Trindade et al. (2020)) with three series of results each were also used. The comparative analysis presented below focused only on steel fibers given the greater availability of experimental data in the literature. This paper aims to contribute to deepen the structural knowledge of FRC and extend the research focused on the dissemination and evolution of this technique.

METHODOLOGY

In this paper, the section analysis method was used to predict the mechanical behavior of SFRC beams. This method allows to simulate the nonlinear response and describe the cracking, post-cracking and post-failure behaviors of sections built with different materials. As a result, it is possible to obtain the load-crack mouth opening displacement (CMOD) diagrams of the critical sections of the elements under study. To conduct the analysis, it has been assumed that:

(i) Sections remain plane after loading or imposed strains (hypothesis of Navier-Bernoulli);

(ii) Strain compatibility: perfect bond between concrete and fibers;

(iii) Shear distortion and stresses are negligible and, for this reason, were not considered (hypothesis of Euler-Bernoulli);

(iv) Internal forces are applied on the symmetrical axis of the section.

Fig. 1 consists in a schematic representation of the element cross-section, the linear strain and the stress distribution suggested in the section analysis method. In the analysis, it was also assumed that the stresses located in the upper part of the section refer to compressive stresses, while the stresses located in the lower part refer to tensile stresses. The section is analyzed by taking the bottom layer of the section as a reference.

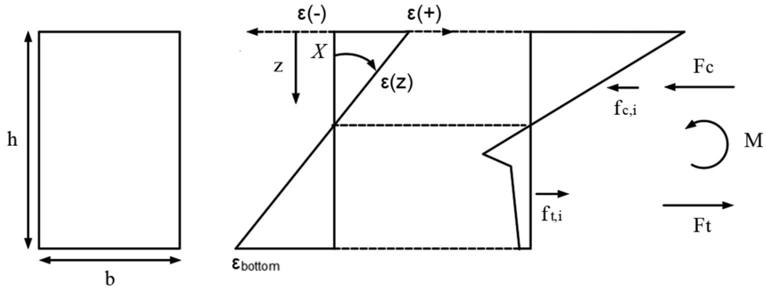


Figure 1 - Schematic representation of the cross-section of a beam and the resulting stress and strain distribution for the RILEM model.

After defining the assumptions, the calculation algorithm is executed based on the analysis of cross-sections. The results that define the flexural behavior of the FRC are obtained analytically. To carry out this study, a sectional analysis model elaborated in Excel was used. The developed program allowed the tests extracted from the literature to be reproduced numerically, thus obtaining the load-CMOD curve for the constitutive equation studied. The calculation procedure, based on conceptual structures similar to those described in previous studies (Yoo and Yoon, 2015; Yoo and Yoon, 2016; Galeote, Blanco, and de la Fuente, 2020), is illustrated in the flowchart in Fig 2.

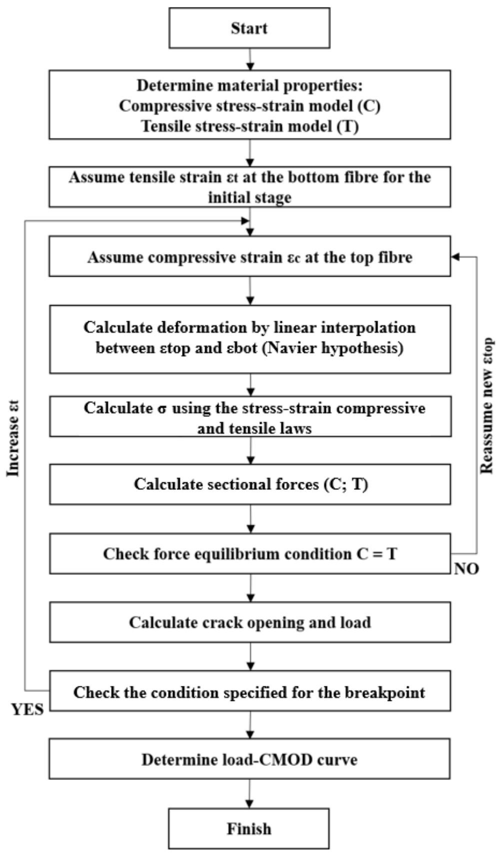


Figure 2 - Flowchart of the analytical cross-section model.

As illustrated in Fig. 2, the model starts by defining the compressive and tensile material properties in light of the adopted constitutive law, which, for the present study, was the RILEM. Afterwards, the section is analyzed taking the bottom layer as a reference, in which a tensile strain is assumed for the initial stage. Then, the compressive strain at the top, as well as the curvature and the strain in any layer, can be calculated according to the combination of the Navier-Bernouilli hypothesis.Thereafter, the stresses corresponding to each strain are calculated with the aid of the predefined constitutive equation. The tensile and compressive forces are calculated by integrating the stress-strain diagram, in order to verify the equilibrium condition. If the equilibrium condition is not satisfied, a new tensile strain is assumed and iterated until the condition is verified. From this step, the bending moment can be calculated by multiplying the resultant of the forces by its respective lever arm and the crack opening at the section. When a result is reached, the tensile strain is increased and the iterative process restarts until the section can no longer establish an equilibrium condition.

The crack opening can be obtained by means of the deformation in the bottom layer and the characteristic length ($w = \varepsilon_{bottom} \cdot l_{cs}$). The characteristic length (l_{cs}), according to Galeote, Blanco and de la Fuente (2020), is an indicator of the crack spacing used in calculations in which the value is influenced by several factors, such as type and volume of fibres, matrix strength, cross-section geometry, load level (service, ultimate) etc. Studies in the literature, as it is the case of De Montaignac, Massicotteand Charron (2012), reveal that there is not a clear consensus to specify l_{cs} and researchers use different criteria to determine its value. However, in this paper, the recommendation of fib Model Code was considered ($l_{cs} = h_{sp}$), being h_{sp} the distance between the top of the notch and the top of the specimen. In accordance with the European beam test, described in EN 14651, h_{sp} should be 125 mm ± 1 mm (see Fig. 3) for specimens with 150 mm height.

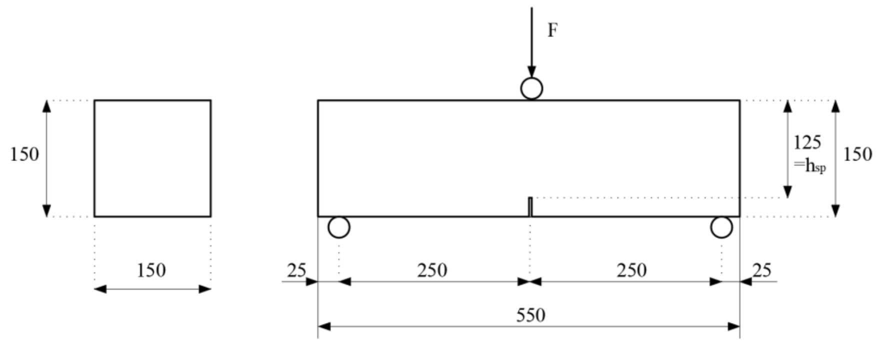


Figure 3 - Position of the notch in the specimen section according to EN 14651 (CEN, 2007).

Although the recommendation of the fib Model Code was used, other recommendations of the value of the l_{cs} are found in the literature, as illustrated in Tab. 1.

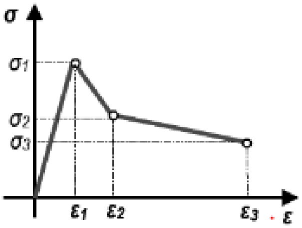
Table 1 – Proposed values for l_{cs} (Montaignac, Massicotte, and Charron, 2012).

Rule	Reference
$\boldsymbol{h_{sp}}/2$	RILEM TC162-TDF
$\boldsymbol{h_{sp}}$	fib Model Code, CNR-DT 204
$2 \cdot \boldsymbol{h_{sp}}$	Strack (2008)

After performing the mentioned procedure, the respective load-CMOD curves were obtained. The solution of each section was integrated to obtain the global behavior of the FRC. Subsequently, a comparison between the results obtained analytically and the experimental results available in the literature was established for different volumes of steel fiber reinforced concrete. This comparison will provide an estimate of how appropriate the constitutive equation studied is to simulate the tensile behavior of the composites under exam.

This paper analyzes the constitutive law proposed by the recommendation of RILEM TC162-TDF. Tab. 2 shows the diagram of the multilinear model proposed by the aforementioned standard and also indicates its respective parameters. The constitutive parameters of the model were calculated with data – concrete and specimen properties – and three-point bending test results from the literature.

Table 2 – Constitutive model proposed by RILEM.

Diagram	Parameters	Reference
	$\boldsymbol{\sigma_1} = 0{,}7 \cdot \boldsymbol{fctm_{fl}} \cdot (1{,}6 - \boldsymbol{d})$ $\boldsymbol{\sigma_2} = 0{,}45 \cdot \boldsymbol{K_h} \cdot \boldsymbol{f_{R,1}}$ $\boldsymbol{\sigma_3} = 0{,}37 \cdot \boldsymbol{K_h} \cdot \boldsymbol{f_{R,4}}$ $\boldsymbol{\varepsilon_1} = \boldsymbol{\sigma_1} / \boldsymbol{E_{HRF}}$ $\boldsymbol{\varepsilon_2} = \boldsymbol{\varepsilon_2} + 0{,}1‰$ $\boldsymbol{\varepsilon_3} = \boldsymbol{\varepsilon_u} = 25‰$	RILEM TC162-TDF

Based on the experimental data collected in the literature, a study of the constitutive equations of RILEM was performed. For this purpose, the numerical model of sectional analysis described above was used, which allows obtaining the load-crack opening curve corresponding to the constitutive equations under analysis. For experimental campaigns, the characterizations of beam elements performed using 3-point bending tests were prioritized. Given this aspect, the experimental reference used in this paper refers to a database composed of results of 3PBT performed at the University of Brescia (Tiberti et al., 2018). This database is the result of 81 series of tests performed for a total of 528 beams. In addition, two other references (Salvador (2012), Trindade et al. (2020)) with three sets of results each were also used.

RESULTS AND DISCUSSIONS

The results of experimental campaigns, taken from the bibliography, were contrasted with the values determined through cross-sectional analysis following the constitutive law proposed by RILEM. Fig. 4 and Fig. 5 present the load-crack opening curves obtained for reinforced concrete with different volumes of steel fibers. In such graphics, the scope of all the results of the experimental tests extracted from the literature were plotted, as well as the curves obtained analytically through the multilinear diagram proposed by the RILEM regulation.

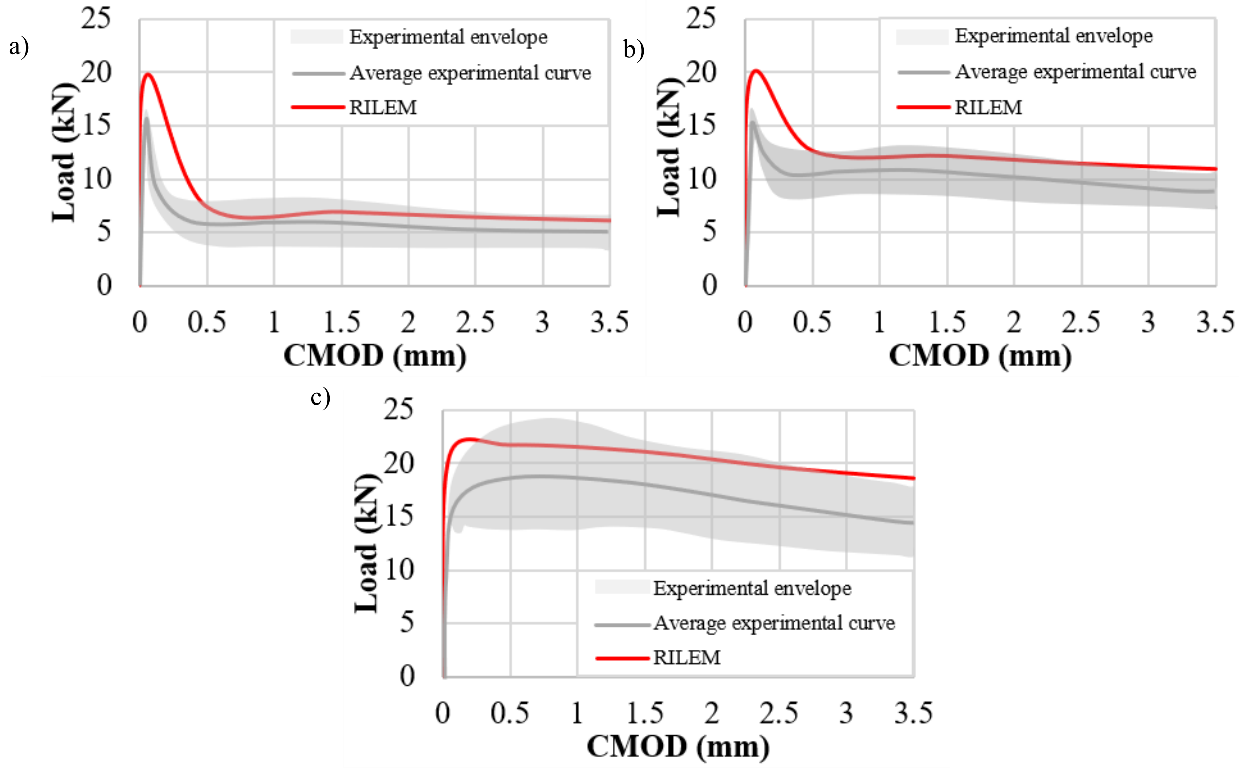


Figure 4 – Load vs. CMOD curves obtained using volumes of steel fibers in the following variations: (a) 15 kg/m³, (b) 30 kg/m³, (c) 45 kg/m³. (Trindade et al., 2020)

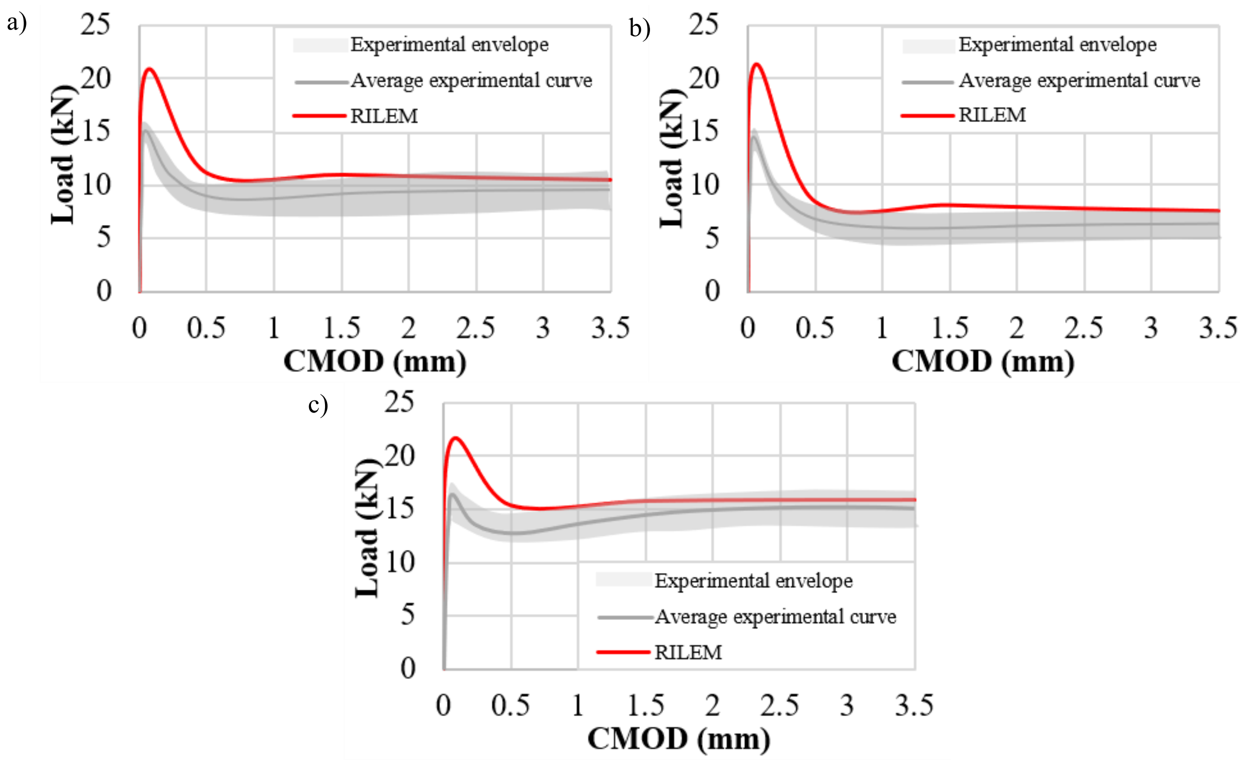


Figure 5 – Load vs. CMOD curves obtained using volumes of steel fibers in the following variations: (a) 15 kg/m³, (b) 25 kg/m³, (c) 35 kg/m³. (Salvador, 2012)

The RILEM model generally predicted loading values that deviate, in an unsafe direction, from that predicted by the experimental results. However, in the comparative analysis presented in Fig. 4 and Fig. 5, it is observed that the RILEM curve approximates the experimental references after peak loading. Furthermore, it is also noted that the constitutive model curve approaches, for all fiber volumes, the upper part of the experimental envelope, thus demonstrating a tendency of the RILEM constitutive law to overestimate the resisted load values. This overestimation of the results becomes even more evident when the high estimated values for the cracking load of the elements are observed. Based on the above, it is evident that the RILEM model presents a certain tendency to overestimate the load values obtained, and this effect is smaller after element cracking. To better understand the difference obtained between the experimental model and the analytical model, a brief statistical analysis was performed in sequence.

To enhance the analysis, residual strength values from 81 tests of FRC beams with varying concrete and fibers characteristics were taken from the work of Tiberti et al. (2018). Next, the collected strength values were transformed into load values, and then the average experimental results obtained by Trindade et al. (2020) and Salvador (2012) were added to this data set (Fig. 4 and Fig. 5). For the five main points ($F_{LOP;}$ $F_{R,1;}$ $F_{R,2;}$ $F_{R,3;}$ $F_{R,4}$) resulting from each of the three-point bending tests, the calculation of the mean absolute error (MAE) was performed. The errors were therefore predicted for the loads corresponding to CMODs of 0.05 mm, 0.5 mm, 1.5 mm, 2.5 mm, and 3.5 mm. Such calculations were performed aiming to better understand the differences between the strength values obtained by the RILEM model and the results from experimental tests for each of the aforementioned crack openings. In this perspective, it is presented in Fig. 6 the illustration of the mean absolute error values obtained for each one of the points mentioned above.

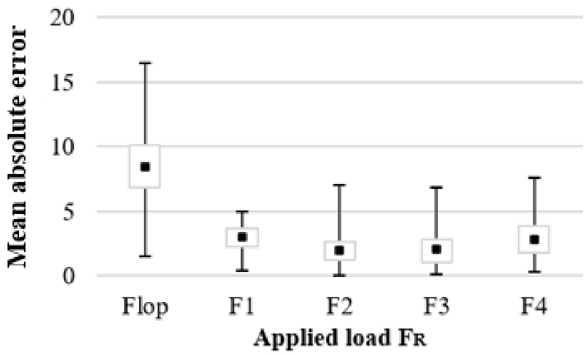


Figure 6 - Mean square error for the load values corresponding to the following CMODs: 0.05 mm (Flop), 0.5 mm (F1), 1.5 mm (F2), 2.5 mm (F3), 3.5 mm (F4)

Figure 6 shows that the force necessary to cause a 0.05 mm crack opening (considered as the limit of proportionality force - Flop) registered the highest average error value, about 3 to 4 times higher than the average MAE value found for the other points. In addition, a significant distance between the minimum point and maximum point, referring to the error of the Flop, about its average error value was noticed. Thus, the RILEM model presents a low accuracy in predicting the Flop value for CRF. This fact is evident not only by the high and varied error values presented in the graph in Fig. 6, but also by the representations indicated in Fig. 4 and Fig. 5, through which it is possible to see that the force value corresponding to the CMOD of 0.05 mm is the point at which the RILEM curve is most distant from the experimental reference.

The other forces predicted by the RILEM model, as mentioned, presented much lower error when compared to those obtained by Flop, especially the forces corresponding to CMODs of 1.5 and 2.5 mm (F2 and F3). Such forces presented an average MAE around 1.5 times smaller than the one obtained for the forces corresponding to the CMODs of 0.5 and 3.5 mm (F1 and F4), and 4 times smaller than the average error obtained for the peak load. Nevertheless, the force that presented the smallest variation between the calculated error values was force F1, which presented its maximum and minimum error values very close to the mean error. The other points analyzed presented error values, especially the maximum value, which were more significantly different from their respective mean errors. Thus, the post-cracking loads presented, in general, lower MAE values when compared to those obtained by Flop. In the same way, the strength values F2 and F3 were the points of better prediction by the model, given the smaller average errors presented.

In order to corroborate with the results found, Álvarez et al. (2010) state that the constitutive equation of RILEM is relatively optimistic for the calculation of SFRC sections, resulting in values that overestimate the experimental results, with emphasis on the peak loading value. Similarly, Álvarez (2013) highlights that the constitutive model suggested by RILEM shows significant differences with the experimental results for small deflections. However, the author also points out that for large deflections, the model accurately reproduces the experimental curves, which was also observed in this study. Finally, it should be noted that, according to Tlemat, Pilakoutas and Neocleous (2006), the overestimation of the results with the RILEM model is due to the use of high values in the parameters that define the post-cracking phase of the constitutive law.

CONCLUSIONS

This paper investigated the constitutive equation proposed by the RILEM recommendation, which is based on three-point bending tests, to predict the tensile behavior of fiber-reinforced concrete. The main objective of this study was to compare the ability of the RILEM model to predict the structural behavior of concrete reinforced with different volumes of steel fibers. For this purpose, the cross-section analysis method based on the Navier-Bernoulli theory and experimental results extracted from the literature were used. The following conclusions may be derived based on the results and the analysis presented here.

- Regarding the cracking load predicted by the RILEM model, the values found were quite optimistic about the experimental studies used as a reference in this study, predicting, on average, twice the experimentally predicted value. Thus, it is observed that, for the prediction of the FRC peak load, RILEM model may lead to results against structural safety, because it significantly overestimates the value of the force required for cracking of the material. Furthermore, the MAE results found in the tests, besides being very high compared to the other points, showed a large variability;
- It was found that the results obtained by the RILEM model, for the residual loads, are closer to the experimental campaigns than those obtained for the Flop value. Although there is a better match, the curve for the residual loads is close only to the highest load values found experimentally, which would also lead to results that are not very conservative and against safety. However, it can also be seen from the analysis of the calculated MAE values that the RILEM model presents a greater assertiveness in the calculation of the residual loads, since these presented a MAE that, on average, was 3 to 4 times smaller than that obtained for the peak load value. Thus, it is verified that despite still presenting overestimated values in the experimental model, the RILEM model presents more accurate results for predicting the loads that occur after cracking, with an emphasis on the forces referring to CMODs of 1.5 and 2.5 mm (F2 and F3), which presented the lowest mean errors.

In conclusion, it should be reiterated that there is currently a relevant normative basis that aims to support and promote the use of FRC. There is not a single constitutive equation that best represents all the situations to be designed with fiber-reinforced concrete. In the case of the RILEM equation, it was found that this standard results in load values that were mostly high in the experimental campaigns studied here. Therefore, it is necessary to perform an analysis similar to the one presented in this paper for the other existing models, to verify their ability to predict the loads that occur in real FRC elements.

It is also worth reiterating that the aforementioned results were obtained by adopting a characteristic length according to the fib Model Code regulations. Nevertheless, it is known that this parameter may influence the analysis performed. In this perspective, it is recommended to carry out this same study, but varying this parameter according to what the other technical recommendations define to assess its influence. In addition, there are also other regulations that present recommendations of constitutive laws that aim to predict the behavior of the FRC. These may be investigated, in future works, to seek a better fit for the points of the load-crack opening graph that did not demonstrate good proximity to the equations analyzed in this work.

ACKNOWLEDGMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001. The authors also wish to express their gratitude and sincere appreciation to the Graduate Program in Civil Engineering at the Federal University of Parana for enabling and encouraging research projects related to fiber-reinforced structures.

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