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Topology Optimization with Volume and Natural Frequency Constraints by Using the TOBS Method

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Abstract. A challenge for topology optimization methods is to incorporate multiple constraints into the structural design. This type of problems can be important for the design of vibrating structures, allowing to change the structural layout so that the natural frequencies of the structure deviates from the operating frequency while maintaining the structural stiffness. In this paper, it is proposed to perform topology optimization with volume and natural frequency constraints to obtain a stiff design. The Topology Optimization of Binary Structures (TOBS) method is employed. The equilibrium equations are solved by using finite element method. The modified SIMP model is used as a material interpolation model to obtain the natural frequency sensitivities. The first four natural frequencies for each case are observed using the Modal Assurance Criterion (MAC)-based mode-tracking seeking to evaluate mode crossing. The algorithm is tested and verified for two bidimensional problems, the biclamped beam and the tower subjected to a horizontal load. Different values of natural frequency constraints are tested, looking at the final topology and mode crossing. The penalty variation effect in final topology is verified. This is the first time a binary topology optimization method is employed to incorporate volume and natural frequency constraints simultaneously. Numerical results show that the TOBS method suits well the present problem, providing optimum layouts for structures subjected to vibrations maintaining the natural frequency in levels established in the optimization process.
Keywords: Topology Optimization, Integer Linear Programming, Natural Frequency, Modified SIMP Model.

INTRODUCTION

Topology optimization is a topic that has gained a lot of importance in recent years, due to the versatility of the method for producing resistant parts, saving material. Topology optimization methods can be classified into density methods, binary methods and boundary variation methods (Pedersen, 2000). Density methods are the most popular and the only ones applied so far in commercial software. The best known density method is Solid Isotropic Material with Penalization (SIMP) which became very popular with the publication of Sigmund's work (Sigmund, 2001). Density methods are currently the most developed and their application has been extended from the structural field to several other fields such as heat transfer, acoustics, dynamics, fluid flow, aeroelasticity (Pedersen, 2000).

Frequency optimization in dynamics system have great importance in many engineering fields such as aerospace, aeronautical and automotive. Despite the extensive applicability of SIMP model, its use in frequency optimization is challenging due to artificial localized modes in lower mass areas (Pedersen, 2000). To overcome this, the modified SIMP model using a discontinuous function has developed and has been applied successfully to solve the frequency optimization problems (Pedersen, 2000; Du and Olhoff, 2007). Almost at the same time, some authors has been proposed to solve the frequency optimization binary method Evolutionary Structural Optimization (ESO) (Xie and Steven, 1996; Zhao et al. 1997; Yang et al. 1999). However, the various deficiencies of the original ESO as pointed out by Rozvany and Querin (2002) also exist in this kind of problem (Huang et al. 2011). Later, the development of the Bidirectional Evolutionary Structural Optimization (BESO) method by Huang and Xie (2007), allowed Huang et al. (2011) to solve the frequency optimization problem using an alternative material interpolation.

From the presented works, most them used the frequency as an objective function to be maximized. Another optimization problem that can be proposed and that has great practical relevance in engineering is to put the natural frequency as a constraint and consider compliance. An interesting example would be in the application of rotating machines, where can one design a structure to have a natural frequency far from the rotor operating frequency and maximizing stiffness. At this point, the proposition of optimization problems with multiple constraints its relevant, e.g., volume and natural frequency.

In this context, the Topology Optimization of Binary Structures (TOBS) method was recently published, which is a binary method developed with formal mathematical programming using an integer optimizer to solve the optimization problem (Sivapuram and Picelli, 2018). This new method has an advantage over BESO by replacing the heuristic process with an integer linear programming (ILP) procedure, which allows define problems with multiple constraints without the need to use multipliers Lagrange (Sivapuram and Picelli, 2018; Sivapuram and Picelli, 2020). Furthermore, the method allows the definition of other types of constraints without having to change the source code. Sivapuram et al. (2018) extended TOBS method to design of binary microstructures, exploring the ability of the TOBS method to work with multiple constraints. In this work, it was proposed several optimization problems considering volume minimization with non-volume constraints, such as bulk and shear moduli, square/cubic symmetry, isotropy, thermal conductivity and a combination of them in two and three dimensions. Recently, Larsson et al. (2022) used the TOBS method to minimize the mass of 2D structures subject to static and time-harmonic loads considering static and dynamic compliance constraints.

In this context, the objective of this work is to perform the topology optimization of structures minimizing compliance subject to both volume and natural frequency constraints. For this, will be employed as optimization method the TOBS algorithm using the modified SIMP model to assist compliance sensitivities calculation and an alternative material interpolation proposed by Huang et al. (2011) to assist natural frequency sensitivities calculation. The classical bicampled beam and a tower with a horizontal load are solved for two different natural frequency constraints. Modes shapes were successfully tracked using MAC-based algorithm. The results show that the TOBS method can be used efficiently to optimize structures considering volume and natural frequency constraints.

TOPOLOGY OPTIMIZATION OF BINARY STRUCTURES

The TOBS method, proposed by Sivapuram and Picelli (2018), employs binary design variables $\{0,1\}$. This methodology linearizes the objective and constraint functions associated with integer linear programming (Williams, 2009). Therefore, the linearized optimization problem to be solved is given by:

$$\begin{aligned}
\underset{\Delta\mathbf{x}^k}{\text{Minimize}} \quad & \left.\frac{\partial f}{\partial \mathbf{x}}\right|_{\mathbf{x}^k} \Delta\mathbf{x}^k \\
\text{Subject to} \quad & \left.\frac{\partial g_i}{\partial \mathbf{x}}\right|_{\mathbf{x}^k} \Delta\mathbf{x}^k \leq \bar{g}_i - g_i^k \quad i \in [1, N_g] \\
& ||\Delta\mathbf{x}^k||_1 \leq \beta N_d \\
& \Delta x_j \in \{-x_j, 1-x_j\} \quad j \in [1, N_d]
\end{aligned} \tag{1}$$

where $f(\mathbf{x})$ is the objective function, bounded by $g_i(\mathbf{x}) \leq \bar{g}_i$. $i \in [1, N_g]$, where N_g and N_d are respectively the number of inequality constraints and elements in the vector of design variables. β is the flip limits and $||\Delta\mathbf{x}^k||_1$ is the truncation error. The term g_i^k is the value of the constraint g_i in the k_{th} optimization iteration. The ILP solver is used to find the optimal change $\Delta\mathbf{x}$ for the integer design variables $\mathbf{x}$. After each iteration, the design variables are updated as $\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$.

OPTIMIZATION PROBLEM AND MATERIAL MODELS

Optimization Problem

The formulation of the binary optimization problem from Eq. (1) is related to minimizing the mean compliance of the structure subject to a given volume and natural frequency constraints. The optimization problem is expressed as:

$$\begin{aligned}
\underset{x}{\text{Minimize}} \quad & C(x) \\
\text{Subject to} \quad & V_i(x) \leq \overline{V}_i, \quad i \in [1, N_g] \\
& \omega_1(x) \geq \overline{\omega}_1, \quad i \in [1, N_g] \\
& x_j \in [0,1], \quad j \in [1, N_d]
\end{aligned} \tag{2}$$

where $C(x)$ is the structural compliance, V_i is the volume fraction of the structure, $\overline{V}_i$ is the constrained volume fraction, $\omega_1(x)$ is the first natural frequency of the structure, and $\overline{\omega}_1$ is the constrained first natural frequency. The examples presented in this work are solved with one volume constraint and one natural frequency constraint, i.e., with N_g = 1.

Material Models

In this work, the modified SIMP and an alternative material interpolation models will be used to calculate compliance and natural frequency sensitivities, respectively. In the modified SIMP material model, proposed by Sigmund (2007), the intermediate material stiffness can be penalized with the power-law as:

$$E(x_j) = E_{min} + x_j^p (E_0 - E_{min}), \tag{3}$$

where p is the penalization factor, E_{min} is the stiffness of soft (void) material, and E_0 is the Young's modulus of solid material. Later, Felix et al (2021) proposed the use of an η variable to express the ratio between the defined E_{min} and E_0. From this, the Eq. (3) becomes:

$$E(x_j) = [\eta + x_j^p (1-\eta)] E_0, \tag{4}$$

Alternatively, the mass and stiffness of the elements as function of the design variables were interpolated using the scheme proposed by Huang et al. (2010). In this model, the density and Young's modulus of the material are expressed by:

$$\rho_(x_j) = \rho_0 x_j, \tag{5}$$

$$E(x_j) = E_0 \left(\frac{x_{min} - x_{min}^p}{1 - x_{min}^p)} (1 - x_j^p) + x_j^p \right), \tag{6}$$

where ρ_0 and E_0 denote the density and Young's modulus of the solid material, respectively.

SENSITIVITY ANALYSIS

From Huang and Xie (2007) and Han et al. (2021) the adjoint method can be used to determine the sensitivity of displacement and force vectors by introducing a Lagrangian multiplier vector λ. The sensitivity of the mean compliance can be expressed as:

$$\frac{dC}{dx_j} = -\frac{1}{2} \mathbf{u}^\mathbf{T} \frac{\partial \mathbf{K}}{\partial x_j} \mathbf{u}, \tag{7}$$

where $\mathbf{u}$ is the global nodal displacement vector, and $\mathbf{K}$ is the global stiffness matrix. Applying the modified SIMP model to obtain the derivative of global stiffness matrix related to design variables ($\partial \mathbf{K} / \partial x_j$):

$$\frac{dC}{dx_j} = -\frac{1}{2} p x_j^{p-1} (1-\eta) \mathbf{u_j}^T \mathbf{K} \mathbf{u_j} \tag{8}$$

As the natural frequency enters as a constraint in the optimization problem, it is necessary to calculate its derivative with respect to the design variables. The dynamic behaviour of a continuum structure using finite element method can be expressed by the following eigenvalues problem (Huang et al, 2010):

$$(\mathbf{K} - \omega_i^2 \mathbf{M}) \mathbf{u_i} = 0 \tag{9}$$

where $\mathbf{M}$ is the global mass matrix, ω_i is the ith natural frequency, and u_i is the eigenvector corresponding to ω_i. From Huang et al (2010), the sensitivities of natural frequency are given by:

$$\frac{d\omega_i}{dx_j} = \frac{1}{2\omega_i \mathbf{u_j}^T \mathbf{M} \mathbf{u_j}} \left[\mathbf{u_j}^T \left(\frac{\partial \mathbf{K}}{\partial x_j} - \omega_i \frac{\partial \mathbf{M}}{\partial x_j} \right) \mathbf{u_j} \right] \tag{10}$$

Substituting the derivatives of the global stiffness $\mathbf{K}$ and global mass $\mathbf{M}$ matrices and assuming that the eigenvector $\mathbf{u}_j$ is normalized with respect to the mass matrix $\mathbf{M}$, the sensitivity of the ith natural frequency can be expressed as:

$$\frac{d\omega_i}{dx_j} = \frac{1}{2\omega_i} \left[\mathbf{u_j}^T \left(\frac{1 - x_{min}}{1 - x_{min}^p} p x_j^{p-1} \mathbf{K_j} - \omega_i \mathbf{M_j} \right) \mathbf{u_j} \right] \tag{11}$$

where x_{min} is the minimum density, used to denote a void element (small value, e.g. 10^{-6}), since that set the element density equal to 0 causes numerical issues in the finite element analysis

Sensitivity Filtering

To avoid the well known problem of obtaining checkerboard solutions, generally a mesh-independent filter is used (Sivapuram and Picelli (2018), Huang and Xie (2007)). The filtered sensitivity of an element e is obtained using a weighted average of element sensitivity over the neighborhood of e defined by a radius r_{min}. The filtered sensitivity filter is given as:

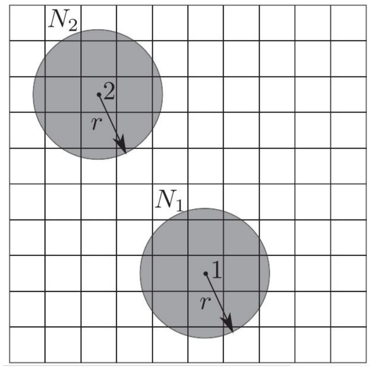


Figure 1 – Filtering: areas of averaging for filtered elemental sensitivities (Elements 1 and 2). Sivapuram and Picelli (2018).

$$\frac{\partial f}{\partial x_e} = \frac{\sum_{m \in N} H_{em} \dfrac{\partial f}{\partial y_m}}{\sum_{m \in N} H_{em}} \tag{12}$$

where $\partial f / \partial x_e$ is the filtered element-based sensitivity, N is the neighborhood around element e as shown in Fig. 1 from where elemental sensitivities are considered for weighted averaging, and the weights H_{em} are given by:

$$H_{em} = \mathtt{max}(0, r_{min} - \mathtt{dist}(x_e, x_m)) \tag{13}$$

where r is the radius of circular (2D) neighborhood region N. The weights are defined such that the nodes nearer to the element contribute higher to its corresponding filtered sensitivities compared to farther nodes. `dist` function computes the Euclidean distance between two nodes in Eq. (13). In some cases, seeking to improve time stabilization, the filtered sensitivity field is calculated in two consecutive iterations as:

$$\frac{\partial f}{\partial x_e} = \frac{\dfrac{\partial f}{\partial x_e}^{k} + \dfrac{\partial f}{\partial x_e}^{k-1}}{2} \tag{14}$$

MODAL ASSURANCE CRITERION-BASED MODE-TRACKING

As the natural frequency is involved in the optimization problem, it is necessary to verify the existence of mode crossing during the optimization process. For this, the MAC-based approach is used. The definition of MAC is given by (Kim and Kim, 2000):

$$MAC(\Phi_a, \Phi_b) = \frac{|\Phi_a^T \Phi_b|}{(\Phi_a^T \Phi_a)(\Phi_b^T \Phi_b)} \tag{15}$$

where Φ_a and Φ_b are the two eigenvectors of interest. The MAC can assume values ranging from 0 to 1, where for value of MAC equal to 1, means that the two eigenvectors Φ_a and Φ_b represent exactly the same mode shape. Based on that, the idea is to get reference mode shapes Φ_{ref}, which are the mode shapes at iteration zero, which will be compared with the mode shapes obtained at each iteration using MAC. Thus, evaluating four mode shapes, where the first is the mode of interest, the eigenvector having the MAC value closest to 1 with respect to Φ_{ref} is selected as the first mode Φ_1.

RESULTS AND DISCUSSION

This section presents the results obtained using the TOBS method. The problems chosen for this analysis are the biclamped beam and a tower subjected to a transversal load at free tip. The goal the problem is to minimize the compliance of the structure subject to a volume fraction constraint of $V = 50$ %. Material models are interpolated considering $p = 3$. In all the examples, the convergence is defined by averaging the changes in the compliance function over 6 consecutive iterations for a tolerance of $\tau = 0.001$.

Biclamped Beam

The design domain of biclamped beam is illustrated in Fig. 2. For optimization the structure is initially considered with 50% of the volume. The properties of the structure are assumed as: Young's modulus of $E = 70$ GPa, Poisson's ratio of $\nu = 0.3$, and density of $\rho = 2700$ kg/m^3. A external load of $F = 1 \cdot 10^6$ N is considered. Due to the symmetry, only half of the design domain is analysed using a mesh of 280 x 40 four-node quadrilateral elements.

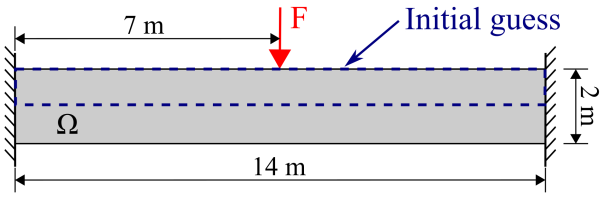
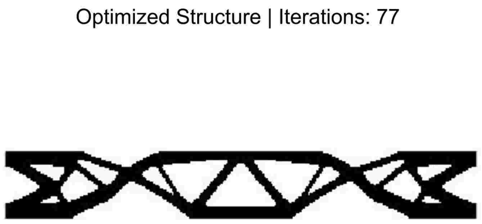
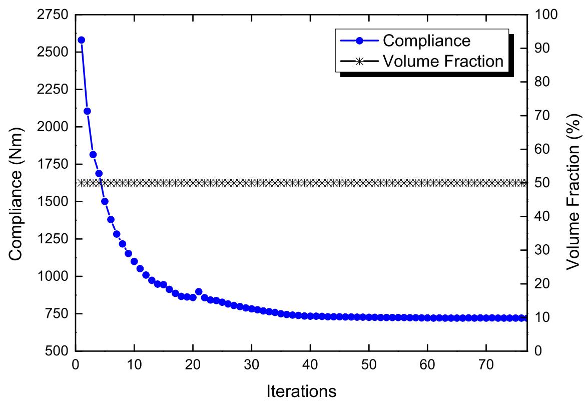


Figure 2 – Design domain for biclamped beam.

The parameters for the TOBS are: $p = 3$, $\varepsilon_v = 0.01$ for volume constraint, $\varepsilon_f = 0.05$ for natural frequency constraint, $\beta = 0.1$, convergence tolerance $\tau = 0.001$, $x_{min} = 10^{-3}$, $\eta = 10^{-9}$, and $r_{min} = 0.15$ m. Plane stress condition is assumed. The Fig. 3 and 4 presents the results of optimized structure with compliance history and frequency history with optimized mode shapes for natural frequency constraint of 55 Hz, respectively.



(a)



(b)

Figure 3 – Optimized structure and compliance history for biclamped beam with natural frequency constraint of 55 Hz: (a) Final topology, and (b) Compliance and volume history.

From Fig. 3(a) it is noted that the final topology presents a result that incorporates both the compliance effect and the natural frequency effect. The shape of the beam refers to the case of maximization of the natural frequency with the addition of the two central bars that are used to support the external load that acts on the top of the center of the beam. It is observed by Fig. 3(b) that compliance over minimization behavior, precisely because an initial guess with volume equal to the constrained volume is used. Thus, the optimizer was able to search for a better distribution of material for that same volume. In the frequency history of Fig. 4 it is observed that the optimizer obeys the frequency constraint that

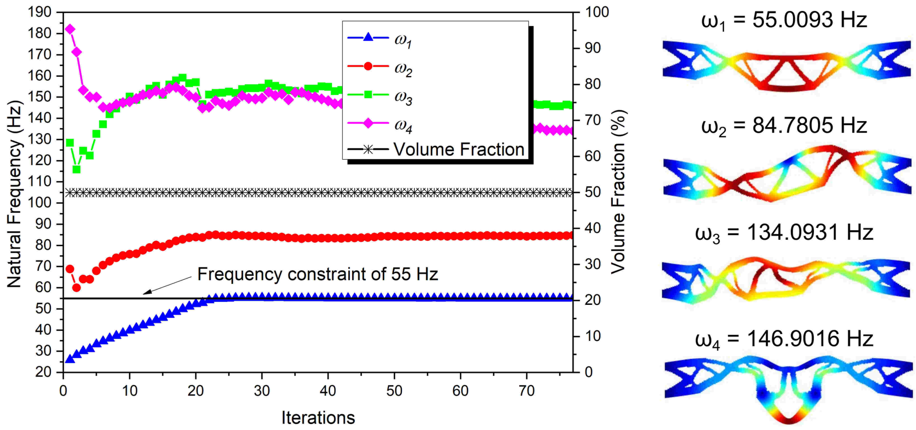
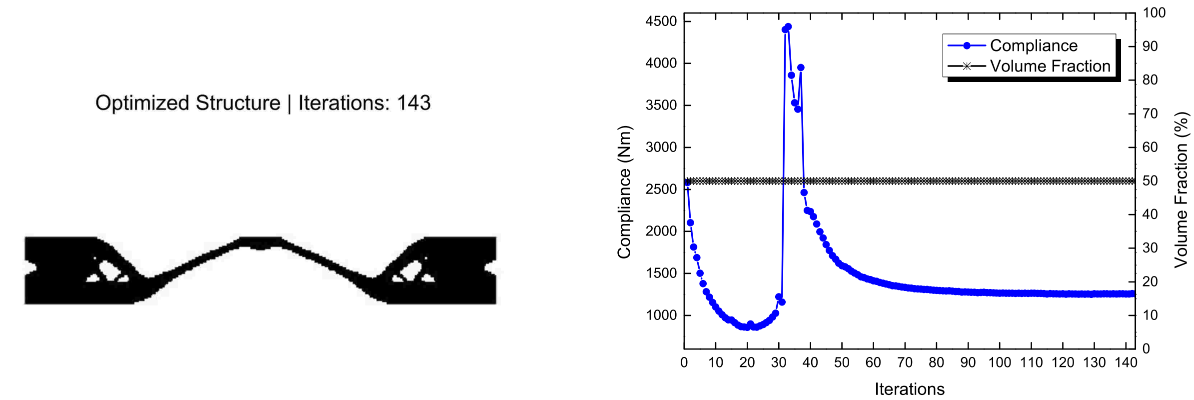


Figure 4 – Frequency history and mode shapes for biclamped beam with natural frequency constraint of 55 Hz.

terminates the optimization in an active way. In addition, the mode crossing between the third and fourth mode shapes is tracked, showing that the implemented MAC algorithm is working correctly, a fact that can also be visualized by the mode shapes in Fig. 4.

Seeking to further explore the natural frequency constraint, it was proposed to solve the biclamped beam problem using a natural frequency constraint of 75 Hz. Here, the same optimization parameters and material properties as in the previous case are used. Fig. 5 and 6 present the results of the optimized structure with the compliance history and frequency history with the mode shape for the 75 Hz frequency constraint.



(a) (b)

Figure 5 – Optimized structure and compliance history for biclamped beam with natural frequency constraint of 75 Hz: (a) Final topology, and (b) Compliance and volume history.

In the Fig. 5(a) it is noted that the optimizer tends to concentrate more material at the ends of the beam, based on the natural frequency constraint, and reduces the material in the center of the beam, which is concentrated more in the region of action of the external load. Furthermore, in Fig. 5(b) it is observed that close to iteration 25 the compliance of the structure begins to increase, a fact that was caused by the loss of the lower central member of the beam. After that, the optimizer is able to look for material distributions that decrease compliance and satisfy the natural frequency constraint at the same time. In the frequency history, Fig. 6, in addition to the mode crossing of the third and fourth mode shapes that was observed in the previous case, there is also the mode crossing between the first and second mode shapes. Again, the frequency curve indicates that the MAC algorithm is working correctly, as the first mode, the mode of interest, continues to be constrained even after it has crossed over with the second mode.

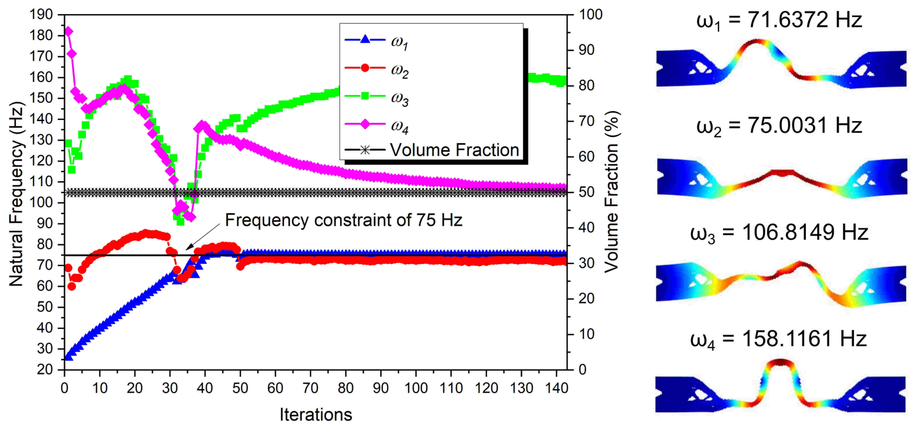


Figure 6 – Frequency history and mode shapes for biclamped beam with natural frequency constraint of 75 Hz.

The Tower

The design domain for the tower is illustrated in Fig. 7. The properties of the structure are assumed as: Young's modulus of $E = 70$ GPa, Poisson's ratio of $\nu = 0.3$, and density of $\rho = 2700$ kg/m^3. A external load of $F = 1 \cdot 10^6$ N is considered. The structure is discretized using a mesh of 60 x 180 four-node quadrilateral elements.

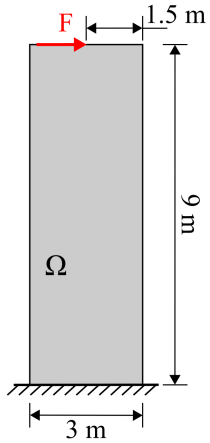
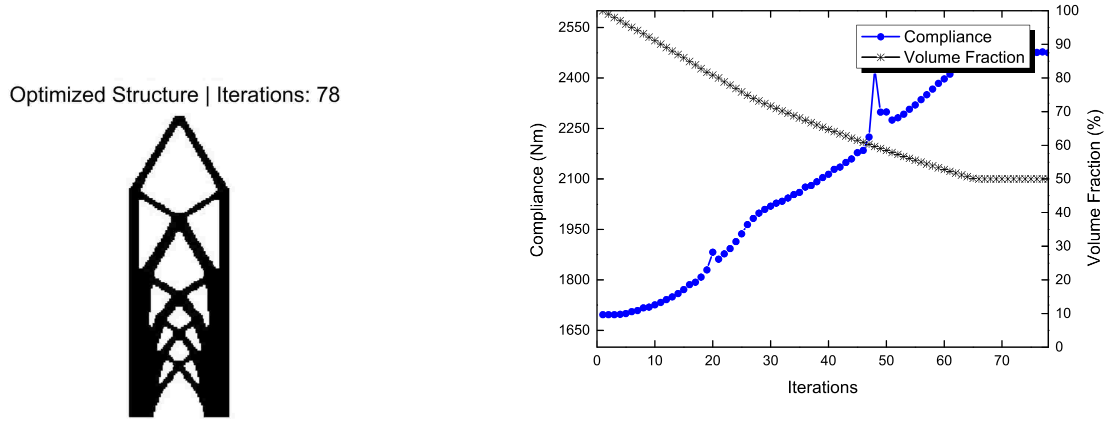


Figure 7 – Design domain for the tower.

The parameters for the TOBS are: $p = 3$, $\varepsilon_v = 0.01$ for volume constraint, $\varepsilon_f = 0.05$ for natural frequency constraint, $\beta = 0.1$, convergence tolerance $\tau = 0.001$, $x_{min} = 10^{-3}$, $\eta = 10^{-9}$, and $r_{min} = 0.15$ m. Plane stress condition is assumed. The Fig. 8 and (9) presents the results of optimized structure with compliance history and frequency history with optimized mode shapes, respectively.

From Fig. 8(a) it follows that the optimizer tends to place more material at the base of the tower, seeking to increase the stability of the structure at the same time as it seeks to increase natural frequency. Instead the case of the biclamped beam, in this example an initial guess with a volume of 50% of the material is not used, so the compliance curve shows a growth, Fig. 8(b). This behavior happens because the structure can not remove the material at the same time decreasing compliance, since removing material decreases the rigidity of the structure. Another point to note is the frequency history, Fig. (9), which, unlike the previous example, does not show mode crossing despite the fourth mode being very close to the third. In addition, the observation of mode shapes, Fig. 9, confirms that there was not mode crossing.

Finally, is proposed to solve the tower problem using a natural frequency constraint of 55 Hz. The objective is to verify if increasing the frequency constraint causes the mode crossing. Here, the same optimization parameters and material properties as in the previous case are used. Fig. 10 and 11 present the results of the optimized structure with the compliance history and frequency history with the mode shape for the 55 Hz frequency constraint.



(a) (b)

Figure 8 – Optimized structure and compliance history for the tower with natural frequency constraint of 45 Hz: (a) Final topology, and (b) Compliance and volume history.

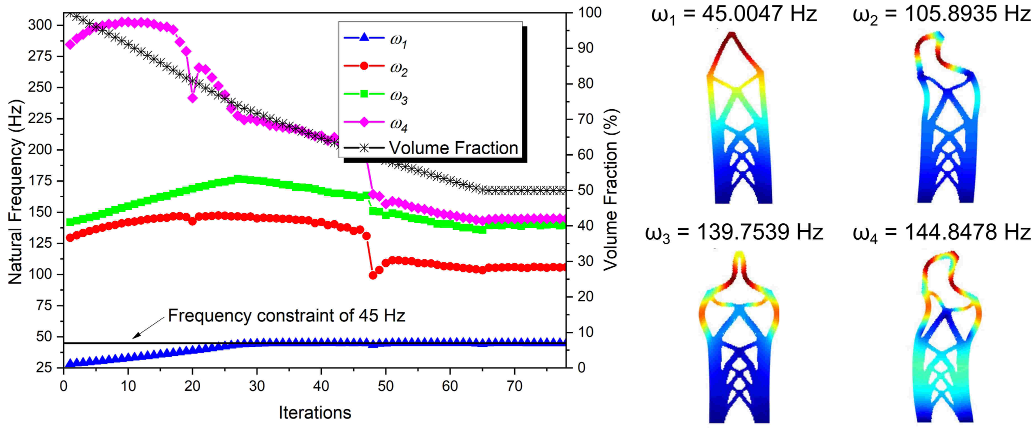
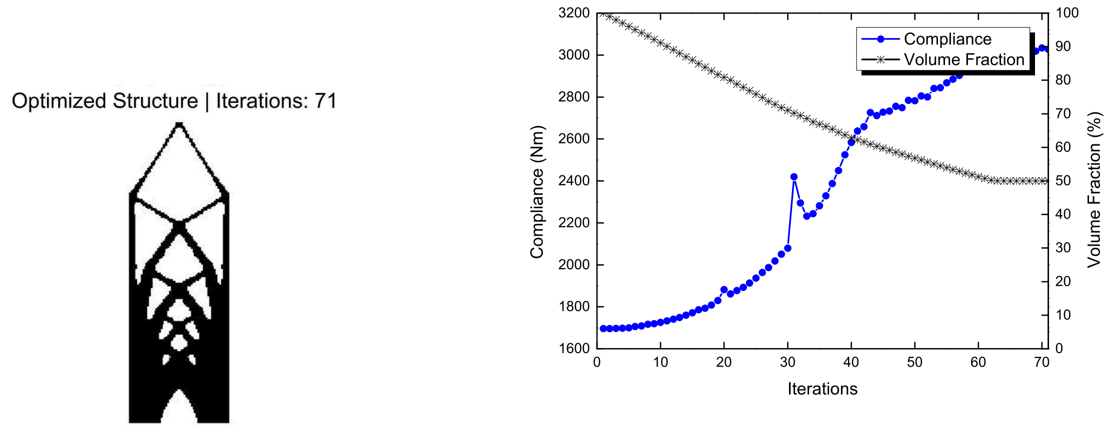


Figure 9 – Frequency history and mode shapes for a tower with natural frequency constraint of 45 Hz.



(a) (b)

Figure 10 – Optimized structure and compliance history for the tower with natural frequency constraint of 55 Hz: (a) Final topology, and (b) Compliance and volume history.

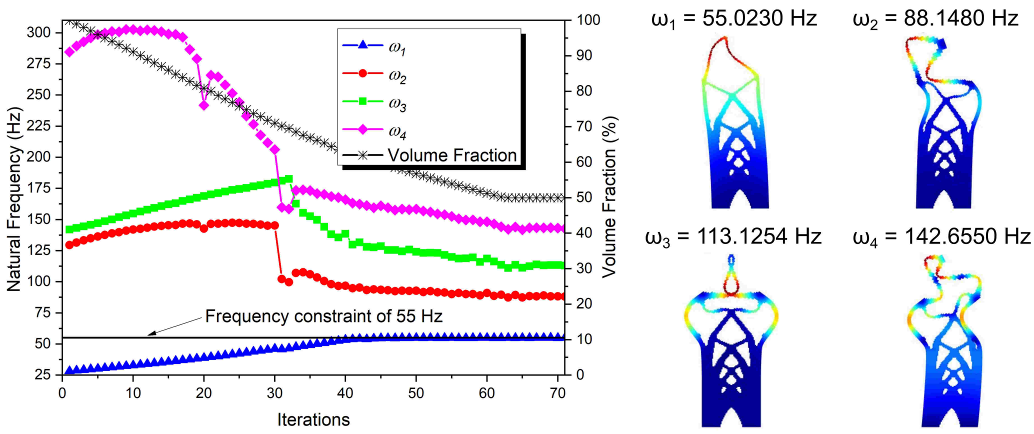


Figure 11 – Frequency history and mode shapes for the tower with natural frequency constraint of 55 Hz.

Similarly to the previous case, the optimizer tends to concentrate more material at the base of the structure, with a higher frequency constraint the upper bars that support the external load become much slender, Fig. 10(a) . This causes a large increase in compliance that reaches 3027 N·m, Fig. 10(b), more than 500 N·m than the final compliance value for the 45 Hz constrained case, Fig. 8(b). So, the optimizer tends to degrade compliance to comply with the natural frequency constraint. Furthermore, a crossing between the fourth and third vibration modes is observed near iteration 30, but the modes return to their original positions, Fig. 11. Observing the mode shapes, it is possible to corroborate that at the end of the optimization there was not mode crossing, Fig. 11.

CONCLUSIONS

A study of topology optimization of minimizing compliance subjected to volume and natural frequency constraint using TOBS method combining with material interpolation scheme proposed by Huang et al. (2010) and SIMP material models is proposed. The results ensure that the method works efficiently working with multiple constraints. In all the cases, the final topology optimization solution has an active natural frequency constraint. Moreover, the implemented MAC algorithm was able to detect the mode crossing allowing to reach the frequency constraint of the first mode even after crossing with the second mode.

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