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Localization of vibration modes in aeronautical turbine blades

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In this work, the phenomenon of localization of vibration modes in quasi-periodic structures with linear behavior is studied. In particular, it is applied in the field of Aeronautical Engineering, in aeronautical gas turbine blades, which have periodic characteristics, weakly coupled to each other through the rotor disk, taking into account possible small imperfections. Linear systems, faced with the disorder caused by small imperfections can lead to the confinement of vibrational energy in certain regions of the structure, a phenomenon known as Mode localization. This phenomenon can cause catastrophic failure due to high vibration amplitude and fatigue. The identification and study of the localization effect from a modal perspective, as well as the response of the structure and its components to dynamic load, is of fundamental importance, as it is a diagnostic tool for possible preventive mitigation actions or even use of this phenomenon in damping of the system. Through the implementation of computer simulation via Matlab software, based on the Finite Element Method, the distribution, interference, and consequence of vibrational energy on the adopted model is verified based on periodic and ordered dynamic characteristics or aperiodic and disordered. The real case considers small variations of the characteristics (length, rigidity, angulation), resulting from manufacturing tolerances or FOD (Foreign Object Debris) impact. The proposed implementation graphically displays the normalized vibrational mode amplitude as a consequence of the emergence of the phenomenon of localization of vibration modes in substructures that may be restricted to one or a few blades of the structure.
Keywords: localization of modes, vibrational energy, turbine blades, finite element method

INTRODUCTION

The study of vibrations is found in several branches of engineering, and recurrently in aeronautics. Even under optimal conditions, any aircraft engine naturally suffers the effect of vibration resulting from its dynamic behavior. To ensure the safe operation of modern turbofan units, it has become essential to assess their vibrational conditions. The main concern in turbines blades are the large dynamic loads applied to the substructure due to vibrations and high rotational speeds. This can lead to a destructive scenario, with deterioration of the systems, fatigue cracks, and failures.

The symmetry of a turbine is a characteristic that defines it as a periodic and ordered structure, due to its composition of identical substructures, i.e., they should have the same stiffness, mass etc., coupled by the turbine shaft. These substructures, the blades, may not be identical structures due to manufacturing imperfections, FOD (Foreign Object Debris) collisions, among other conditions that can change the dynamic characteristics of these blades. These variations may lead to the blades deviating from their nominal design, which is called mistuining. Dynamic systems are very sensitive to structural parameters, since natural frequencies and vibration modes are functions of the geometric characteristics and properties of the materials that compose them.

Whereas the ideal model distributes the vibrational energy through all the components of the system, when mode localization occurs, vibrational energy may be confined to just a few or even one component. Thus, design and maintenance analysis must include effects of the random or deliberate presence of disorder in their characteristics which results in a phenomenon known as vibration mode localization. This phenomenon was first described by Anderson (1958) in the context of Solid Physics a work that earned him a Nobel Prize.

Dye and Henry (1969) developed an approximated equation for the turbine blades response, using a discrete parameter model. El-Bayoumy and Srinivasan (1975) concluded that the blade stress was frequency dependent, and Ewins (1969) performed a theoretical-experimental study and found a good agreement between the calculated and measured frequencies and mode shapes. Bendiksen and Valero (1987) investigated the case of the mode localization for unsynchronized cyclical symmetrical structures. The mode localization phenomenon in periodic structures of linear behavior was extensively discussed by Reyolando MLRF Brasil and Mazzilli (1995) also worked on the subject.

The classic mistunings affect usually mechanical systems. They are length, stiffness, and mass, arising from manufacturing tolerances. This work adds the occurrence of mistuning as a result of blade twist angle deviation and mistunings caused by a blade impact situation with a FOD - Foreign Object Debris. The study of the origin of mistuning initiated by Armstrong (1956) and Tobias and Arnold (1957). Whitehead (1966) focused on the influence for the forced vibration behavior of systems with blades. Authors Ewins (1969) and Dye and Henry (1969) followed the same rationale. Authors Yuan et al. (2017) and Castanier and Pierre (2006) present a recent work with a literature review on the problems of mistuning in systems of a blisk, and its implications for the propagation of uncertainty associated with dynamics of aeronautical engine systems.

Pierre and Murthy (1992), Madden, Epureanu, and Filippi (2012), Chan and Liu (2000), Duffield and Agnes (2001), Fang et al. (2006), Repetckii, Nguyen, and Ryzhikov (2017),Rodriguez and Kauffman (2019), Baker and Capece (2002) who addressed the vibration localization modes due to high degree of mistuning of mass, stiffness and geometry. R. Brasil and Hawwa (1995); Dhar and Sharan (1997);Fang et al. (2006) put forward the mistuning of rigidity.

Dye and Henry (1969); El-Bayoumy and Srinivasan (1975), Bendiksen and Valero (1987), Hemberger, Filsinger, and Bauer (2014), Capiez-Lernout et al. (2005) highlighted mis tuning due to manufacturing tolerances or different blade attachments that cause abnormal stresses due to vibrational confinement. Finally, authors Judge, Pierre, and Mehmed (2001), Reyolando MLRF Brasil and Costa (2007), Chen et al. (2019) discuss the effects of varying the magnitude of mass and its position in the system, generating energy confinement of vibrational energy.

In this work, we develop an algorithm to analyze the localization of vibration modes in periodic or non-periodic structures, particularly in turbofan blades, simplifying them as a cantilever beam with light coupling, their dynamic properties approximated by Rayleigh's Method, also considering the effect of deviation on length, stiffness, and angle of twist caused by impact with an FOD.

Case Study:CFM56

The study of the phenomenon of localization of modes is applied to the fan's blade of an aeronautical turbine, the CFM56 engine was chosen as a case study. The CFM56 is a family of aeronautical engines with turbofan reaction technology that uses gas turbines, produced by CFM International, adopted in aircraft such as the Boeing 737, the Airbus A320, 340-300. About fan blades, Center (2003) informs that there are 44 titanium alloy blades. The adopted alloy is used for its high strength and weight saving characteristics. The geometric dimensions of the blades from CFM56-2 are showed by Lane (1989), see Fig. 1, $L = 0.5999$ m, $W_1 = 0.1161$ m, $W_2 = 0.1470$ m, $T_1 = 0.075$ m, $T_2 = 0.0040$ m.

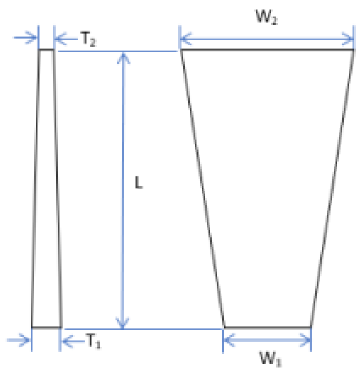


Figure 1: *Fan* blade geometry (no pitch, attack or camber)

Vibrational Dynamics

The well-known system of linear ordinary differential equations of motion of a discrete mechanical system is

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\mathbf{u} + \mathbf{K}\dot{\mathbf{u}} = \mathbf{P} \quad (1)$$

where $\mathbf{u}$ is a displacement vector *nx1*, $\dot{\mathbf{u}}$ is a velocity vector *nx1*, $\ddot{\mathbf{u}}$ is an acceleration vector *nx1*, $\mathbf{M}$ is the mass matrix *nxn*, $\mathbf{C}$ is the damping matrix *nxn*, $\mathbf{P}$ loading vector and $\mathbf{K}$ stiffness matrix *nxn* given by Eq. (2)

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_G \quad (2)$$

where $\mathbf{K_0}$ constant initial elastic stiffness matrix *nxn* and $\mathbf{K_G}$ geometric matrix *nxn*.

The focus of this paper is the study of free vibrations, not forced vibrations whose amplitudes would be affected by the amount of damping present. Real aeronautical structures are usually very under critically damped, with ratios lower than 1%, Clough and Penzien (1993). These ratios will not affect the obtained undamped modal shapes. A particular case of Eq.(1) called undamped free vibrations, is presented in Eq. (3), where damping and loading are neglected, so that the system is set in motion only due to the initial conditions. Thus, we have a homogeneous differential equation of motion:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = 0 \tag{3}$$

The harmonic response of the system given by Eq. (3), that is, the undamped free vibration modes are of the form $\hat{\mathbf{u}}$, which considers that all coordinates of the system vary harmonically in time, at the same frequency, called damped vibration frequencies and in the same phase,

$$\mathbf{u} = \hat{\mathbf{u}}\cos(\omega t + \theta), \tag{4}$$

where θ is the phase angle and ω is one of the n undamped circular frequencies of the system.

Submitting to Eq. (4) the double derivation in time and substituting in Eq.(3), we have the system of homogeneous algebraic equations:

$$(\mathbf{K} - \mathbf{M}\omega^2)\hat{\mathbf{u}} = 0, \tag{5}$$

a linear algebraic eigenvalue and eigenvectors problem.

In the system studied, the set of blades of an aeronautical turbine, the modeling is simplified to 10 substructures, so the stiffness matrix $\mathbf{K}$ has the generic form given by Eq. (6):

$$\text{K=} \begin{bmatrix} K_1+k & -k & & & & & & & & \\ -k & K_2+k & -k & & & & & & & \\ & -k & K_3+2k & -k & & & & & & \\ & & -k & K_4+2k & -k & & & & & \\ & & & -k & K_5+2k & -k & & & & \\ & & & & -k & K_6+2k & & & & \\ & & & & & -k & K_7+2k & -k & & \\ & & & & & & -k & K_8+2k & -k & \\ & & & & & & & -k & K_9+2k & -k \\ & & & & & & & & -k & K_{10}+k \end{bmatrix} \tag{6}$$

where K_p (p = 1, ,10) are the equivalent stiffnesses of each of the substructures, obviously equal to each other for the tuned case, and k corresponds to the stiffness, of small magnitude, of coupling between the substructures. Matrix $\mathbf{K}$ is formed, see Eq. 2, by the matrix $\mathbf{K_G}$, the geometric stiffness matrix depending on the axial load applied to the system. In our case, the centrifugal inertial force has a stabilizing effect in matrix $\mathbf{K_G}$, increasing stiffness and frequencies.

Rayleigh's Method

The system being investigated consists of aeronautical engine turbine blades. This is a cyclic problem, i.e, it does not start or end in a certain substructure. Mathematical modeling simplified the model as an "open" clamped beam structure In this work, we use Rayleigh's method to approximate the dynamic characteristics of the variable section blades.

Rayleigh (1896) is a forerunner of the Finite Element Method, both discretization technique of continuous media. The method proposes that the exact solution displacement function $u(x)$ be replaced by an approximate function $\phi_i(x)$, called shape function. Thus, Eq. 3 is replaced by a single degree of freedom ODE:

$$M\ddot{q}(t) + K_0 q(t) - K_G q(t) = 0, \tag{7}$$

where M, K_0 and K_G are the mass and generalized stiffnesses, and $q(t)$ is the single generalized coordinate that multiplies the shape function..

In Rayleigh's method, the choice of shape function is arbitrary, provided it satisfies the geometric boundary conditions. The adopted cubic function is the exact solution for a prismatic cantilever beam and a reasonable basis for our models. Eq 8 gives the cubic shape function.

$$\phi = \frac{3x^2}{2L^2} - \frac{x^3}{2L^3} \quad \phi' = \frac{3x}{L^2} - \frac{3x^2}{2L^3} \qquad \phi'' = \frac{3}{L^3}(L-x) \tag{8}$$

The generalized mass is given by

$$M = \int_0^L \rho A \phi^2 dx \tag{9}$$

where ρ is the density, $A = wt$ is the area, with the plate thickness $t = t_1 - \frac{t_1 - t_2}{L}x$ and the plate width $w = w_1 - \frac{w_1 - w_2}{L}x$, as shown in Fig. 1.

Substituting the variables, the mass of this model is given by

$$M = \frac{\rho L}{10080}[(1630t_2 + 305t_1)w_2 + (305t_2 + 136t_1)w_1] \tag{10}$$

The generalized elastic stiffness is given by

$$K_E = \int_0^L EI(\ddot{\phi})^2 dx \tag{11}$$

Substituting the variables, the elastic stiffness matrix of this model is given by

$$K_E = \frac{E}{560L^3}[(4t_2^3 + 9t_1t_2^2 + 12t_1^2t_2 + 10t_1^3)w_2 + (3t_2^3 + 12t_1t_2^2 + 30t_1^2t_2 + 60t_1^3)w_1] \tag{12}$$

and the geometric stiffness matrix, in turn, is given by

$$K_G = \int_0^L F(x)(\dot{\phi})^2 dx \tag{13}$$

where the normal force (tension) in each section is $F(x) = \rho A \omega^2 \frac{(L^2 - x^2)}{2}$

Substituting the variables, the geometric stiffness matrix of this model is given by

$$K_G = \frac{\rho \omega^2 L}{2240}[(236t_2 + 133t_1)w_2 + (133t_2 + 146t_1)w_1] \tag{14}$$

The total generalized stiffness of the system, considering the normal force (tension) positive, is then

$$K = K_E + K_G \tag{15}$$

Frequency expression that takes into account the influence of axial force, in Hertz

$$f = \frac{1}{2\pi}\left(\frac{K}{M}\right)^{\frac{1}{2}} \tag{16}$$

METHODOLOGY

In this paper, ”mistuning” is considered as the small changes in physical properties of the substructures, the turbine themselves, that lead to significant mode localization. The system undergoes the mistuning in length, mistuning stiffness, mistuning stagger angle, and mistuning due to impact with a FOD. As considered, in order to display the phenomenon, a random choice between $\pm 7.5\%$ of nominal values of those properties was adopted, as shown in Fig. 2. Length mistuning is performed by entering a $\pm\Delta l$ at the nominal length l. The stiffness mistuning occurs with introducing a $\pm\Delta E$ in the elastic modulus E. The mistuning of the blade's stagger angle occurs with the introduction of a $\pm\Delta I$ in the inertia I. The magnitude of all disturbances is randomly generated.

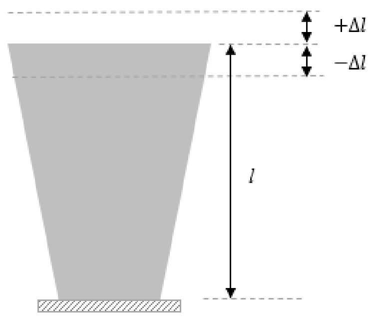


Figure 2: Introducing a $\pm\Delta$ in the nominal features, in this case a length mistuning.

Based on the characterization of each substructure using Rayleigh's Method and the Finite Element Method, we define the localization of the vibration modes in aeronautical turbine blades, using MATLAB, which is a high performance interactive software for matrix analyses. The analysis of the problem involves the construction of specific codes for each deviation, but there are fundamental steps inherent to all codes. They are: input data; definition of ideal or actual analysis; definition of the random vector of disturbance; generation of the local stiffness matrix for each element of the structure; generation of the global stiffness matrix of the structure; generation of the local mass matrix for each element of the structure; generation of the structure's global mass matrix; solving the problem of eigenvalues and eigenvectors (natural frequencies and vibration modes) and output results.

NUMERICAL RESULTS

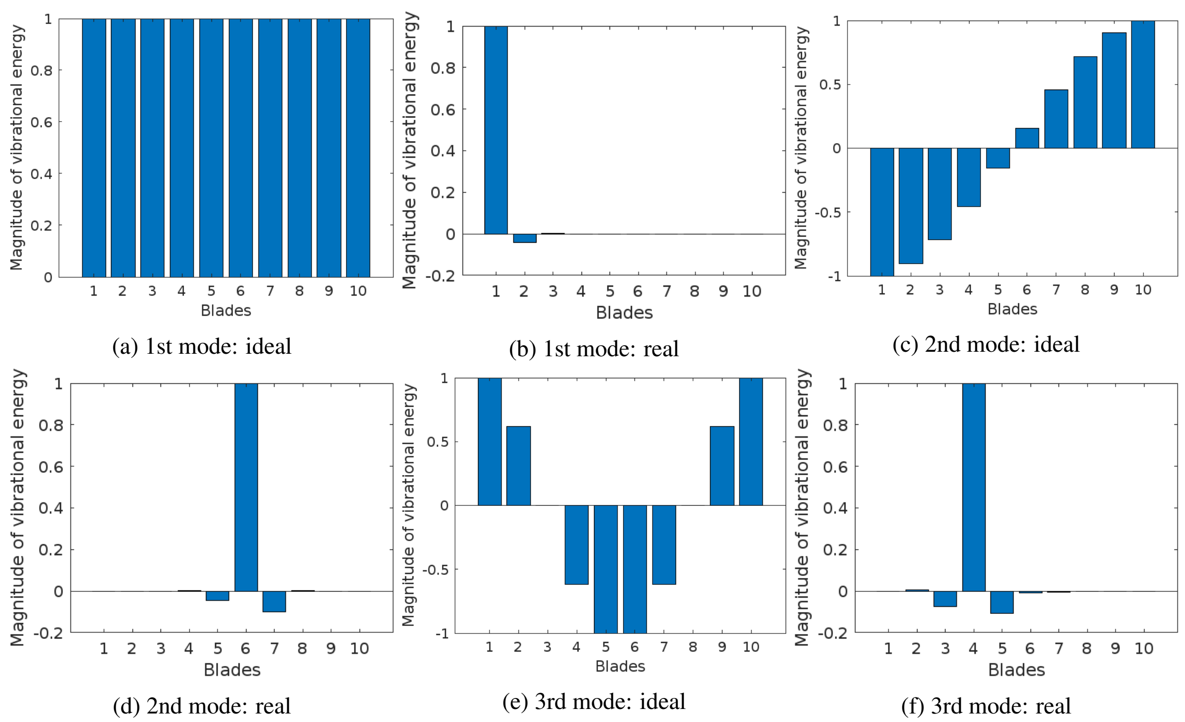
The simulation, for the ideal or real analysis, introduces the cases of mistuning of length, stiffness, angle, and impact with FOD, in the system, through an algorithm. Results show that when the substructures are the same, without deviation, the vibration modes are homogeneously extended throughout the system, and the frequencies are repeated with approximately the same values for each of the substructures. When small mistuning and a slight coupling are present, vibration modes are localized in some or even in one of the substructures, and the respective frequencies change significantly between substructures, as shown in Tab. 1.

The graph results display normalized modal displacements of the 10 blades of the fan, for the three first vibration modes, for ideal (perfect tuning) and real (mistuning) situations. In both Figures, for each case, non-mistuning shows replicability of results despite particularities in each algorithm and symmetrical or anti-symmetrical patterns from a source blade. Fig. 3 shows the phenomenon under length mistuning and non-mistuning. This mistuning shows a strong confine vibration in comparison with the other cases. Fig. 4 displays that the system undergoes the stiffness mistuning and non-mistuning. Fig. 5 demonstrates stagger angle mistuning and non-mistuning. Fig. 6 indicates the phenomenon of localization originated from debris collision.

The confinement factor of vibrational energy is the relational ratio between the disorder of the system and the degree of coupling. The coupling factor is constant in each case, isolating the effect of disorder on the results. It could infer that the length, compared to the others, has the highest disorder force, and therefore the highest degree of localization, while the impact with FOD has the lowest degree of confinement. The difference in frequency between the blades is most affected in the case of angle mistuning.

Table 1: Numerical results for frequency [Hz] in the case of mistunings of length, stiffness, angle and impact with FOD

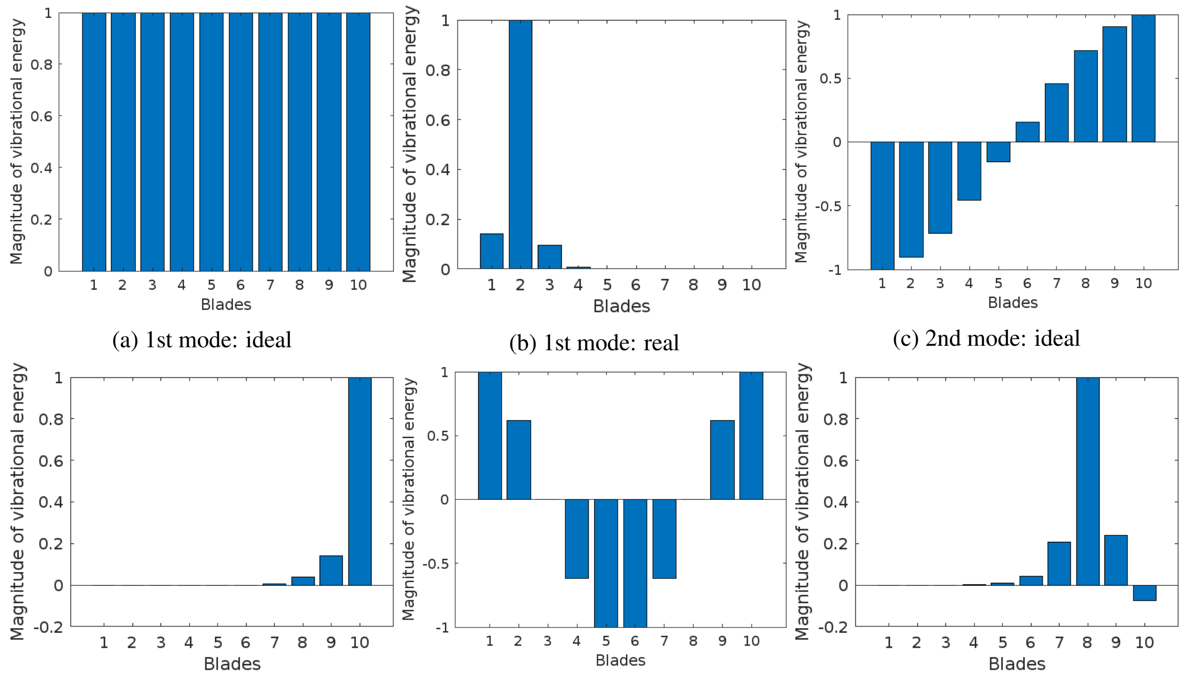
Blade	Without mistuning	Mistuning of the length	Mistuning of the stiffness	Mistuning of the angle	FOD
1	115.5833	105.2160	113.8734	287.3656	113.6947
2	115.6805	130.4781	114.3108	288.9083	113.8934
3	115.9621	111.6699	116.8424	300.5923	116.1127
4	116.3993	128.0449	117.1173	301.6433	117.7362
5	116.9478	113.7640	118.6106	311.7201	117.9414
6	117.5529	123.6882	119.9809	315.1769	118.8469
7	118.1549	117.5990	120.3084	318.1176	119.3176
8	118.6954	114.8617	120.4053	318.1479	119.7818
9	119.1225	122.6369	121.4939	320.7045	120.6751
10	119.3960	125.4527	122.0146	323.4081	121.6856



(a) 1st mode: ideal (b) 1st mode: real (c) 2nd mode: ideal

(d) 2nd mode: real (e) 3rd mode: ideal (f) 3rd mode: real

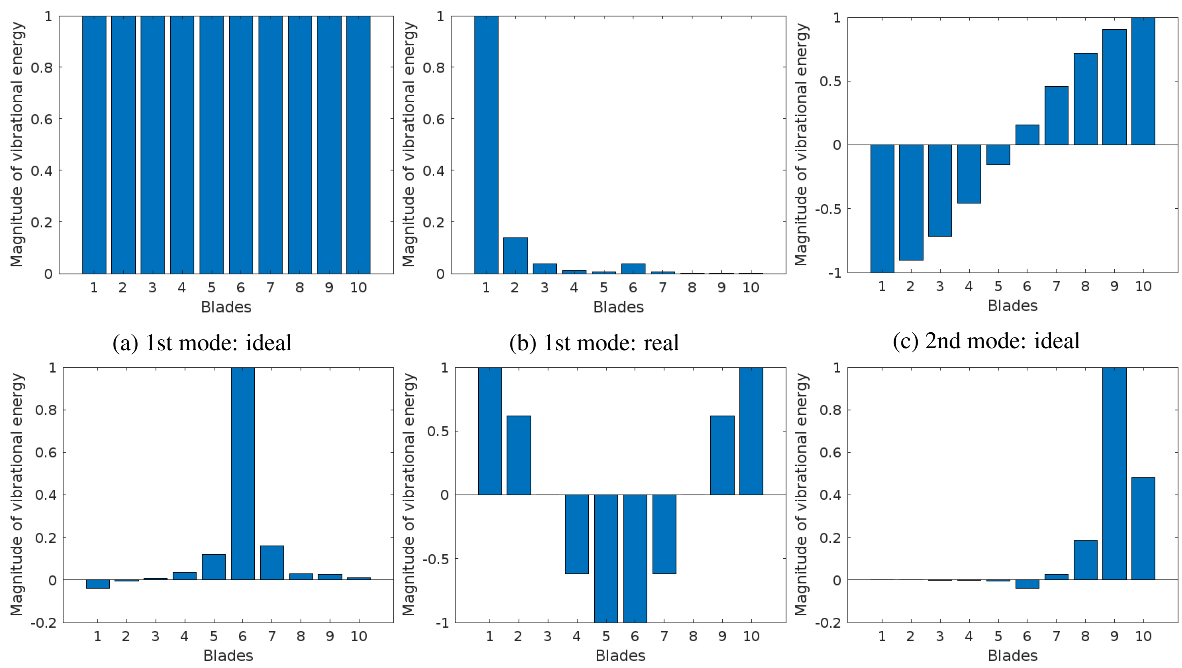
Figure 3: Length mistuining case: Blade vibration amplitudes vs. number of bldes. The periodic in the left side and quasi-periodic in the right side, for the ten vibration modes.



(a) 1st mode: ideal (b) 1st mode: real (c) 2nd mode: ideal

(d) 2nd mode: real (e) 3rd mode: ideal (f) 3rd mode: real

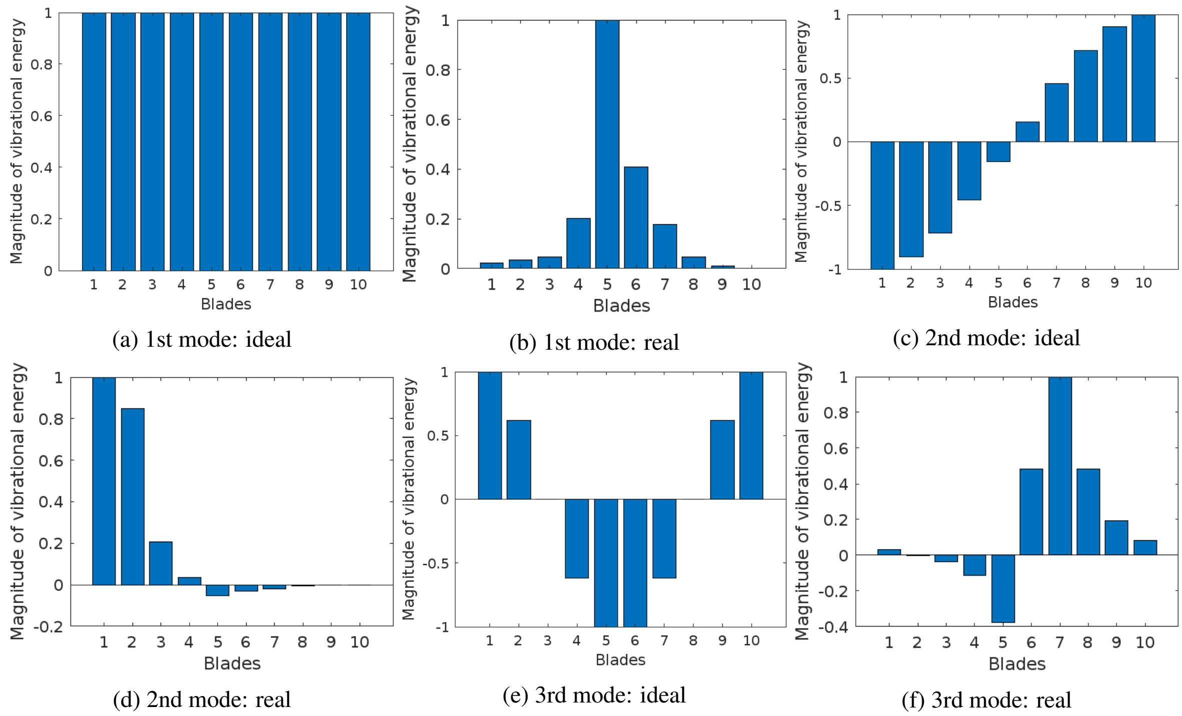
Figure 4: Stiffness mistuining case: Blade vibration amplitudes vs. number of bldes. The periodic in the left side and quasi-periodic in the right side, for the ten vibration modes.



(a) 1st mode: ideal (b) 1st mode: real (c) 2nd mode: ideal

(d) 2nd mode: real (e) 3rd mode: ideal (f) 3rd mode: real

Figure 5: Stagger mistuining case: Blade vibration amplitudes vs. number of bldes. The periodic in the left side and quasi-periodic in the right side, for the ten vibration modes.



(a) 1st mode: ideal (b) 1st mode: real (c) 2nd mode: ideal

(d) 2nd mode: real (e) 3rd mode: ideal (f) 3rd mode: real

Figure 6: Mistuining due to FOD's impact case: Blade vibration amplitudes vs. number of bldes. The periodic in the left side and quasi-periodic in the right side, for the ten vibration modes.

CONCLUSIONS

This work was successful in obtaining the phenomenon of localization of vibration modes in aeronautical turbine blades under imputed mistunging. The determination of frequencies and modes of vibration is a generalized problem of eigenvalues and eigenvectors solved numerically. Dynamic characteristics of the variable section blades were obtained via Rayleigh's method.

The mathematical model was implemented via a computational algorithm to evaluate the phenomenon of mode localization in turbine blades, taking into account the elastic and geometric stiffness, due to large centrifugal forces. It was successfully implemented in the MATLAB environment. This can be used generically for several turbine models and the insertion of random perturbations.

The occurrence of mode localization in a quasi-periodic aeronautical system such as a turbine is still little studied. As far as we know, assessment of the effect of angle and debris collision disturbances on this phenomenon is a novel contribuition to this study.

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