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MODEL AND ANALYSIS OF SEVERO´S PAX DIRIGIBLE

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Abstract: *The cold morning of the 12th of May of 1902 was to be a special one in the life of Augusto Severo. The semirigid dirigible Pax, that he conceived and designed, was ready for its inaugural flight. After months of construction, the event called great attention. After greeting the public, Severo and his mechanic, cut the ropes that tied the ballon to ground. A fast rising, followed by a fire, took place next. In instants the Pax was destroyed leaving no survivals. Despite the accident the ballon was considered advanced for its time with many innovations. This work addresses then the modeling and analysis of this semi-rigid structure, from drawings encountered in the literature, to very its strength. A finite element model is used to qualify the structure that sustained the ballon. The strength is found as being acceptable to various figures of merit.*
Keywords: *Pax airship, motion, structure, finite element model, safety factor*

1. INTRODUCTION

By 1881 Augusto Severo was already interested in aerostats. In that year he designed the Potyguarania, a dirigible with some new ideas. The name was a tribute to the state of Rio Grande do Norte, where he was born. Even though never constructed, in 1892, a grant of the government allowed him to try check ideas in a new balloon, the Bartolomeu de Gusmão, named in deference to the priest born in Santos, a pioneer in the area with his Passarola (Louro, F.,2014). Gusmão was a semi-rigid dirigible that used an internal structure capable of avoiding the pitch moment present in the designs of that time. Constructed in France, this aerostat with a structure build with bamboo, lack strength.

In 1901, Severo after the previous experience, designed a new airship, the Pax, looking for the 100 thousand Francs of award from a competition put forth by the *Aeroclub de France*. Henry Deutch of la Meurthe, a French entrepreneur, offered one hundred thousand francs for the first pilot able to depart from Paris, that could, in a non-stop flight, travel in less than half an hour a distance of 11 km, having the Eiffel Tower in the route, [1]. For that sake Severo improved his knowledge about aerostats, using a semi-rigid solution to avoid pitch, with maneuverability obtained with from a set of engines and propellers. The balloon was huge, with a volume of 2 344 m^3, 13 m radius in the central region and 20 m height. It was, equipped with two internal combustion engines, of 16 and 24 Hp, 7 propellers being one in the bow, one around the barge and four around the main body, working for propulsion. No rudder was used. The main propellers were held in the main beam of the supporting structure, so as to help pitch moments control. An internal structure was created to support the barge, where crew and load were placed, the balloon envelope plus the thrust and control equipment. The barge had a platform that was 15 m long by 1 m wide, Fig. 1.

On May, 12 at 5:30 am, with a large crowed present, Severo and one mechanic started the flight. The voyage began well with the craft reaching about 100 m in that cold morning, very fast. Then one of the propellers stopped. Trying probably reach higher altitudes, Severo threw away some ballast, common practice on those days. In a few moments the airship reached an altitude of about 400 m. It was then that a fire started in the balloon – possibly from overheating and self-ignition of hydrogen. The exact causes of the problem are not known, but didn't seem precipitated by any structural failure. In a few minutes without the envelope, the main structure fell to the ground, with the crew trapped in the barge. The fall point is at avenue du Maine, and made headlines worldwide. This fatality made the republic deputy Severo the first victim of the airship history.

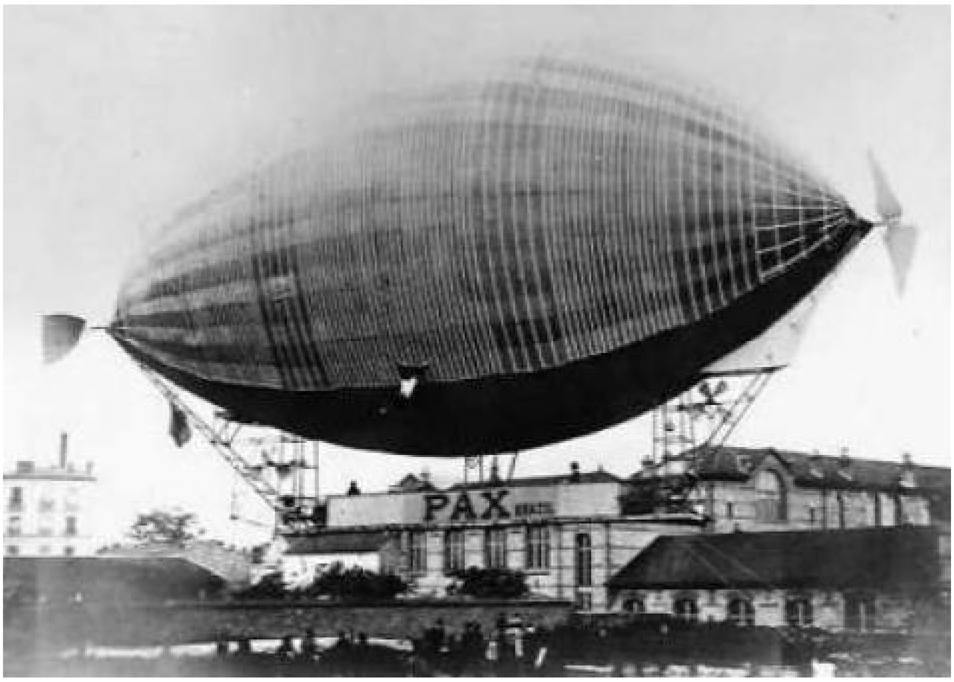

Fig. 1 Airship PAX at the gas loading phase

2. MODEL PARAMETERS

In order to evaluate the design of airship PAX, geometry of the balloon, obtained from the loading process coupled to the anchorage and fixing to the main structure, needs to be considered for form and contact pressures. Loading requires that the flight equations be reviewed, for vertical ascension as well horizontal plane drift. This is so because of the many scenarios in a voyage.

2.1 Loading Phase and Envelope Geometry

The Pax was constructed in Paris, in separate units, being the envelope built in a closed place, hangar, and the vertical structure of the airship in an open space. The constructor was Henri Lachambre. The construction took many months and consumed all the resources that Severo had, about 150 thousand francs. Many times equipment and structural parts had to be exchanged for defects after testing. This consumed time and resources. Months went away and only in April of 1902 the airship was ready.

The envelope was loaded first with hydrogen in a procedure that took five days, ending only in May first. This was a simultaneous process as hydrogen, at low temperatures, was delivered to site as the balloon was being inflated. The thermodynamic expansion from low temperatures to the ambient one, about 0ºC, occurred continuously. In all, more than 2000 m^3 of filling gas were employed. During this assemblage phase, Severo participated somewhere else in the ascensions of three balloons as he had to master the art of the guiding balloons. He was to be the pilot of his own invention.

Tying the loaded envelope to the structural part required use of ropes and ballast. Normally the balloons would be lowered with ropes, then when the touching the main structure starts, a fitting deformation process would ensue, with the presence of contact pressure. An upper value for these pressures may be estimated by considering the burst strains of the fabric used in the envelope and the tear resistance of the ropes. The form of the balloon changes in the process. Once tied to the barge of the main structure, ballast is used to equilibrate the airship. Control and propelling equipment is assembled next. For the Pax, beam elements were assumed having rectangular-circular forms, with 150x50 mm size, whereas the bamboo bars were supposed hollow round, with diameters in the 50-80 mm range and thickness of 25% external diameter.

Loading a large balloon required many workers. As the process evolved, the mix of cylindrical and elongated elliptical forms of Pax arose. On preparation days prior to the flight, many people visited the hangar of PAX, as its size was impressive. The geometry of a balloon envelope depends of the fabric used, its thickness, filling gas and the way it is tied to the central structure. The form and dimensions used in this work were produced from pictures of *époque.*

Two ascensions trials of Pax took place in May, with the airship kept fasten to the ground by ropes, avoiding the lateral wind as the days were of calm air. The load tests of May, 4 and May, 7 approved the balloon. George Sache, a French mechanic that had worked with engines Bucket that propelled the airship was aboard everyday helping in the control of the air ship. He would day with Severo days later.

2.2 Flight Equations

Motion of the airship may be divided into two parts: rising and drifting motion. Once ready, with positive buoyancy, the airship is launched with release of tying ropes. On the first phase, the unbalance of vertical buoyancy B and weight W causes the ship to accelerate ascending against a mounting drag force D_v acting in the vertical direction, if wind and lateral thrust are not present. Vertical equilibrium requires an inertia force, with vertical acceleration $a = v_{v,t}$ being v_v the instantaneous velocity (Mingireanu el all, 2014) :

$$m\frac{dv_v}{dt} = B - W - D; \quad B = L - W_e \tag{1}$$

where the $B = (\rho_{air} - \rho_{gas})V_b$, with $V_b = V_b^c + V_b^e$ the cylindrical and elliptical volume contributions. Air and balloon gas densities are denoted as ρ_{air} and ρ_{gas}, respectively. The weight term $W = mg$ comprises the mass contributions from the supporting structure m_s, the mechanical equipment m_m, crew m_c and load m_l, fuel and ballast, for example. Total mass is denoted as m. The drag for motion in the vertical direction comprises the effect of horizontal plan area $A_h = A_c + A_e$, cylindrical and elliptical, air density, ascension velocity and vertical drag coefficient, C_d^v, $D_v = 0.5\rho_{air}A_h C_d^v {v_v}^2$.

If the variation of temperature in the ascension is not large, variations in balloon geometry that affect drag, may be disregarded, and Eq. (1) then leads to a differential equation in terms of the velocity:

$$m\frac{dv_v}{dt} = a - bv^2; \quad a = B - W; \quad b = 0.5\rho_{air}A_h C_d^v \tag{2}$$

whose solution will depend on the explicit integral:

$$\int_0^{v_v} \frac{dv_v}{a - b{v_v}^2} = \int_0^t \frac{dt}{m} \tag{3}$$

for constant drag coefficients. The first integral results in:

$$I = \ln\left|\tan[\frac{1}{2}\mathrm{arc}(\cos(\sqrt{\frac{b}{a}}\,v_v)]\right| \tag{4}$$

That needs to be evaluated between initial, $v_v(0) = 0$, and present velocity $v_v(t)$. Once obtained the vertical velocity field, the vertical displacement from initial position $d_v(0) = 0$ may be obtained from further integration:

$$d_v = \int_0^{v_v} v_v dt \tag{5}$$

This condition, however, prevails for a short while, as the drag increases fast with the increase of velocities, ending when a limit ascension velocity is reached:

$$v_v = \sqrt{\frac{a}{b}} \tag{6}$$

As the temperature of air decreases as the height increases, if no filling gas losses are allowed, this constant velocity condition may be kept with use of the ballast.

The horizontal drift problem follows similar path; as lateral propulsion or gusty winds may occur. Equilibrium in this case is written as (Ling, 2015):

$$m\frac{dv_h}{dt} = T \pm F_w; \quad F_w = \frac{1}{2}\rho_{air}A_h C_d^h {v_h}^2 \tag{7}$$

The same sort of solution procedure used in Eq. (1) may be used. The velocity caused by the actuation of the propellers, with no wind leads to:

$$v_h = \frac{-1}{{v_{ho}}^{-1} + \alpha m^{-1} t}; \quad \alpha = 0.5 \rho_{air} A_h C_d^h \tag{8}$$

2.2 Structural Response

The design procedure for the required dimensions of the vertical structured devised by Severo for semi-rigid Pax needed measurement of the long dimensions of the parts of the frame, Fig. 2. This was obtained by photo mapping and comparison with some known dimensions, like basic length and height for example. Sectional dimensions had to be computed from frame analysis, after initial choice of material and guess of profiles and sectional parameters. A design scenario was chosen, with quasi-static conditions prevailing. As the structure was essentially 1D, with presence of beams and bars, with some flexible ropes, the finite element method as applied to 1D elements was chosen.

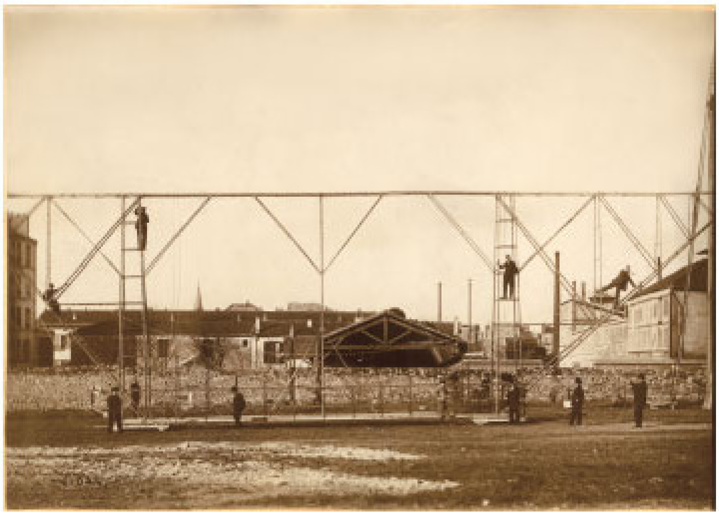

Figure 2. Pax structure in construction out of the hangar of Vaugirard, Paris, 1901

The principle of virtual work is the basis of the finite element method: the internal virtual work produced by internal resultants - normal n, shear s and bending moment m - work over the increments of associated virtual displacements $< \delta u, \delta v, \delta\theta >$ to render the internal virtual work (Zienkiewicz, 2013):

$$\delta W_i = \sum_{1}^{n_p} \int_{l_i} [n \partial \delta u + s \partial \delta v + m \partial \delta\theta] \tag{9}$$

for the n_p parts that comprise the structure, during the loading time. The external virtual work is performed by surface tractions, of axial and lateral components $< a, l >$ plus body forces, $< b_a, b_l >$ of the same type:

$$\delta W_e = \sum_{1}^{n_p} \{ \int_{l_i} [a \partial \delta u + p \partial \delta v] + \int_{l_i} [b_a \partial \delta u + b_l \partial \delta v] \} \tag{10}$$

for plane conditions. Point loads are treated as special case. Equilibrium requires equating these pair of equations:

$$\sum_{j}^{n_p} \int_{l_i} [n\, s\, m]^T \; [\delta u \; \delta v \; \delta\theta] \; dl = \sum_{j}^{l} \int_{l_j} [a\, l]^T [\delta u \; \delta v] dl + \sum_{j}^{n_p} \int_{l_j} [b_a \; b_l]^T [\delta u \; \delta v] A dl \tag{11}$$

Discretized, each part may have the displacements written in terms of interpolation functions and end of the values, once discretization of the structure is applied:

$$\boldsymbol{u} = N^n \boldsymbol{u}^n; \quad \boldsymbol{v} = N^n \boldsymbol{v}^n \tag{12}$$

being N^n the interpolation functions, and u^n, v^n the components of nodal displacements. Applying this to the above:

$$\sum_{j}^{n_p} \delta d_j^{n} \overset{T}{\int_{l_j}} N^{n\,T} i \,dl = \sum_{j}^{n_p} \delta d_j^{n} \overset{T}{\int_{l_j}} N^{n} (t+b)\,dl \tag{13}$$

where $d^n = [u^n v^n \theta^n]$ with $\theta^n = v_{,xx}{}^n$ are nodal displacements, $i^t = [EAu_{,x}\ GAv_{,x}\ EIv_{,xx}]$ are internal efforts, $t^T = [a\,l]$ and $b = [b_a\ b_l]$ are body forces per unit length. Considering equilibrium on an element by element basis, the virtual displacements cancel out, leaving the equilibrium of internal resultants i and external efforts $t+b$ in the form:

$$\sum_{j}^{n_p} k_j d^n{}_j = \sum_{j}^{n_p} (t_j + b_j) \tag{14}$$

with axial, shear and flexural stiffness matrices making up matrix k . Normally this equation can't be solved as it stands, because beams have different orientations. Considering a global coordinate system, with transformation matrix T^j pre-multiplying each element stiffness matrix, followed by collection of an all element sum, gives:

$$KD^n = T; \quad K = \sum_{j}^{n_p} K_j; \quad T = \sum_{j}^{n_p} T_j + B_j \tag{15}$$

Being that $K_j = T^j k_j T_j^T$ is the beam element stiffness in global coordinates whereas $< T\,B._j = T^j < t_j\,b_j >$ is the resulting traction vector for the element. This process is known as assemblage, being constructed with due consideration of the interfacing elements in the moment of building the structural matrix K Solution for the nodal displacement field, virtually would take, $D^n = K^{-1}T$.

There is no assurance that the main structure of Pax was constructed of bamboo, reason why it was assumed that the upper beam and barge floor beams were made of light wood. The rest is assumed as being bamboo. The reason for this choice is that the bad experience of Severo with in failure of previous airship Gusmão, would make him take such a care. Table 1, includes the principal mechanical properties of the structure of Pax.

Tabela 1. **Mechanical Properties of Structural Materials of Pax**

Material	Bamboo	White Pine
Elastic Modulus E, GPa	14.28	10.7
Poisson's ratio ν	0.25	0.30
Yield Strength S_{yt}, MPa	50.2	25.1
Density ρ, kg/m^3	960	458

Loading of the structure consisted of the fixing load provided by the ropes in the tying procedure, the loading coming from the contact and accommodation of the envelope to the main beam – supposing the upper value relative to the tearing condition. Ropes were also supposed not to tear. One propeller working at the extremity of the beam, with its thrust applied was also considered, in the quasi static analysis.

The drawing of the structure required several planes of drawing being used, and lead to in plane intersections. In a 3D drawing, using solids the same technique had to be used, providing more details, but only good under the view point of analysis. They are not included here, but do corroborate the trends.

In the design procedure, initial values for the sectional properties were assumed. The solver a finite element program (Abaqus^{TM} ,2015) was used to obtain the displacement, strain and stress field. Discretization considered 0.25 m length elements. Values of Mises stress were used to ascertain presence of any inelasticity occurrence. Several runs were considered, with checking to the mass of the structure and required ballast of the aerostat. Figure 3 shows the primary structure with loading and boundary conditions. In Figure 4, the stress field, in the augmented deformed configuration of half structure is presented. The main beam at the top of the structure, in its central part shows larger stresses, with a safety factor around 2. Critical sections occurred close to the upper beam in central area, where the main pillar is. Flexibility was larger in this beam. Hardly bamboo with its variation of properties would be indicated for such an important part. This corroborates the fact that the structure was quite strong, being able to support the impact derived from the accident.

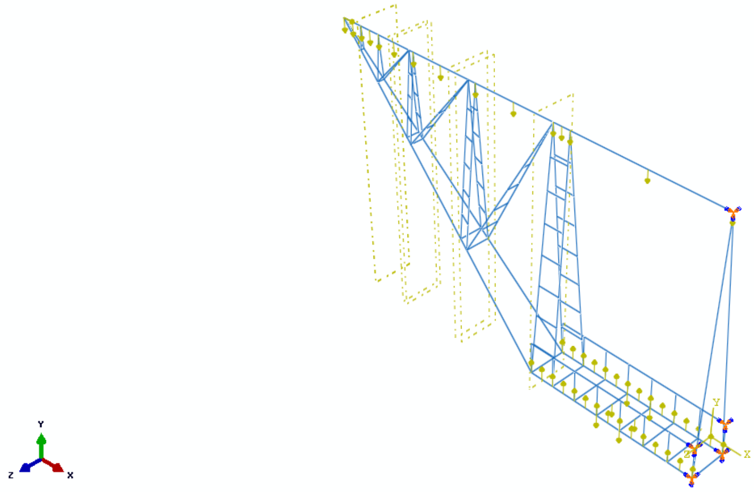

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Figure 3. Half Pax primary structure under loading

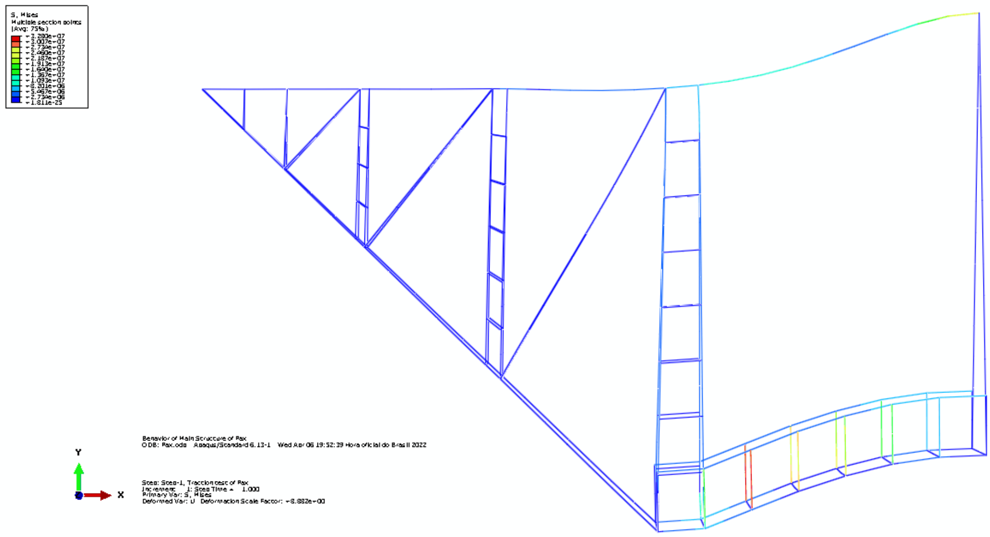

Figure 4. PAX Mises stress field on deformed half structure

2.3 Conclusions

This work was developed to review one of the important designs in the engineering history of Brazil. It took many decisions as there aren't but images left of the Pax. However, it was possible to get a possible Pax design, that the finite element design and analysis procedure verifies as reliable. Loading process with the deformation of the envelope to accommodate the main structures was not solved here. It would as well as the simulation of flight bring additional facts to the solution.

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4. RESPONSABILITY

The authors are the only responsible by the contents of this work.