# Fatigue Study of Notched Components in the Frequency-Domain

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*Abstract. Structures are built up to support a variety of in-service loading specific to their function. Random vibration is observed in most structural components in many areas such as offshore, aerospace, civil and automotive industries. Fatigue life prediction plays a key role for verifying the design and assessing the structural integrity during service. The fatigue damage is commonly calculated by a linear rule proposed by Palmgrem-Miner. The scatter of fatigue testing data suggests that a probabilistic characterization of the material resistance should be considered during fatigue life calculations. In this work, the inherent uncertainties of the fatigue phenomenon and the influence of a notch are studied in the fatigue life prediction of a component subjected to random loading. The fatigue life estimated using the Dirlik's frequency domain counting method is compared with the experimental random tests. The agreement is good after setting correctly the structural damping ratio $\zeta$ and the so-called fatigue notch factor $k_f$.*
***Keywords: Frequency Domain, Fatigue, FEM, Random Loading, Notched samples.***

## INTRODUCTION

Fatigue as a technical problem became evident around the middle of the 19th century, August Wohler performed systematic fatigue tests of smooth and notched railway axles in the 1850s. In addition to the introduction of the S-N diagram, a plot of the number of cycles to failure at a given stress level, his work also led directly to the concept of a fatigue (or endurance) limit which represents the theoretical maximum cyclic load a material can withstand indefinitely without risk of fatigue failure (Suresh,1991).

Significant advances in the fatigue research were achieved with the mean stress effect studies by Gerber and Goodman, the development of fatigue safety diagrams by Haigh, investigation of reversed loading phenomena by Bauschinger, investigation of the notch effect on fatigue limit by Heyn, formulation of empirical laws to characterize fatigue limit by Basquin, the introduction of the crack growth energy balance by Griffith, life estimation under variable loading by Palmgren and the recognition of the statistical nature of fatigue by Weibull (Dowling,1999). The fatigue damage is traditionally determined from time histories of loading, usually in the form of stress or strain. This approach is satisfactory but requires extensive time records to describe a random loading process accurately. Alternatively, a frequency domain approach has a significant advantage in terms of computational time when finite element analysis is used. Random loading and responses are characterized using power spectral densities (PSDs).

The fatigue damage assessment for components subjected to random excitation is an important concern in engineering. Fatigue damage evolves with the applied load in a cumulative manner which may lead to failure. In 1920, Palmgren employed the concept of damage accumulation, which is known as the Palmgren-Miner linear rule. The concept in mathematical form is expressed as $D = \sum(ni/N)$, where the measure of the damage is the cycle ratio $ni/N_i^f$ with the assumption of constant work absorption per cycle ($n_i$) and characteristic amount of work absorbed at failure $N_i^f$. The energy accumulation leads to a linear summation which at failure equals one. Despite of Palmgren-Miner's rule well-known limitations, such as not accounting for load sequence and interaction effects, it is still dominantly used in design due to its simplicity (Fatemi and Yang, 1998).

The frequency-domain approach offers a significant advantage in terms of calculation time, which can be used to solve much larger and complex problems using finite element analysis. Therefore, this work aims to review and identify modeling parameters that increase the accuracy of the results compared to the experimental tests. The fatigue life of a 6061-T6 aluminum notched specimen is estimated for a random loading profile. The fatigue damage and life are estimated in frequency domains for different S-N curve percentiles. Also, the signal in the time domain was analyzed and the corresponding fatigue life was calculated. By correcting the S-N curve by the notch effect and modeling the damping behavior of the model, the frequency domain approach yields fatigue life prediction that is in close agreement with the experimental random vibration tests.

# 1 EXPERIMENTAL RANDOM TESTS

A notched 6061-T6 aluminum specimen illustrated in Figure 1 is subjected to a random vibration profile in the vertical direction(Z) in two steps. The first PSD profile was defined with a specific time duration; subsequently, the component is subjected to a second PSD profile with an increased root mean square acceleration where the component was tested to failure. Based on the cumulative damage process, the objective of the first test is to pre-damage the structure. Both PSD profiles are shown in figure 4. Three samples were tested simultaneously to measure the fatigue life variations among the specimens. Also, the specimens mounted on the shaker table can be seen in Figure 2.
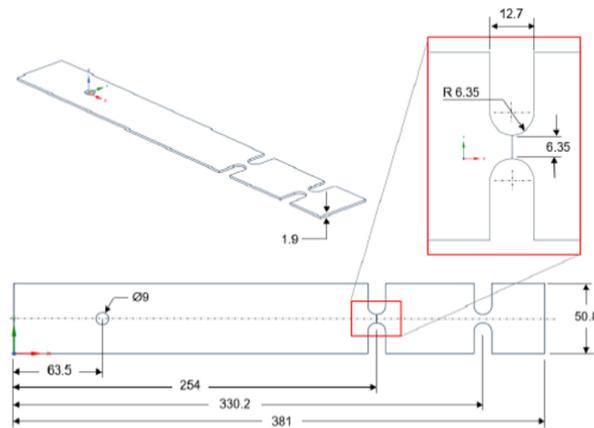
**Figure 1 – Main dimension of tested sample.**

Both applied PSD profiles are shown in Figure 2. The first PSD was applied during 7800 s and the second one for 2500 s. A DSA-4K - GW Gearing  Watson Electronics Limited electrodynamic shaker and three Bruel  Kjær DeltaTron 4519-003 accelerometers were used to excite and measure the acceleration responses at the notches on the test specimens. Both PSD profiles were defined to create a broadband excitation with stress responses below the yield point of the AL 6061-T6 material.

The material properties, engineering stress-strain curve and unnotched fatigue data, for 6061-T6 aluminum are shown in Figure 3. The tensile stress-strain curve and fatigue S-N curve were taken from the military handbook (MIL-HDBK-5G,1994) and the results correspond to unnotched specimens. The fatigue strength reduction factor $k_f$ was calculated using the volumetric approach to correct the S-N curve for the notched condition, as explained by Pascualinotto (2021). The fatigue strength reduction factor $k_f$ is then applied to the unnotched S-N percentile curves. The corrected P-S-N curves in a log-log scale are illustrated in Figure 3 (b). It is noticeable the impact of the notch on the fatigue strength of the specimens.

Table 1 displays the results of the experimental random fatigue. The failure was defined when the complete fracture (separation) has occurred based on the assumption that takes most of the fatigue life to initiate a dominant crack. None of the samples failed during the application of the first loading PSD profile. As can be seen in the table, the failure occurred after more than 1500 seconds of duration of the second PSD function for all three samples. The mean time to failure was calculated as 9667 seconds.

**Table 1 – Experimental fatigue results.**

| Specimen | STEP 1 | STEP 2 | Total Time to failure (s) | Mean time to failure (s) |
|---|---|---|---|---|
|  | Exposure time (s) | Exposure time (s) |  | $\mu \pm \sigma$ |
| Sample 1 | 7800 | 1772 | 9572 |  |
| Sample 2 | 7800 | 2324 | 10124 | 9667±228.5 |
| Sample 3 | 7800 | 1534 | 9334 |  |

To properly define the transient response and obtain the Frequency Response Functions (FRFs), the damping must be defined, which is a very difficult parameter to obtain. When testing data is not available, a constant damping ratio
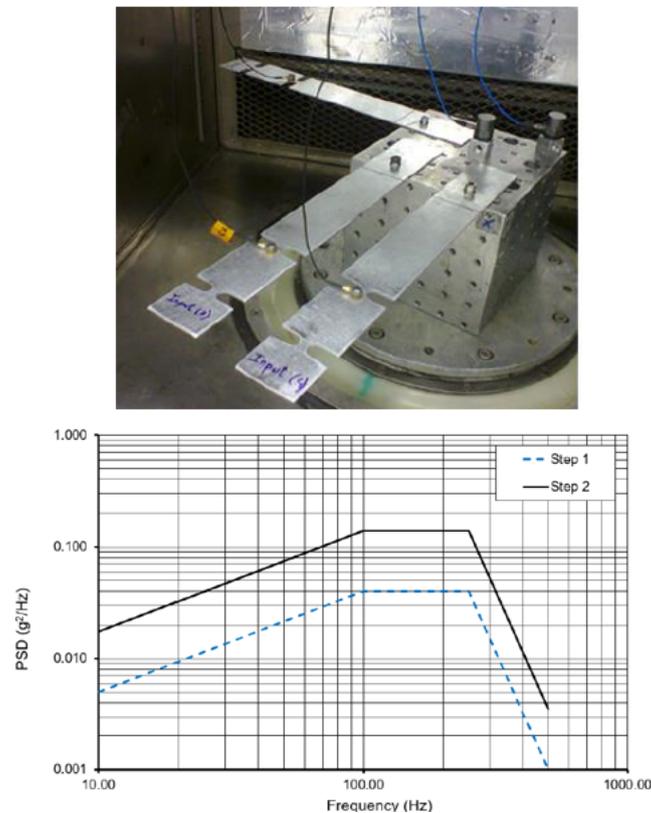
**Figure 2 – Acceleration PSD base excitation input and samples mounted on the shaker table.**

is usually estimated based on previous experience with same or similar material or based on the information available in the literature. In this work, the damping was modeled using the half power bandwidth method obtained from testing data. Accelerometers were used to measure the acceleration responses at the notches on the tested specimens when the specimens were subjected to the second vibration profile (Step 2 shown in Fig. 2). The acceleration response spectrum densities are shown in Figure 4. For each resonance peak the damping ratio $\zeta$ was calculated based on the quality factor $Q$ using half power bandwidth method (Pascualinotto,2020). The Rayleigh proportional damping was used in the dynamic analysis. Details of the calibration procedure for the constants representing the mass ($\alpha =4.188$) and stiffness ($\beta = 8.49 \times 10^{-7}$) proportional damping coefficients can be found in (Pascualinotto,2020).

## NUMERICAL METHODOLOGY

The fatigue life in the time domain is estimated by applying the time-series load history to the component in the FEM environment; a transient structural analysis is performed to generate the stress histories at all discretized points of the structure, defined by the mesh size in the finite element modeling. Then, Rainflow cycle counting and linear cumulative damage, per Palmgreen-Miner's rule, are applied to estimate the component's total damage and fatigue life. On the other hand, in the frequency domain approach, illustrated in Figure 5, the dynamic characterization of the discretized structure is made by its frequency response functions (FRFs), which are calculated based on a unit load excitation (e.g. for a PSD given in unit of acceleration $(g^2/Hz)$, the structure is harmonically excited with one (1) g acceleration at the correspondent excitation direction).

The PSD input load multiplies the square of FRFs stress tensor resulting in the stress response spectrum (PSD). Figure 5 illustrates an input PSD and the corresponding PSD of one stress tensor component. The fatigue life is then estimated by calculating the spectral moments of the PSD of stress, probability density function (PDF), and duration of the vibration profile. The obtained number of cycles subjected to the structure is compared to the material S-N curves via linear cumulative damage.

By definition, a random time history cannot be periodic; however, if the time history is taken from an ergodic stationary
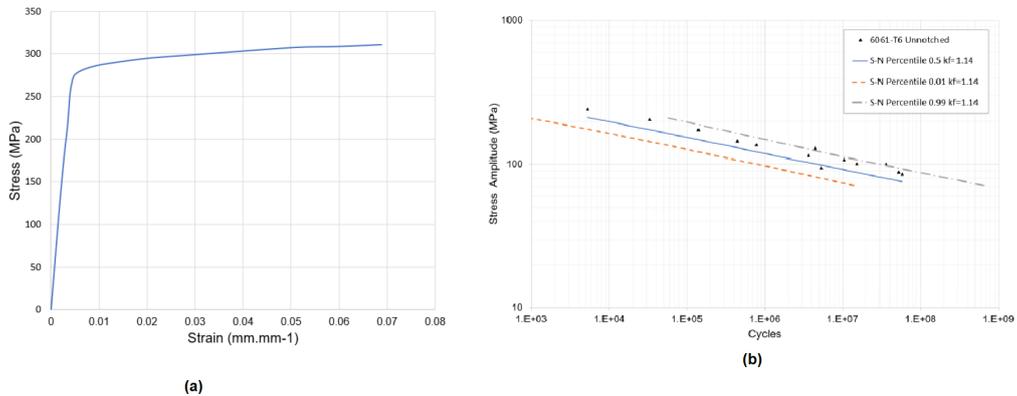
**Figure 3 – Tensile and fatigue properties of the 6061-t6 Al.**
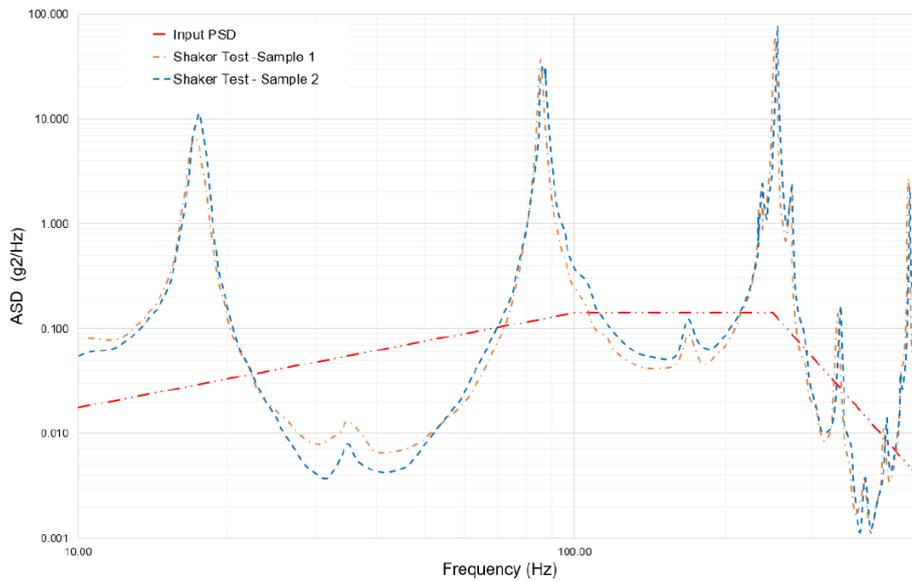


**Figure 4 – Acceleration Response spectrum.**

Gaussian random process, then it may be expressed in the frequency domain. A random process is said to be stationary if the probability distributions of the ensemble remain the same for each time instant [3]. If the ensemble probability distribution function is Gaussian, then the process is known as a Gaussian random process. A stationary process is called ergodic if the statistics taken from one sample are the same as those obtained for the ensemble. For nonstationary processes, the statistics obtained from a sampled time history would not be representative of those of the whole random process as they would be continuously changing.

The difficulties with the computation of the rainflow cycles distributions from time-domain data have led to the development of frequency-domain techniques. The seminal work by Rice (Rice, 1998) and research since has shown that the PDF of peak response can be determined from moments of the frequency domain response power spectral density (PSD) and that the peak PDF is a combination of a Gaussian and a Rayleigh PDF.

In 1985, Dirlik developed a practical closed-form solution for the wideband random problem., He proposed a pdf that considered the sum of two Rayleigh and one exponential distribution generated from an extensive Monte Carlo simulation (Dirlik, 1985). Although apparently more complicated than some alternative methods, it is only a function of the four moments of area of the PSD of the stress. This method is widely applicable and constantly outperforms all the other available methods for wideband vibration problems (Lee, 2005).

The Dirlik's pdf of stress amplitude is given by:

$$p(s) = \frac{\frac{D_1}{Q}e^{\left(-\frac{s}{2Q\sqrt{m_0}}\right)} + D_3\frac{s}{2\sqrt{m_0}}e^{\left(-\frac{s^2}{8m_0}\right)} + \frac{D_2}{R^2}\frac{s}{2\sqrt{m_0}}e^{\left(-\frac{s^2}{8R^2m_0}\right)}}{2\sqrt{m_0}} \tag{1}$$

where the parameters D1, D2, D3, Q, a R are defined as a function of the PSD moments as shown in Vagner and Sarzosa (2020, 2022). $m_0$ is the root mean square (RMS) of the PSD profile. Note that the coefficients $D_i$ must satisfy the relation $D_1 + D_2 + D_3 = 1$.

The number of cycles at a specific value of stress $S_i$ are obtained for a given time of vibration exposure $T$ by

$$n_i = p(S_i)dS \times E[P] \times T \tag{2}$$

The damage is then calculated generally using the 50% or P50 percentile S-N curve considering the fatigue notch factor. Figure 6 shows the mesh of the finite element model used in the study. A shell model with the element formulation SHELL181 was used in the Ansys software. A mesh independence study was carried out to verify the sensibility of the results to the mesh refinement. The final adopted mesh size has square elements of 0.125 mm close to the notch area. The boundary conditions were defined to emulate the fixation of the specimen in the shaker fixture.

## RESULTS

The dynamic characterization of the structure was carried out by performing a modal analysis solving the natural frequencies and vibration modes of the system. Seven natural modes were identified in the frequency range up to 500Hz, which is the upper limit of the vibration profiles. The lower and higher natural frequencies were $f_1$=17.29 Hz and $f_7$=453.8 Hz, respectively. Figure 7 displays the shapes of the first seven modes of free vibration.These modes are used to obtain the frequency response functions (FRFs) at the modal frequency response step for a unit harmonic acceleration input.
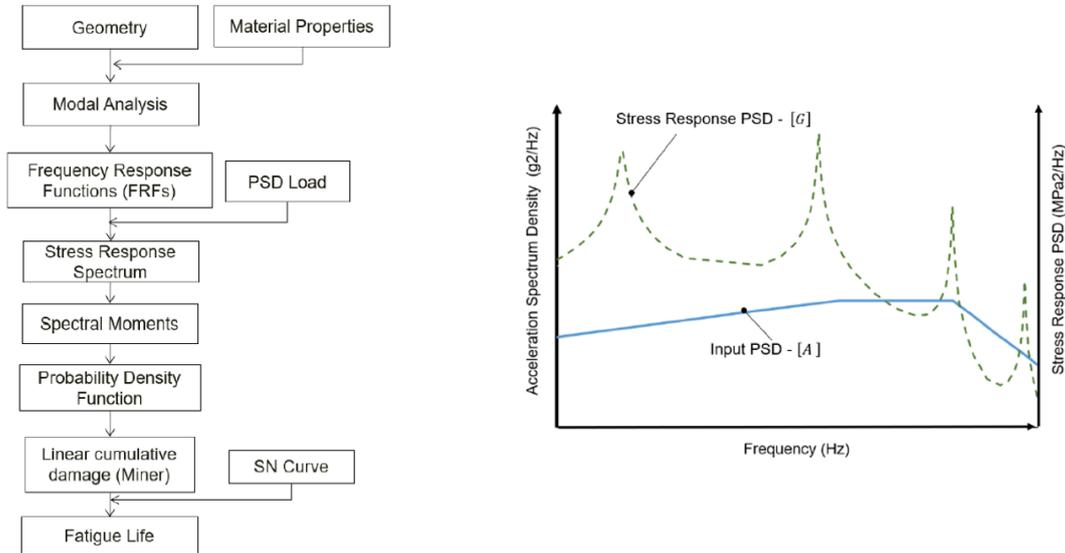


**Figure 5 – Flowchart for fatigue life estimation based on the frequency domain and illustration of input and output PSDs.**

If the mechanical system is characterized by its FRFs and the input random excitation described by its PSD, the system stress response can be evaluated by scaling the PSD of the input signal by the magnitude squared of the stress FRFs as follows in matrix notation:

$$[G] = [Q]^T[A][Q] \tag{3}$$

where [G] is the stress response PSD, [A] is the input PSD excitation and [Q] is the dynamic response matrix. The dynamic response matrix is independent of PSD excitation and for the case being explored in this work, vibration in z-axis direction only, [Q] can be written as a vector (Voigt notation) at the free surface:

$$[Q]_{3x1} = [\sigma_x(f) \ \sigma_y(f) \ \sigma_{xy}(f)] \tag{4}$$
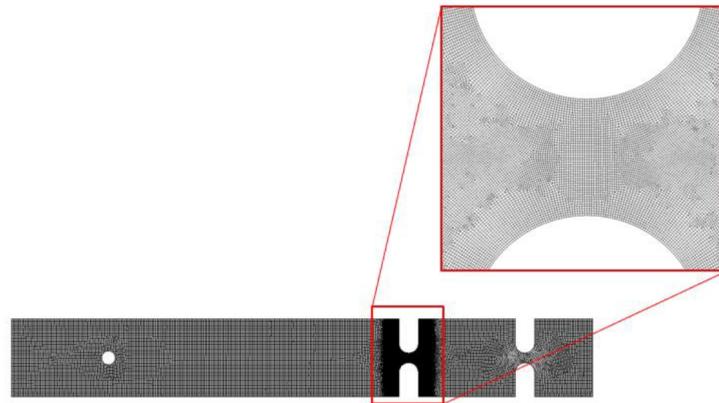
**Figure 6 – Finite element model.**

Figures 8 illustrates the dynamic response matrix measured at the root of the notch. The FRFs show a predominantly axial stresses, with peak responses at the first and second modes. Shear stresses are significantly lower for the uniaxial excitation case. Furthermore, the response peaks up to 500Hz are observed at the modes 1, 2, 4 and 5. The vibration mode 3 in torsion is an antiresonance, thus not being evident on the FRF responses.

The obtained stress response PSD [G] for the given input acceleration spectrum is shown in Fig.4. The critical plane method is used to combine the stress response PSD components. From the combined stress response PSD, number of cycles is then estimated based on the probability density function. The input PSD exposure time and the expected number of positive peaks. The PSD has no time duration, as it is simply a representation of the frequency content and energy of the time signal, therefore, the total number of cycles must be scaled by the period that the structure is exposed to the vibration profile.

Similarly, to the definition of the acceleration spectrum magnitudes, the time durations were defined to create a cumulative damage process in the first step not leading to failure, followed by a sufficient exposure time to the second profile leading to failure. The exposure time were defined as 7800s for the first PSD and 2500s for the second PSD. Then, the fatigue life was estimated based on the linear cumulative damage calculated for the defined S-N curve percentiles. The linear cumulative damage rule was calculated based on the material fatigue testing data's S-N percentiles 0.01, 0.5, and 0.95.

The time domain approach was also used to calculate the failure time. Acceleration time histories were synthesized following the acceleration spectrum densities specifications by using the inverse Fourier transform, and setting random values for the phase of each periodic signal that is used to define the time signal. The synthesized acceleration time signals were applied as a transient base excitation to the notched specimen at the fixed support defined in the modal analysis. The time transient analysis is computationally expensive analysis as the stress response tensor is calculated for each time step increment which for the total signal length of 25 seconds gives a total of 131,095 acceleration records. Details of the time domain analysis can be found in (Pascualinotto,2021).

The number of cycles at each stress level is then calculated from the stress response PSD, given the duration of the exposure to the vibration profile, see Eq. 2. The usage of the 0.95 percentile would rate the damage as only 14% compared to the experiment. The upper limit of the S-N curve, which corresponds to p=0.99, is non-conservative for fatigue predictions. Therefore, it is not recommended to be used for design purposes.

The results in Table 2 show good agreement between testing and analytical results when the meantime to failure from the test results is compared to the results using the percentile 0.5 of the S-N curve. As one would expect, the predictions using p=0.01 S-N are highly conservative in all cases. Finally, a summary of damage and time to failure is summarized in Table 2 for both methods. A good agreement of both methods, time and frequency domain, is observed in table 2 when compared to the mean failure time from the fatigue experiments. Moreover, using an Intel i7 CPU, 3.3 GHz, 16 MB RAM with 6 cores, the time domain solution took 42 h versus only 2.5 h in the frequency domain, being clear the advantages in the frequency domain approach.

Despite the well-known limitations of the linear cumulative damage, the proper dynamic characterization of the structure via a representative damping modeling and the consideration of geometrical factors (notch) which affect the fatigue behavior were found in this work to influence the fatigue life prediction under random vibration loading.
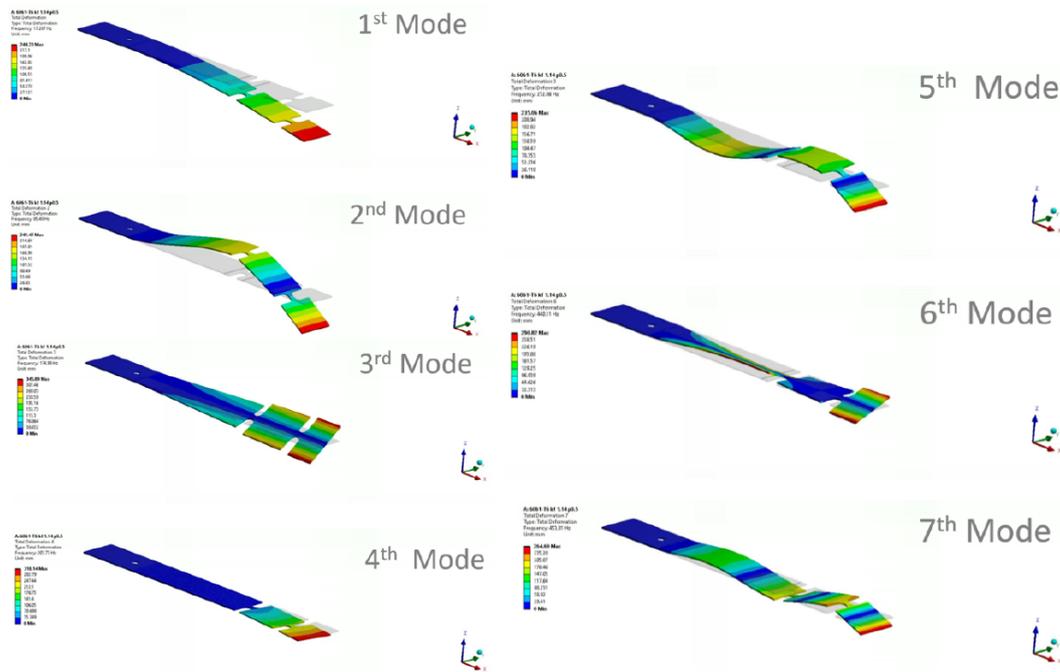
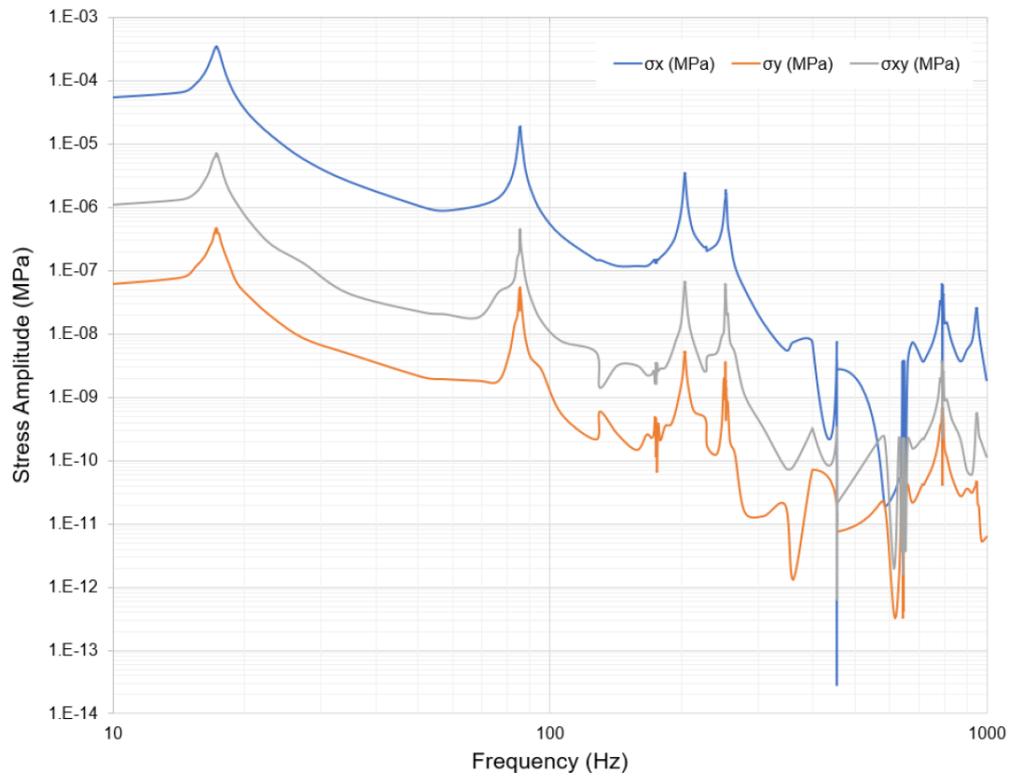**Figure 7 – Vibration profiles of the notched sample.**



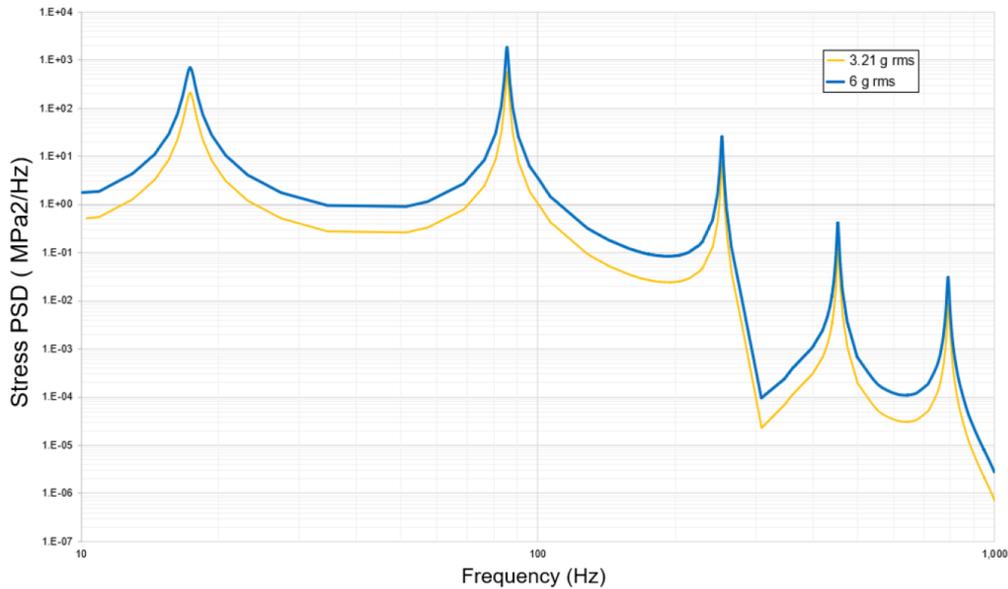**Figure 8 – FRFs measured at the notch root for a 1g base harmonic excitation.**

**Figure 9 – Stress Response PSD for both input PSDs.**

**Table 2 – Fatigue Life and Damage relative to the S-N curve percentile.**

| S-N curve percentiles | Damage $(PSD_1+PSD_2)$ | Life (repeats) $(PSD_1+PSD_2)$ | Frequency domain Dirlik (s) | Time domain Time to failure (s) | Experiment Mean time (s) |
|---|---|---|---|---|---|
| 0.01 | 6.5 | 0.15 | 8146 | 8129 | |
| 0.5 | 1.35 | 0.74 | 9835 | 9647 | 9667 |
| 0.95 | 0.14 | 6.84 | No-failure | No-failure | |

## CONCLUSIONS

In this work, the fatigue life estimation of a notched structural component under random vibration was calculated using a probabilistic linear cumulative damage model and frequency domain technique. There are various sources of variation when modeling fatigue in a random environment. However, using the linear cumulative damage with p=0.5 produced results in good agreement with testing after the appropriate characterization of the effect of the notch and modeling of the damping. The fatigue strength reduction in the presence of a notch was estimated based on the volumetric approach. The proper correction of the base S-N curve (p=0.5) influenced the fatigue life estimation of approximately 69%, after that, including the effects of geometric discontinuities is critical to the accuracy of the prediction. Another fact contributing to the good agreement of the predictions is the primarily uniaxial behavior of the stress tensor, which approximates the material response to the one obtained in the standard S-N test.

## REFERENCES

SURESH, S.; Fatigue of Materials, Cambridge University Press (Cambridge), 1991.

DOWLING, N. E.; Mechanical Behavior of Materials, Prentice Hall (Upper Saddle River, NJ), 1999.

FATEMI, A.; YANG, L.; Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials. International Journal of Fatigue, Vol 20, No. 1, pp 9-34, 1998.

MIL-HDBK-5G, Military Handbook – Metallic Materials and Elements for Aerospace Vehicles Structures, Vol. 1 and 2, 1994.

Vagner Pascualinotto Jr. Fatigue life estimation of a notched component using frequency domain technique and probabilistic linear cumulative damage model. Master thesis, University of São Paulo, Brazil, 2021.

T. Dirlik. Application of Computers in Fatigue Analysis. PhD thesis, The University of Warwick, 1985.

Rice, S.O.; Mathematical Analysis of random noise. Bell Syst. Tech. J., vol 23, pp.282-332, 1944.

Yung-Li Lee, Jwo Pan, R. Hathaway, M. Barkey. Fatigue Testing and Analysis: Theory and practie. Elsevier, 2005.

Vagner Pascualinotto Junior, Diego Felipe Sarzosa Burgos. Fatigue Life Estimation Using Frequency Domain Technique and Probabilistic Linear Cumulative Damage Model. Pressure Vessels and Piping Conference. PVP2020-21536,2020.

Sarzosa, D., Pascualinotto, V. Fatigue life estimation of notched components using frequency domain approach. Proceedings of the 27th Offshore Symposium, February 22nd, 2022, Houston, Texas, Texas Section of the Society of Naval Architects and Marine Engineers

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