MECSOL 2022 - Proceedings of the 8th International Symposium on Solid Mechanics
M.L. Bittencourt, J. Labaki, L.C.M. Vieira Jr. and E. Mesquita (Editors), Campinas SP, Brazil, October 17th to 19th, 2022

Aircraft Structural Reliability/Risk Estimate with Limited Data Using Possibility Theory

Mateus C C Z Oliveira [1a], Moacyr M Cardoso Jr [1b], and Mariano A Arbelo[1c]

[1a] ITA, Aeronautics Institute of Technology, São José dos Campos, SP, Brazil - mateusmcczo@ita.br

[1b] ITA, Aeronautics Institute of Technology, São José dos Campos, SP, Brazil - moacyr@ita.br

[1c] ITA, Aeronautics Institute of Technology, São José dos Campos, SP, Brazil - marbelo@ita.br

Abstract: One of the most important field of aircraft structural reliability is related to fatigue failures. Although several analysis and predictions are made during the design phase, only Full-Scale tests or fleet occurrences can identify some failures modes on specific details. Test data (and, in some cases, also fleet data) can usually be a limited set, due to the inherent complexity, costs and time that a complete Full-Scale test demands, or due to a single isolated fleet leader occurrence. This work proposes an approach to estimate structural reliability when limited data is available. While there is a known (possible) range, but not a properly known distribution for parameters as material properties, geometrical variation due to manufacturing tolerances, loading variation, etc, typical analysis proceeds by picking one value inside this range (it may be the mean, or the most conservative value). As predictions and conclusions made by a single observation point may carry high amount of uncertainty, it will be useful to somehow quantify the effect related to the assumed parameters. Probabilistic calculation depends on well characterized distributions for inputs, which often is not available, so, alternatively, the Possibility Theory was used to explore the parameters that are usually estimated. A typical scenario was modelled and evaluated. Uncertainty propagation was performed with Independent Random Sampling (IRS) algorithm from R/RStudio software (HYRISK library). The analysis consists of a trunnion collar failure from a Main Landing Gear during a Full-Scale Fatigue Test, and risk prediction for fleet. The results are assessed and compared with typical calculation from reference literature. The proposed approach helps to better quantify and understand uncertainty in the prediction of aircraft structural reliability/risk when only limited data is available. This gives a better and most realistic picture of reliability/risk ranges and each factor contributions, allowing for more appropriated and sound decisions.

Keywords: Risk, Structural Reliability, Fatigue, Possibility Theory, Uncertainty

NOMECLATURE

α – Weibull Distribution Shape Factor or Fuzzy Membership/Confidence Levels (depending on context)

AI – Aversion Index

Bel – Belief Function

CDF – Cumulative Distribution Function

FC – Flight Cycles

FMLG – Forward Main Landing Gear

F_{Tu} – Material Tension Strength (Allowable)

h – Hazard Rate Function

Kt – Stress Concentration Factor

MLG – Main Landing Gear

N – Necessity Function

$SFPOF$ – Single Flight Probability of Failure

Pl – Plausibility Function

$\prod$ – Possibility Function

π – Possibility Distribution

INTRODUCTION

Globally there has been noticed an increase of the use of probabilistic methods applied to structural analysis. The probabilistic approach has the advantage of making possible the quantification of the risk (or reliability) associated to a component or process, taking into account the variability of each parameter involved.

However, the use of probabilistic methods is difficult for practical application in the industry, as it depends on a complex and complete characterization of the variability of each parameter, requiring a large volume of data and tests that are often not easily available.

One of the most important field of aircraft structural reliability is related to fatigue failures. Although several analysis, evaluation and predictions are made during the design phase, only Full-Scale tests or fleet occurrences can bring to light some failures modes on specific details.

Test data (and, in some cases, fleet data as well) can often be a limited set, due to the inherent complexity, costs and time that a full-scale test requires, or due to a single isolated fleet leader occurrence.

The use of possibilistic approaches aims to bring alternatives to these situations, making it possible to quantify variability and uncertainties, even when only a limited amount of data or information is available. At these stages, the variability/uncertainties can also be classified as aleatory (which are inherent of the physical aspects of the system) or epistemic, which are due to lack of data and/or knowledge to fully understand and quantify all variations.

In this work, a typical practical example was considered: as indicated on Tuegel et al. (2018), when limited data is available, as a single point, e.g., a common practice is to assume this value as the mean and to use a Weibull distribution for which the shape factor (α) is assumed based on typical values for the material that the component is made.

Typical range of shape factor (α) cames from Tuegel et al. (2018), Freudenthal (1975) and Whittaker and Besuner (1969):

• High strength steels (F_{Tu} > 1379 Mpa): α = 2.0 to 2.5,

• Titanium alloys: α = 2.5 to 3.0,

• Low strength steels (F_{Tu} < 1379 Mpa): α = 3.0 to 3.5,

• Aluminum alloys: α = 3.5 to 4.5.

At Tuegel et al. (2018), probability calculations and risk/reliability predictions are then performed based on these assumptions, while the epistemic uncertainties are not explicitly calculated and considered.

OBJECTIVES

This works proposes a new approach to estimate structural reliability when limited data is available. While there is a known possible range for parameters as material shape factor (α), current analysis typical proceed by picking one value inside this range, like the mean value, or, in some cases, the most conservative one. As predictions and conclusions made by a single observation point may carry a high amount of uncertainty, it will be useful to somehow quantify the uncertainty related to the assumed inputs, by exploring the results throughout their range.

Other sources of variabilities and uncertainties, which have a known range, but not a properly known distribution, can also be considered in an integrated analysis, as typical geometrical variation due to manufacturing tolerances, residual stresses due to manufacturing process, and in-service loading/stress variation.

METHODS

This section brings a brief review of basic and important definitions. For a more in depth understanding of each concept, the mentioned references are recommended for further reading.

Probability Density Functions (PDF)

A PDF, $f(x)$, gives how likely (in terms of a probability P) is to have the occurrence of a specified value x. Its mathematical definition can be stated as, (Tuegel et al. (2018)):

$$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f(x)dx \qquad (1)$$

Cumulative Distribution Functions (CDF)

A CDF, *F(x)*, gives the cumulative occurrences up to a specified value *x*. It is the integral of the PDF, from $-\infty$ up to *x*, (Tuegel et al. (2018)):

$$F(x) = \int_{-\infty}^{x} f(x)dx \quad (2)$$

Reliability (R) and Hazard Rate Function (HRF)

When the occurrences are related to failures, it may be defined the reliability, $R\ (x)$, as the opposite of the cumulative failure values. It gives the probability that an item will survive up to a specified value *x*. It follows that:

$$R\ (x) = 1 - F(x) = 1 - \int_{-\infty}^{x} f(x)\, dx = \int_{x}^{\infty} f(x)\, dx \quad (3)$$

The hazard function or hazard rate $h(x)$ is an instantaneous failure rate, or, the conditional probability of failure in the interval *x* to (*x* + *dx*), given that there was no failure by *x* (O'Connor and Kleyner (2012) and Tuegel et al. (2018)). It is mathematically defined as:

$$h = \frac{f(x)}{R(x)} = \frac{f(x)}{1-F(x)} \quad (4)$$

The expected proportion of items of age *x* that fail in a short time Δ*x* is equal to $\Delta\, x \cdot h(x)$ (Tuegel et al. (2018)).

Single Flight Probability of Failure (SFPOF)

The $SFPOF$ is probability of failure at one flight given that the aircraft has survived up to that point. It is the product of the hazard rate at a specified moment (t) and a time increment (Δt) of one flight (Tuegel et al. (2018)).

$$SFPOF = h(t).\Delta t \quad (5)$$

Uncertainty Theories

Wierman (2010) brings a comprehensive presentation of uncertainties and its different characterization. Kiureghian and Ditlevsen (2009) presents and discuss the two most important categorizations of uncertainties: random (or aleatory, or stochastic) and epistemic.

Random uncertainties are natural, inherent variability of a system or a property. They are not reducible through studies, additional information, and knowledge.

Epistemic uncertainties result from the lack of knowledge, which may be related to a phenomenon, a model and even the actual value of a random variability of parameter, due to incomplete/imprecise nature of available information. They can be reduced by studies and additional data. But they can also be limited by the frontier of current knowledge.

Total uncertainty is the combination of random and epistemic uncertainties. In risk analysis there are ways to consider and quantify both, separately or integrated.

As pointed out by Dubois and Guyonnet (2011), while stochastic uncertainty is adequately addressed using classical probability theory, several uncertainty theories have been developed to explicitly handle incomplete/imprecise information.

Baudrit, Dubois and Guyonnet (2006) recalls that it is necessary to distinguish between situations where uncertainty is due to the variability of the observed phenomenon, from situations where it is due to a mere lack of knowledge. While the first case is handled by means of probability theory, the second is more appropriately approached by set-valued representations whereby all that is known is that a certain value belongs to a certain set, which is possibly fuzzy. This is the idea of Possibility Theory.

Possibility Theory is a special branch of Evidence Theory that deals with bodies of evidence (Zio and Pedroni (2013)). It is devoted to the handling of incomplete information.

A few advantages of Possibility Theory, quoted from Zio and Pedroni (2013):

- It works with non-statistical (non-probabilistic) uncertainty; thus, it is applicable to all kind of uncertainty;
- It needs fewer arbitrary assumptions than probability theory;
- It is intermediate in conservatism between analogous Monte Carlo and interval approaches;
- Possibility distributions maintain conservatism under uncertainty about dependencies among variables;

Possibility distributions are very robust representation when empirical information is very sparse. In other words, final results have a small sensibility to the actual details of the input possibility functions shapes (Zio and Pedroni (2013)).

Possibility Theory

Possibility Theory is convenient to represent imprecise knowledge (Baudrit, Dubois and Guyonnet (2006)). A possibility distribution (π) describes the plausible values of some uncertain variable X. Possibility Theory provides two evaluations of the likelihood of an event, meaning that the value of a real variable X should lie within a certain interval:

the possibility $\prod$ and the necessity *N*. The normalized measure of possibility $\prod$ (respectively necessity N) is defined from the possibility distribution π: R → [0; 1] such that $sup_{x\in R}$ π(x) = 1, as follows (Baudrit, Dubois and Guyonnet (2006)):

$$\prod(\mathrm{A}) = sup_{x\in A}\, \pi(\mathrm{x}) \tag{6}$$

$$\mathrm{N(A)} = 1 - \prod(\overline{\mathrm{A}}) = inf_{x\notin A}\, \pi(\mathrm{x}) \tag{7}$$

A possibility distribution may also be viewed as a nested set of confidence intervals, which are the α -cuts of π [$\underline{x}_\alpha$; $\overline{x}_\alpha$] = {x, π(x) ≥ α};. The degree of certainty that [$\underline{x}_\alpha$; $\overline{x}_\alpha$] contains *X* is *N*([$\underline{x}_\alpha$; $\overline{x}_\alpha$]) (= 1 - α if π is continuous). Degrees of Necessity are equated to lower probability bounds and degrees of Possibility are then equated to upper probability bounds (Baudrit, Dubois and Guyonnet (2006)).

Evidence Theory

Evidence theory allows for the incorporation and representation of incomplete information: its motivation is to be able to treat situations where there is more information than an interval, but less than a single specific probability distribution. The theory can produce epistemic-based uncertainty descriptions and in particular probability intervals (Zio and Pedroni (2013)). It allows imprecision and variability to be treated separately within a single finite framework, providing mathematical tools to process information which is at the same time of random and imprecise nature (Baudrit, Dubois and Guyonnet (2006)).

As in Possibility Theory, Evidence Theory provides two indicators, plausibility *Pl* and belief *Bel*, to qualify the validity of a proposition stating that the value of variable *X* should lie within a set A (a certain interval for example). Plausibility *Pl* and belief *Bel* measures are defined from the mass distribution (Baudrit, Dubois and Guyonnet (2006)):

$$v: \mathcal{P}(\Omega) \to [0,1]\ such\ that\ \sum_{E\in\mathcal{P}(\Omega)} v(E) = 1 \tag{8}$$

$$Bel(A) = \sum_{E,E\subseteq A} v(E) \tag{9}$$

$$Pl(A) = \sum_{E,E\cap A\neq\emptyset} v(E) = 1 - Bel(\overline{A}) \tag{10}$$

Where $\mathcal{P}(\Omega)$ is the power set of Ω and E is called focal element of $\mathcal{P}(\Omega)$ when $v(E)$ > 1.

Bel(*A*) gathers the imprecise evidence that asserts *A;* it is the minimal amount of probability that can be assigned to *A*. *Pl*(*A*) gathers the imprecise evidence that does not contradict *A*; it is the maximal amount of probability that can be assigned to *A* (Baudrit, Dubois and Guyonnet (2006)).

Evidence Theory encompasses possibility and probability theory (Baudrit, Dubois and Guyonnet (2006)):

- When focal elements are nested, a belief measure *Bel* is a necessity measure, that is *Bel* = *N*. A Plausibility measure *Pl* is a possibility measure, that is *Pl* = $\prod$.
- When focal elements are some disjoint intervals, plausibility *Pl* and belief *Bel* measures are both probability measures, that is we have *Bel* = *P* = *Pl*, for unions of such intervals.

Thus, all probability distributions and all possibility distributions may be interpreted by mass functions. Hence, one may work in a common framework to treat the information of imprecise and random nature (Baudrit, Dubois and Guyonnet (2006)).

A mass distribution (v) can be built from a probability distribution function (p) or a possibility distribution (π) as follows (Baudrit, Dubois and Guyonnet (2006)):

1) Probability → Belief function.

Let *X* be a real random variable with a probability density p_x. By discretizing it into *m* intervals, we define, as focal elements, disjoint intervals $(]a_i;\ a_{i+1}])_{i=1:::m}$ and we can build the mass distribution $(v_i)_{i=1:::m}$ as follows $\forall i = 1:::m$ (Baudrit, Dubois and Guyonnet (2006)):

$$v(]a_i;\ a_{i+1}]) = v_i = P(X \in]a_i;\ a_{i+1}]) \tag{11}$$

2) Possibility → Belief function.

Let *Y* be a possibilistic variable. We denote by π the possibility distribution of *Y* and π_α the α -cuts of π. Focal elements for *Y* corresponding to α -cuts are denoted $(\pi_{\alpha j})_{j=1:::q}$ with $\alpha_0 = \alpha_1 = 1 > \alpha_2 > ... > \alpha_q > \alpha_{q+1} = 0$ and are nested. We denote by $(v_j = \alpha_j - \alpha_{j+1})_{j=1:::q}$ the mass distribution associated to $(\pi_{\alpha j})_{j=1:::q}$ (Baudrit, Dubois and Guyonnet (2006)).

Imprecise Probability

An objective probabilistic representation is incomplete if a family of probability functions $\mathcal{P}$ is used in place of a single distribution P because the available information is not sufficient for selecting a single one in $\mathcal{P}$ (Dubois and Guyonnet (2011)).

Let $\mathcal{P}$ be a probability family on the referential Ω. For all $A \subseteq \Omega$ measurable, we can define (Baudrit, Dubois and Guyonnet (2006)):

$$\text{its upper probability } \overline{P}(A) = sup_{P\in\mathcal{P}}P(A) \tag{12}$$

$$\text{its lower probability } \underline{P}(A) = inf_{P\in\mathcal{P}}P(A) \tag{13}$$

The upper bound $\overline{P}(A)$ can be used to measure the degree of plausibility of A, evaluating to what extent A is not impossible, i.e., there is no reason against the occurrence of A. The lower bound $\underline{P}(A)$ can be used to measure the degree of certainty of A (Dubois and Guyonnet (2011)).

At Ω, the follow property holds: $\underline{P}(A) = 1 - \overline{P}(\overline{A})$, where $\overline{A}$ is the opposite event of A. It expresses the idea that an event A is certain if and only if its opposite is impossible. Each event A is then assigned an interval $[\overline{P}(A), \underline{P}(A)]$, which is larger as information is lacking. In the face of ignorance, the consistent representation consists of using the trivial bounds [0, 1] (Dubois and Guyonnet (2011)).

Let $\mathcal{P}(\underline{P} < \overline{P}) = \{P, \forall A \subseteq \Omega, \underline{P}(A) \leq \mathrm{P(A)} \leq \overline{P}(A)\}$ be the family probability induced from upper $\overline{P}$ and lower $\underline{P}$ probability induced from $\mathcal{P}$. The notion of cumulative distribution function becomes a pair of upper & lower cumulative distribution functions $\overline{F}$ and $\underline{F}$ defined as follows (Baudrit, Dubois and Guyonnet (2006)):

$$\forall x \in \mathbb{R} \;\; \overline{F}(x) = \overline{P}(X \in]-\infty, x]) \tag{14}$$

$$\forall x \in \mathbb{R} \;\; \underline{F}(x) = \underline{P}(X \in]-\infty, x]) \tag{15}$$

Where X is a random variable associated to P. The gap between $\overline{F}$ and $\underline{F}$ reflects the incomplete nature of the knowledge, thus picturing what is unknown (Baudrit, Dubois and Guyonnet (2006)).

We can interpret any pair of dual functions necessity/possibility [*N*; ∏], or belief/plausibility [*Bel*; *Pl*] as upper and lower probabilities induced from specific probability families (Baudrit, Dubois and Guyonnet (2006)).

Let π be a possibility distribution inducing a pair of functions [*N*; ∏]. We define the probability family $\mathrm{P}(\pi) = \{P, \forall A\ measurable,\ N(A) \leq P(A)\} = \{P, \forall A\ measurable,\ P(A) \leq \Pi(A)\}$. In this case, $sup_{P\in\mathcal{P}(\pi)}P(A) = \Pi(A)$ and $inf_{P\in\mathcal{P}(\pi)}P(A) = \mathrm{N}(A)$ (Baudrit, Dubois and Guyonnet (2006)).

Conversely, given $A_1 \subseteq A_2 \subseteq \cdots \subseteq A_n$ some measurable subsets of Ω with their confidence degrees $1 - \alpha_1 \leq \cdots 1 - \alpha_n$, we define the probability family as follows (Baudrit, Dubois and Guyonnet (2006)):

$$\mathcal{P} = \{P, \forall A_i, 1 - \alpha_1 \leq P(A_i)\} \tag{16}$$

We thus know that $\overline{P} = \Pi$ and $\underline{P} = N$. We hence can define upper $\overline{F}$ and lower $\underline{F}$ cumulative distribution functions such that $\forall x \in \mathbb{R} \;\; \underline{F}(x) \leq F(x) \leq \overline{F}(x)$ with (Baudrit, Dubois and Guyonnet (2006)):

$$\overline{F}(x) = \Pi(X \in]-\infty, x]) \tag{17}$$

$$\underline{F}(x) = N(X \in]-\infty, x]) \tag{18}$$

A mass distribution v may encode probability family $\mathcal{P} = \{P, \forall A\ measurable,\ Bel(A) \leq \mathrm{P}(A)\} = \{P, \forall A\ measurable,\ P(A) \leq \mathrm{Pl}(A)\}$. In this case we have: $\overline{P} = \mathrm{Pl}$ and $\underline{P} = Bel$, so that (Baudrit, Dubois and Guyonnet (2006)):

$$\forall P \in \mathcal{P}, Bel \leq P \leq Pl \tag{19}$$

Hence, we can define upper $\overline{F}$ and $\underline{F}$ cumulative distribution functions such that $\forall x \in \mathbb{R} \;\; \underline{F}(x) \leq F(x) \leq \overline{F}(x)$ with (Baudrit, Dubois and Guyonnet (2006)):

$$\overline{F}(x) = \mathrm{Pl}(X \in]-\infty, x]) \tag{20}$$

$$\underline{F}(x) = Bel(X \in]-\infty, x]) \tag{21}$$

Therefore, both Possibility Theory and Evidence Theory into can be worked with a single probabilistic framework respectful of the incompleteness of the available information. Possibility distributions π and mass distributions v then encode probability families $\mathcal{P}(\pi)$ and thus allow to represent incomplete probabilistic knowledge. The intervals [*N*; ∏] induced from π and [*Bel*; *Pl*] induced from v thus provide some bracketing of ill-known probabilities (Baudrit, Dubois and Guyonnet (2006)).

Uncertainty Propagation

Typical probabilistic calculation can be performed by the Monte Carlo method for stochastic uncertainty (variability). However, when epistemic uncertainty is also presented, specific or hybrid methods shall be used for proper handling of each type of variable.

R/RStudio software was used and, more specifically, the HYRISK library to perform the required calculation regarding Possibility Theory and uncertainty propagations. Independent Random Sampling (IRS) algorithm was used to perform uncertainty propagation, which is briefly described in this section. Further information can be found on Rohmer et al. (2017) and Dubois and Guyonnet (2011).

As described by Dubois and Guyonnet (2011), the IRS method exploits the fact that the theory of evidence encompasses both Possibility and Probability Theories. A combined Monte Carlo random sampling procedure is applied not only to the probability distributions (stochastic variables), but also to the fuzzy intervals (possibility distributions $\pi_1, \ldots, \pi_m$ of each imprecise parameter (epistemic variables)), generating a random fuzzy interval as the system output.

The procedure is as follows (from Dubois and Guyonnet (2011)):

1. Generate $n + m$ random numbers $(X_1, \ldots, X_{n+m})$ in $[0,1]^{n+m}$ from a uniform distribution on $[0,1]$

2. Sample the n PDF's to obtain a realization of the n random variables: $r_1, \ldots, r_m$

3. Sample the m fuzzy intervals $\pi_1, \ldots, \pi_m$ each at different level X_{n+i} to obtain m intervals: $I_1, \ldots, I_m$, where $I_1 = \{r, \pi_i(r) \geq X_{n+i}\}$.

4. Calculate the *Inf* (least) and *Sup* (greatest) values of $z = f\{p_1, \ldots, p_n, I_1, \ldots, I_m\}$, using interval analysis, considering all values located within the intervals $I_1, \ldots, I_m$.

5. Return to step 1 to generate a new realization of the random variables and the fuzzy sets. Repeat ω times.

Finally, a random interval is obtained.

APPLICATION AND RESULTS

Example 5.2.2 from Tuegel et al. (2018) was analyzed using the new proposed approach of this paper.

A trunnion collar from a Main Landing Gear (Fig. 1) failed during the full-scale fatigue test after 2,310 simulated landings.

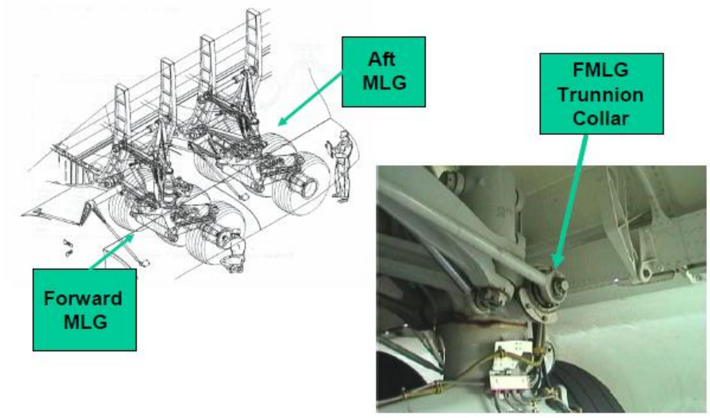

Figure 1 – MLG trunnion collar location, from Tuegel et al. (2018)

The trunnion collar was made from 300M steel with $F_{Tu} = 1930\ MPa$. Due manufacturing tolerances the stress concentration factor (Kt) ranges from 10 to 13. The objective is to determine when this structure will reach a $SFPOF$ of 10^{-4}.

Using Possibility Theory approach and following the flowchart proposed by Dubois and Guyonnet (2011) for variables categorization, α was modeled with a fuzzy triangular distribution and Kt with a fuzzy uniform distribution (Fig. 2).

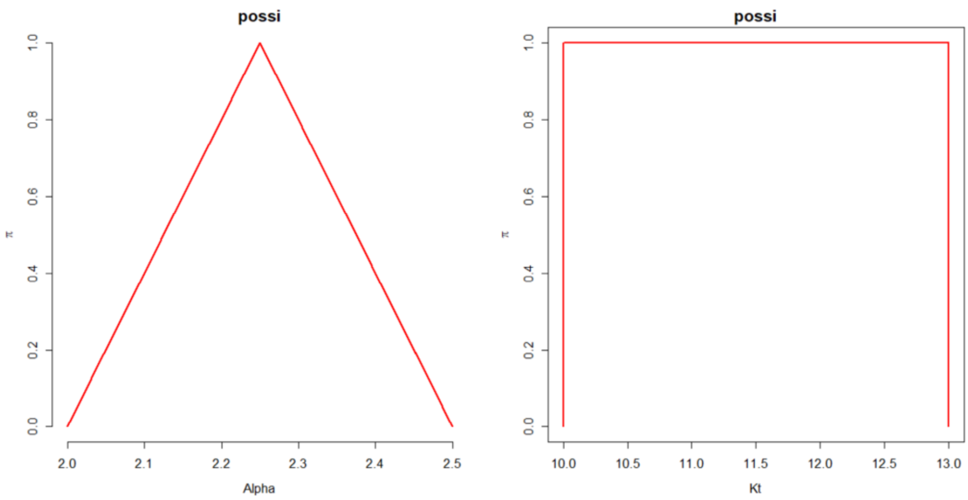

Figure 2 – α and Kt possibility distribution

When a single failure occurs, it is reasonable to interpret the observed test life as the mean of the probability distribution (μ) (Tuegel et al. (2018)). For the test failure at 2310 FC, it is determined an equivalent fatigue stress using a SN curve from MMPDS-11, Chapter 2, page 2-69 (2016). The mean life of 2310 FC ($N_{f\mu}$) and its derived fatigue stress (Seq_{μ}) are then associated with Kt mean value (Kt_{avg}).

The Kt variability is assumed to linearly affect the equivalent fatigue stress as follows:

$$Seq_{Kt} = Seq_{\mu}\frac{Kt_{\pi}}{Kt_{avg}} \tag{22}$$

Where:

$Kt_{avg} = mean\ Kt\ from\ possibilisty\ distibution$ (= 11.5)

$Kt_{\pi} = variable\ Kt\ from\ possibilisty\ distibution$

$Seq_{\mu} = Equivalent\ Fatigue\ Stress\ for\ meand\ life(N_{f\mu})\ and\ Kt_{avg}$

$Seq_{Kt} =\ Equivalent\ Fatigue\ Stress\ a\ given\ Kt\ of\ possibilisty\ distibution$

For any given equivalent fatigue stress associated with Kt variation, it is calculated its associated mean fatigue life (N_f) using the SN curve from MMPDS-11 (2016).

The Weibull scale parameter (β) can be calculated from the test-estimated mean (N_f) as follows (Tuegel et al. (2018)):

$$\beta = \frac{N_f}{\Gamma\left(1+\frac{1}{\alpha}\right)} \tag{23}$$

The Hazard Rate from a Weibull distribution is (Tuegel et al. (2018)):

$$h(t) = \left(\frac{\alpha}{\beta}\right).\left(\frac{t}{\beta}\right)^{(\alpha-1)} \tag{24}$$

So, the $SFPOF$ is:

$$SFPOF = h(t).\Delta t = \left(\frac{\alpha}{\beta}\right).\left(\frac{t}{\beta}\right)^{(\alpha-1)}.\Delta t = \left(\frac{\alpha}{\beta}\right).\left(\frac{t}{\beta}\right)^{(\alpha-1)}.1 = \left(\frac{\alpha}{\beta}\right).\left(\frac{t}{\beta}\right)^{(\alpha-1)} \tag{25}$$

For a given $SFPOF$ target, the moment of this occurrence can be defined by rearranging Eq (25), resulting in:

$$t = \beta.\left(SFPOF.\left(\frac{\beta}{\alpha}\right)\right)^{(1-\alpha)} \tag{26}$$

The time (t) that a given $SFPOF$ is reached depends both on α (a possibilistic variable) and β (which depends on Kt, another possibilistic variable).

R scripts was constructed implementing the presented formulation using IRS algorithm from R HYRISK package for the uncertainty propagation calculation.

Results are present on Fig. 3 where the CDF for the moment t (in FC) that $SFPOF$ = 10^{-4} is reached. The interval for CDF=95% ranges from 221 FC to 1244 FC, considering the Possibility and Necessity curves. Considering conservatively the lower bound, the 221 FC would be the point for a safe intervention on fleet.

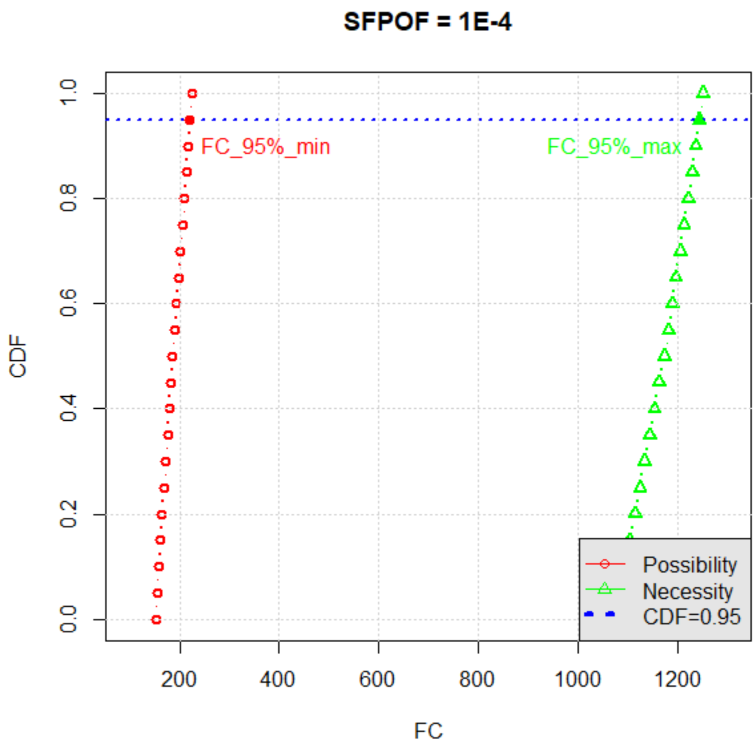

Figure 3 – CDF of $SFPOF$ = 10^{-4} point in FC

The use of the bounds can lead to overly conservative or overly non-conservative results. While in some cases the overly conservative results may be acceptable, for other it may be desirable to look for more reasonable alternatives. In this context, Dubois and Guyonnet (2011) proposed the "Confidence Index" (CI), which weight in both Possibility (or Plausibility) and Necessity (or Belief) outputs. At HYRISK package (Rohmer et al. (2017)), the similar "Aversion Index" (AI) is used, where $AI = 1 - CI, and$:

$$P_{AI} = (1 - AI).\Pi + AI.N \tag{27}$$

An $AI = 0.5$ results in an average curve between Π and N. An $AI < 0.5$ yields higher weight in Π results, and vice versa.

Dubois and Guyonnet (2011) suggest giving more weight to the pessimistic probability bound, than to the optimistic bound (1/3), justifying that "in a context of aversion to risk, it would seem normal to privilege the pessimistic limit, but without completely obliterating the optimistic one".

Figure 4 shows the curves achieved for $AI = 0.5$ and 0.05, resulting respectively in an average curve and a 95% lower bound for the epistemic uncertainty interval. Those curves may have a similar interpretation of confidence levels of probabilistic calculations, but with the important differentiation that confidence levels are derived from stochastic variation, while these current bounds are related to the epistemic uncertainties.

Considering 95% epistemic lower bound and 95% level of CDF, the 265 FC would be the point for a safe intervention on fleet.

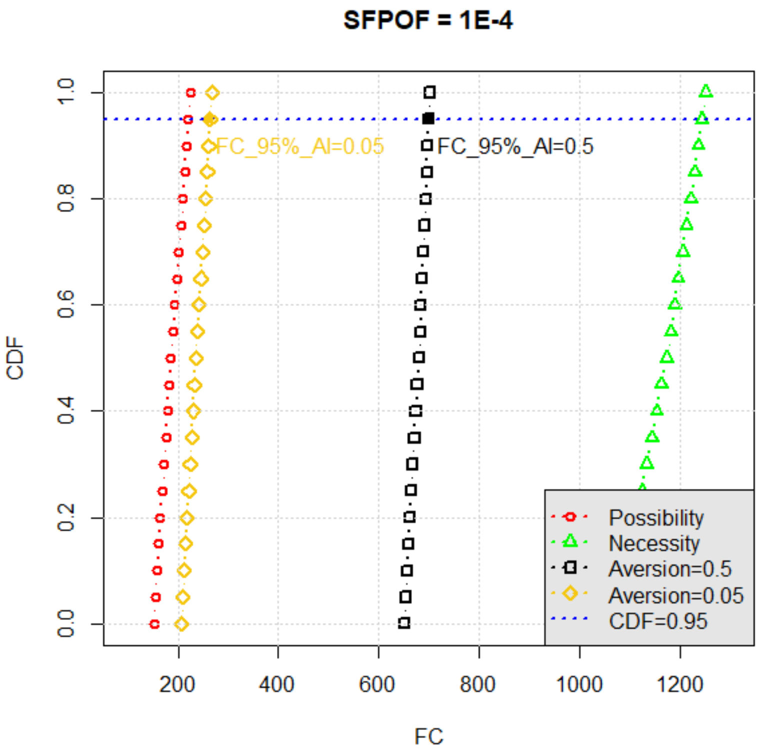

Figure 4 – CDF of $SFPOF$ = 10^{-4} point in FC – Aversion Curves

The effect on variability due to only material variation (α) can also be evaluated by "pinching" (fixing) Kt at its average level of 11.5. The minimum value on Possibility pinched curve is 340 FC (Fig. 5), which matches the calculation from Tuegel et al. (2018), that disregarded Kt variability and took the minimum (most conservative) value of α (α =2). It can be noted that, for this case, the epistemic uncertainty is greatly reduced once the Kt variability is suppressed (as both Possibility and Necessity curves approaches each other).

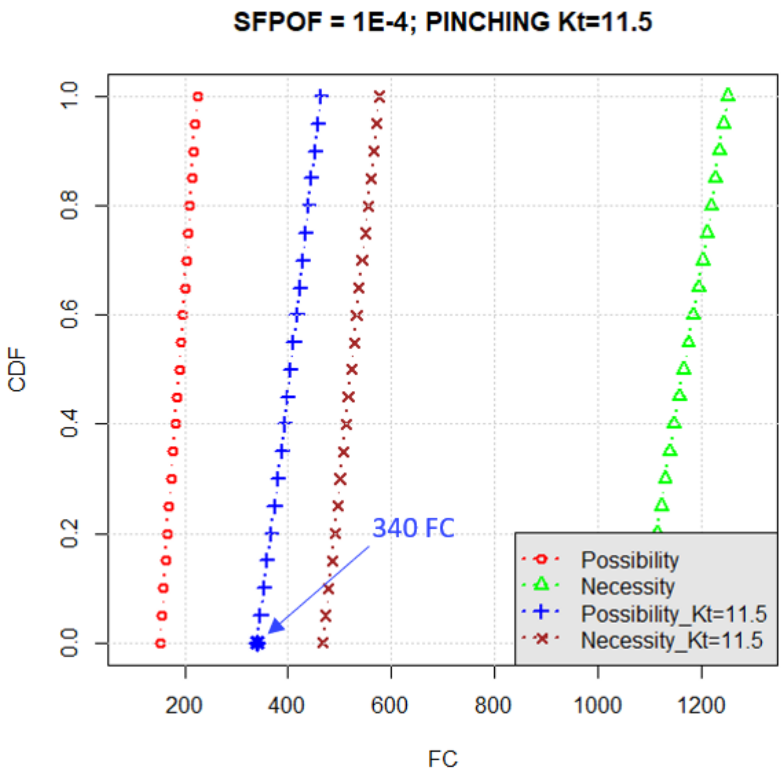

Figure 5 – CDF of $SFPOF$ =10^{-4} point in FC; Kt Pinching Results

CONCLUSIONS

The proposed approach helps to better quantify uncertainty in the prediction of aircraft structural reliability/risk when only limited data is available, by using the Possibility Theory and indicating a range of possible outcomes. This gives a better and most realistic picture of reliability/risk ranges and each factor contributions, allowing for more appropriated and sound decisions.

For the given example, while the original analysis of Tuegel et al. (2018) disregarded the Kt variability, the performed simulation indicated this one as the most relevant for results variability. Even with conservative assumptions on the material variability (α), that case could led to non-conservative results by suppressing Kt variability. The Possibility Theory approach brings a practical way to combine those inputs, calculating a reasonable and useful output.

While this article presented a simple example, further works can explore more complex and complete scenarios involving a mixed interaction of both probabilistic (distributions well characterized) and possibilistic (with epistemic uncertainties) inputs, as well as more realistic/complex formulations (as the use of Kf – Fatigue Notch Factor – instead of Kt, e.g.). Material, geometry, loading, operational profile, etc can be modeled and considered using its proper classification and mathematical handling, yielding a complete and well described risk panorama.

ACKNOWLEDGMENTS

This work was supported by CNPq Grant No. 311972/2020-9.

REFERENCES

Baudrit, C., Dubois, D. and Guyonnet, D. 2006. Joint propagation and exploitation of probabilistic and possibilistic information in risk assessment. IEEE transactions on fuzzy systems. pp. 593-608, 2006.

Dubois, D. and Guyonnet, D. 2011. “Risk-informed decision-making in the presence of epistemic uncertainty.” International Journal of General Systems. Taylor & Francis, 2011, Vols. pp.145-167.

Freudenthal, A. M., 1975, "Reliability Assessment of Aircraft Structures Based on Probabilistic Interpretation of the Scatter Factor," Air Force Materials Laboratory, Wright-Patterson AFB, OH.

Kiureghian, A. D. and Ditlevsen, O. 2009. “Aleatory or epistemic? Does it matter?” Structural Safety 31., pp. 105-112.

MMPDS-11, 2016 “Metallic Materials Properties Development and Standardization”. Battelle, Columbus, Ohio

Tuegel, E. J., Bell, R. P., Berens, A. A. P., Brussat, T., Cardinal, J. W., Gallagher; J. P. and Rudd, J., 2018, “Aircraft Structural Reliability And Risk Analysis Handbook. Volume 1: Basic Analysis Methods.” Wright-Patterson Air Force Base, OH: Air Force Research Laboratory, v.1. (AFRL-RQ-WP-TR-2018-0180)

O’Connor, P. D. T. and Kleyner, A. 2012. “Practical Reliability Engineering”. John Wiley & Sons, Ltd.

Rohmer, J., et al. 2017. Package ‘HYRISK’ - Hybrid Methods for Addressing Uncertainty in RISK Assessments. R Software.

Whittaker, I. C. and Besuner, P. M., 1969, “A reliability analysis approach to fatigue life variability of aircraft structures.” Boeing Company.

Wierman, M. J., 2010. “An Introduction to the Mathematics of Uncertainty.” Creighton University.

Zio, E. and Pedroni, P. 2013. “Methods for representing uncertainty - A literature review”. Toulouse, France : Fondation pour une Culture de Sécurité Industrielle, 2013. ISSN 2100-3874.

RESPONSIBILITY NOTICE