

## A numerical study on fatigue crack propagation in bonded joint under spectrum loads

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*Abstract: Adhesively bonded joints are used in several branches of industry. Despite the numerous studies concerning the fatigue failure of these structures, the phenomenon of crack propagation under truly random loads remains largely unexplored. This work proposes a numerical methodology for fatigue life calculation of propagating cracks inside the adhesive layer. Load-structure interactions are considered, since spectral loads usually are coupled with structural response, and the probability density function of the loadings actuating on the structure are obtained from frequency-domain techniques. Linear Elastic Fracture Mechanics (LEFM) is employed, and the fatigue life is obtained by integration of the  $da/dN$  curve, which for the case study herein is given as a function of the Strain Energy Release Rate of the joint, which is obtained from finite element analysis. The results showed the strong influence of modal damping on the estimated fatigue life. An increase in modal damping from 0.25% to 0.33% resulted in a 10 times higher fatigue life.*

**Keywords:** Spectrum loads, Vibration fatigue, Crack growth, Frequency domain

### INTRODUCTION

Adhesively bonded joints present some very interesting characteristics as a joining method, such as lightweight, conformability to complex geometry, and good fatigue resistance. This type of joint finds application in several branches of industry such as naval, automotive, and aeronautics. In any case, these structures may be subjected to spectrum loads, which can be caused, for instance, by bumps in a road, ocean waves, or wind gusts, and can lead to vibration fatigue. Due to the random nature of these excitation loads, the traditional fatigue analysis in time-domain becomes computationally expensive, and this type of problem is usually analyzed in the frequency-domain, using for instance Dirlik's method (Dirlik, 1985) or the narrow-band solution. Frequency-domain methods are traditionally employed in the safe-life approach, and it has been shown to be a reliable solution for the case of metallic structures under spectral loads, as shown by Han et al (2019).

The fatigue life of adhesively bonded joints can be divided into two phases: nucleation and propagation. The predominance of one phase over the other depends on parameters such as the joint configuration and type of loading. Recently, the authors proposed a methodology for crack nucleation in the frequency-domain (Marques, Ribeiro, and Tita, 2021). The present work will focus solely in the phenomenon of crack propagation.

The problem of adhesive crack growth under variable amplitude has been extensively studied over the past decades. Ashcroft (2004) observed crack growth acceleration and unstable crack growth in CFRP bonded joints under intermittent overloads and proposed a model to predict crack growth based on LEFM under this loading condition. He proposed the addition of a parameter that is a function of the number of overloads and the overload magnitude, which would be responsible for increasing the rate of crack growth due to the observed overloads. Nolting, Underhill, and DuQuesnay (2007) also reported that the presence of overload cycles caused an increase in damage of subsequent smaller cycles. They proposed an effective stress range S-N approach to calculate the fatigue life of bonded aluminum joints under variable amplitude. Khoramishad et al (2011) proposed the use of a cohesive zone model to predict the fatigue response under variable amplitude loading. They included in their model a correction factor to account for R-ratio effects based on the Gerber constant life diagram.

Despite the vast literature on crack propagation in adhesively bonded joints, there are virtually no works regarding crack propagation in the adhesive layer of these types of structures under random stationary Gaussian loads. This work is thus an attempt to fill this gap.

## METHODOLOGY

The random vibration of a deterministic linear system happens when such system is exposed to random (stochastic) loads. The main characteristic of such a process is that its behavior cannot be predicted in advance, and thus can only be described in terms of probabilities (Wijker, 2009).

Random loads can be completely described in the frequency-domain by their Power Spectral Density (PSD), which represents the spread of the mean square amplitude over a frequency range (Mrsnik, Slavic, and Boltezar, 2013). Such description is advantageous because the load response from a selected geometrical location can be expressed by combining the FRF of the structure and the applied random load PSD, as shown:

$$PSD_o = |FRF|^2 PSD_i \quad (1)$$

where  $PSD_o$  is the PSD of the load response,  $FRF$  is the frequency response function which relates the response of the structure to the input load, and  $PSD_i$  is the input load PSD. One important concept of random processes is that of spectral moments. Considering the one-sided PSD of a process as  $G_{XX}(f)$ , where  $f$  denotes frequency. The  $i$ -th spectral moment  $m_i$  is then given by:

$$m_i = \int_0^{\infty} f^i G_{XX}(f) df \quad (2)$$

Due to its randomness, the time history of the spectral loads cannot be retrieved. However, the Probability Density Function (PDF) of the load cycles is related to its PSD, and can be estimated. For a stationary Gaussian process that is restricted to a narrow band range, the PDF can be obtained based on the assumption that the load history assumes the form of a Rayleigh distribution, such that:

$$pdf(P) = \frac{P}{\sigma^2} \exp\left(-\frac{P^2}{2\sigma^2}\right) \quad (3)$$

where  $pdf(P)$  is the probability density function of the load cycles;  $P$  is the load amplitude; and  $\sigma$  is the standard deviation, which is given by the square root of zeroth-order spectral moment of the process. More details of the methodology can be found in Marques, Vandepitte and Tita (2021).

## Geometry and Finite Element Model

To study the debond growth in an adhesively bonded joint, the geometry chosen was that of a skin with a bonded stiffener, which is modeled in finite elements using Abaqus<sup>TM</sup>. This particular structure was used in stiffener pull-off tests performed by Freitas and Sinke (2015), who studied the behavior of two different types of adhesives under monotonic loads. The skin is made of GLARE 5-3/2-0.3, a fiber metal laminate made of three 2024-T3 aluminum alloy layers 0.3 mm thick along with two glass fiber reinforced plastic (GFRP) layers with the layup [0/90/90/0]. The final layup of the skin is thus [Al/[0/90/90/0]/Al/[0/90/90/0]/Al], with a total thickness of 3.134 mm. The stiffener is an inverted T made of 2024-T3 aluminum alloy, 1.5 mm thick. This configuration results in a metal-to-metal bonded joint. The mechanical properties of the skin and stiffener were obtained from the work of Freitas and Sinke (2015), and are summarized in Tab. 1 below:

**Table 1 – Mechanical properties of the materials: skin and stiffener**

Material	Parameter	Units	Value
GFRP	$E_{11}$	GPa	48.9
	$E_{22}, E_{33}$	GPa	5.5
	$G_{12}, G_{13}, G_{23}$	GPa	5.5
	$\nu_{12}, \nu_{13}, \nu_{23}$	-	0.33
	$\rho$	kg/m <sup>3</sup>	2.48
Al 2024-T3	$E$	GPa	72.4
	$\nu$	-	0.33
	$\rho$	kg/m <sup>3</sup>	2.78

The skin has a span 300 mm long, as shown in Fig. 1, and a depth of 100 mm. The skin is fixed at its ends by four bolted steel blocks. In the FEM, a fixed boundary condition is applied to the blocks, while perfect adhesion is assumed between the blocks and the skin. The adhesive layer is assumed to be 0.35 mm thick, with perfect adhesion between the adhesive and the adherent's surfaces.

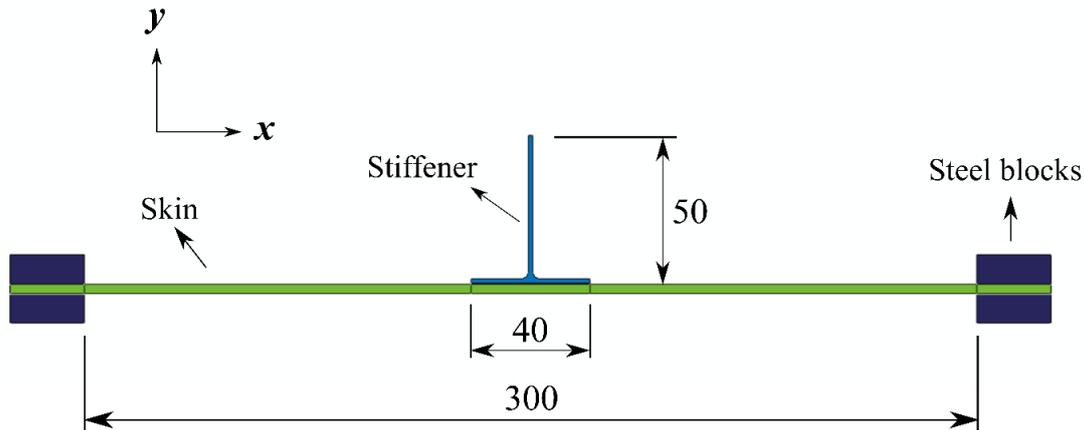


Figure 1 – Dimensions of the skin-to-stiffener geometry used in the present work.

The adhesive's Young modulus and fatigue properties were obtained from the work of Akhavan-Safar et al (2020), while the Poisson ratio and density were assumed based on general properties found in the literature. These values are summarized in Tab. 2 and Tab. 3:

Table 2 – Adhesive mechanical properties.

Parameter	Units	Value
$E$	MPa	1159
$\nu$	-	0.4
$\rho$	kg/m <sup>3</sup>	1.2

Table 3 – Paris law parameters.

Parameter	Units	Value
Intercept ( $C$ )	mm/cycle	0.0568
Slope ( $m$ )	-	7.82
Fracture Energy ( $G_{Ic}$ )	N/mm	2.2

The load case considered will be that of a spectral load applied to the top corner of the stiffener (as illustrated in Fig. 2). The input load PSD will be arbitrarily assumed as constant in the frequency range from 105 Hz to 165 Hz, with an amplitude of 0.02 N<sup>2</sup>/Hz. A preliminary analysis revealed that the first natural frequency of the intact structure happens at 137.6 Hz, which corresponds to its first bending mode, while the second vibrational mode appears at around 230 Hz, and therefore the spectrum load excites only the first mode of the component.

The first step of the analysis consists of obtaining the FRF of the pristine structure, when no damage is present. Thus, an impact load is applied in the top corner of the stiffener and the FRF of the vertical reaction forces (y-direction) on the steel blocks is obtained. An important parameter in the analysis is the damping of the structure. This information is usually obtained experimentally from modal tests performed in the structure. To investigate how damping affects the analyses, three arbitrary values of damping will be used on the numerical simulations: 0.25%, 0.30% and 0.33%. These values are inserted in the numerical model as modal damping applied to the first vibrational mode of the structure. The PSD of the reaction forces is then obtained by means of Eq. 1.

The damage scenario considered is the debond growth through cohesive failure of the adhesive layer between the stiffener and skin. An initially debond area is assumed to exist on the structure. The initial debond (or crack) is taken to be 20 mm long, with a straight crack front throughout the stiffener's base, as shown in Fig. 2.

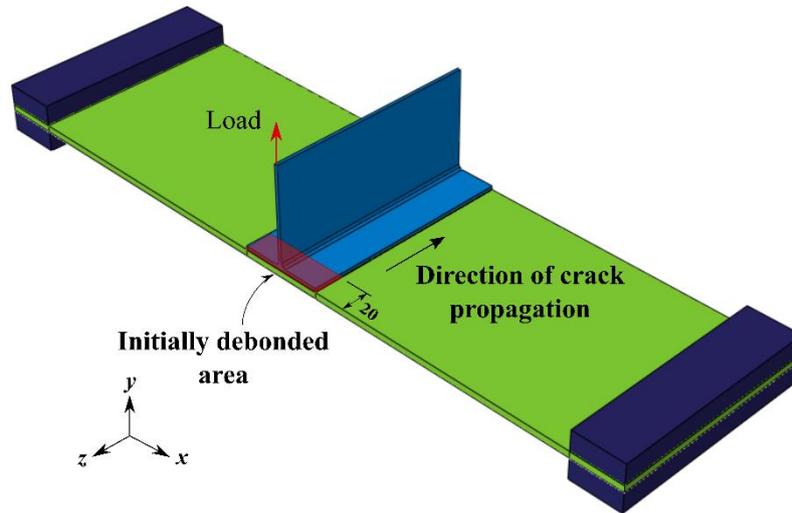


Figure 2 – Damage scenario considered in the analysis.

The second step then consists of obtaining the SERR (Strain Energy Release Rate) of the structure as a function of the crack size. Two initial hypothesis are that: 1. The crack propagates uniformly, i.e., the crack front remains a straight line until the end of load application; and 2. Mode I is the predominant mode of crack propagation, and therefore the values of the J-integral are assumed to be always equal to  $G_I$ . These hypotheses are used to help simplify the analysis of crack propagation. The structure is modeled using 113,604 second order quadratic elements with reduced integration (C3D20R), while the adhesive layer is modeled by approximately 900,000 linear elements (C3D8R), according to convergence analysis. The J-integral is calculated in 5 contours around the crack front. A detailed region of the mesh around the crack tip is shown in Fig. 3, where point A indicates the crack front, which is perpendicular to the image (in the x-direction).

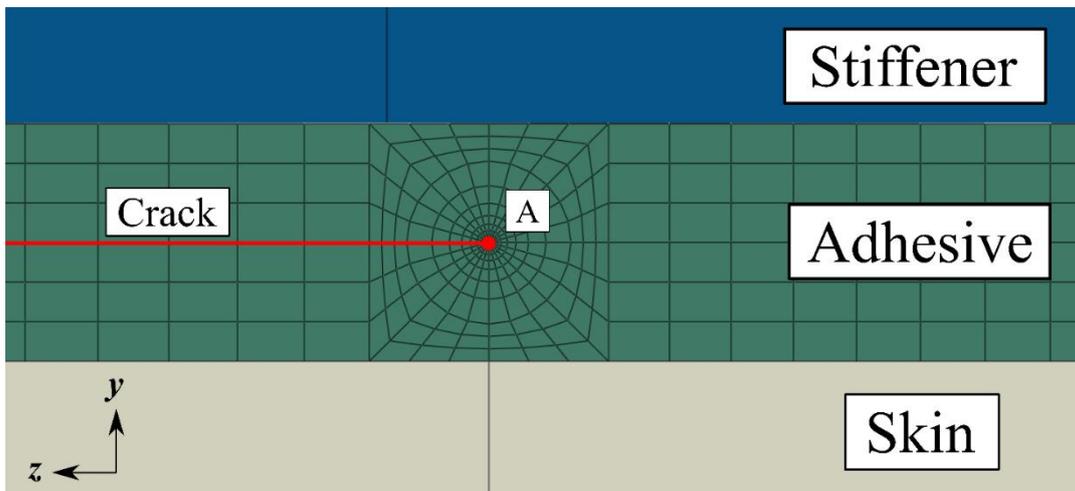


Figure 3 – Detailed view of the mesh around the crack.

$G_I$  values are then obtained for a point load applied in the top corner of the stiffener for discrete values of crack size, ranging from the initial length of 20 mm to a final length of 65 mm, and are normalized by the reaction force on the steel blocks. The graph created in this step is later extended to intermediate crack sizes by means of a cubic interpolation.

### Crack Propagation Under Random Loads

Crack propagation in the adhesive layer is often modeled by the Paris law or variations of it, following a Linear Elastic Fracture Mechanics. The driving force is expressed as a function of the strain energy release rate  $G$ , however, different authors have proposed different expressions to modeled the rate of crack propagation, usually as a function of  $\Delta G$ ,  $(\Delta\sqrt{G})^2$ , or  $G_{max}$ , as discussed by Pascoe et al (2017). In this work, a normalized Paris law is used, based on the work by Akhavan-Safar et al (2020), who combined LEFM and CZM techniques to predict the tensile fatigue life of bonded structures. The Paris law equation used is given by:

$$\frac{da}{dN} = C \left( \frac{G_{max}}{G_{Ic}} \right)^m \quad (4)$$

where  $C$  and  $m$  are the Paris law fitting parameters,  $G_{max}$  is the maximum value of the SERR, and  $G_{Ic}$  is the static fracture energy. The SERR is obtained from finite element simulations. It is assumed that the spectrum loads follow a R-ratio of -1, the maximum SERR at any given cycle is given by the amplitude of the applied load,  $P$ . It is then possible to write the SERR as a function of the load amplitude and the normalized function  $f(a)$  obtained from the finite element analyses:

$$G_{max} = P^2 \cdot f(a) \tag{5}$$

Let's now consider a given load cycle with amplitude  $P_i$ . The Paris law equation can be re-written considering the probability of occurrence of this cycle, as shown:

$$\frac{da}{N_t P df(P_i) dP} = C \left( \frac{P_i^2 f(a)}{G_{Ic}} \right)^m \tag{6}$$

where  $N_t$  indicates the total number of stress cycles applied to the structure. Separation of variables and integration on both sides of the equation yields:

$$\int_{a_i}^{a_f} \frac{da}{f^m(a)} = \frac{N_t C}{G_{Ic}^m} \int_0^{P_f} P^{2m} p df(P) dP \tag{7}$$

where the left-hand side of the equation is integrated from an initial crack (debond) length  $a_i$ , to a final length  $a_f$ ; and the right-hand side is integrated from zero to an upper limit of truncation,  $P_f$ , of the load PDF. Equation (7) can then be numerically solved for the total number of load cycles  $N_t$ .

For complex geometries, analytical expressions for SERR are not readily available, and thus numerical solutions must be employed. In this work, a Finite Element Model (FEM) is built and the SERR versus crack length is obtained from J-integral calculations (further details are given in the next section). A summary of the methodology used herein is given in Fig. 4:

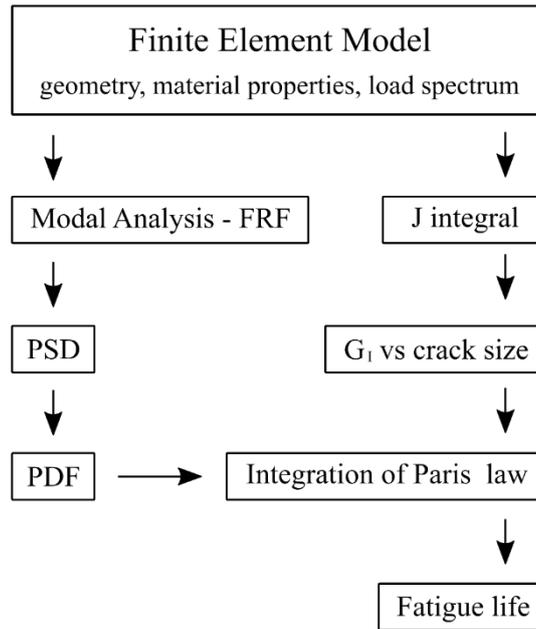


Figure 4 – Flow chart of the methodology used in this work.

## RESULTS

As it was explained, the applied spectral loads are coupled with the dynamic response of the structure. Obtaining the FRF of the structure is thus the first step to obtain the PSD of the loads actuating on the structure. Figure 5 shows the FRF of the reaction forces by an impact load applied at the top corner of the stiffener.

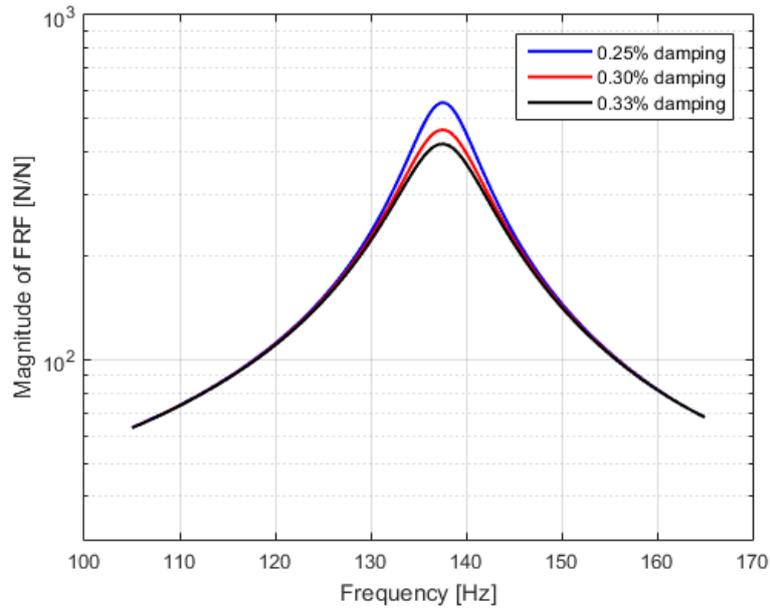


Figure 5 – FRF of the reaction force to a point load applied on the top corner of the stiffener.

### SERR vs Crack

The fatigue life for crack propagation can be obtained by numerical integration of the Paris law. Thus, the values of the SERR must be known for the interval between initial and final crack length. Figure 6 shows the normalized values of the J-integral as calculated by Abaqus. The values obtained are attributed exclusively to mode I of crack propagation.

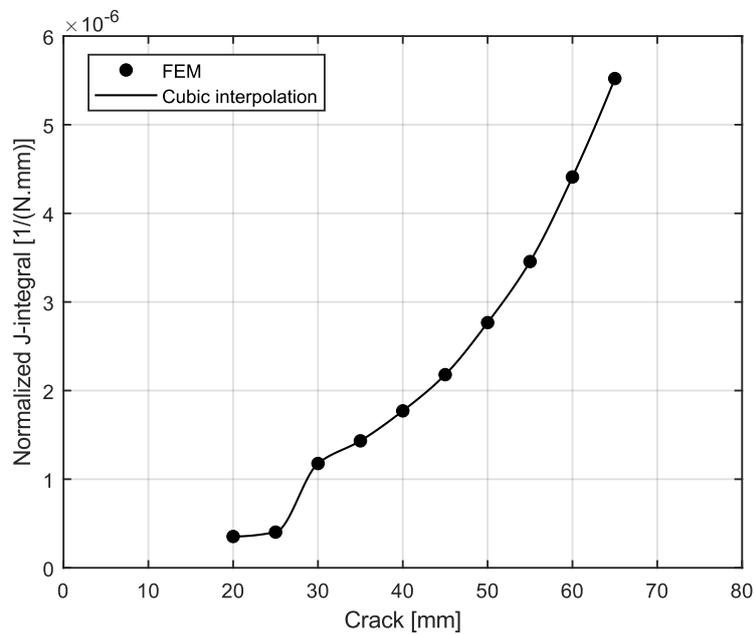


Figure 6 – J-integral values versus crack size.

### Fatigue Life

Once the PSD of the reaction force is calculated, the narrow band solution is used to calculate the PDF of the load amplitude. Figure 7 shows the PDFs obtained for the three different levels of modal damping studied herein.

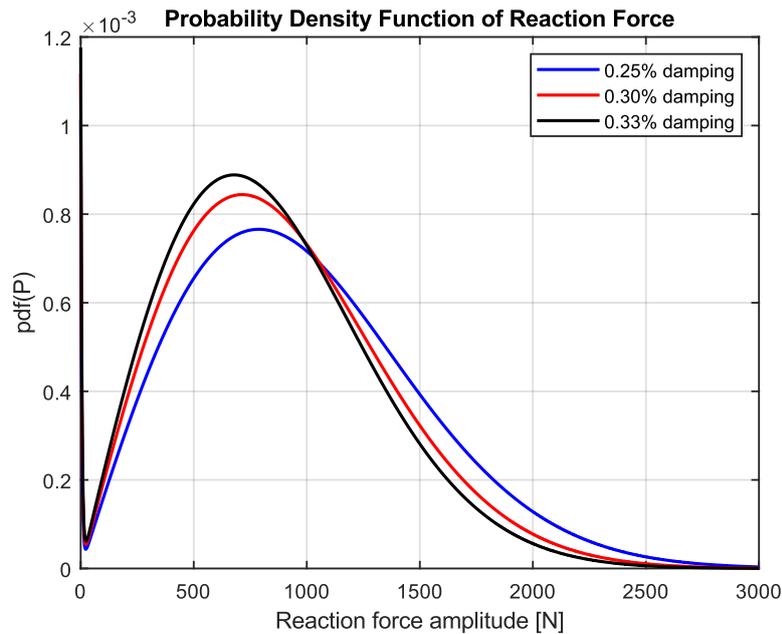


Figure 7 – PDF for different damping levels.

Fatigue life for crack propagation can now be estimated from the Paris law. Table 4 gives the number of cycles for the debond region to grow from an initial length of 20 mm to a final length of 65 mm. It is possible to observe the impact of damping in the fatigue life: an increase in damping from 0.25% to 0.33% results in a fatigue life that is one order of magnitude higher.

Table 4 – Fatigue life for the crack to propagate from 20 mm to 65 mm.

Damping	Fatigue life
0.25%	$8.1 \times 10^4$ cycles
0.30%	$3.7 \times 10^5$ cycles
0.33%	$8.4 \times 10^5$ cycles

## CONCLUSIONS

In the present work it was proposed a methodology for fatigue crack propagation of adhesively bonded structures under Gaussian random loads. These type of spectrum loads are naturally random and although cannot be described in time domain, their PDF can be estimated from frequency domain analysis. The obtained PDF is then combined with Paris law to estimate the fatigue life of the bonded joint. The numerical results show the important effect of damping in the predicted fatigue life of the structure. Future works will focus on the experimental validation of the proposed approach, observing the hypotheses used during the crack propagation analysis.

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