MECSOL 2022 - Proceedings of the 8th International Symposium on Solid Mechanics
M.L. Bittencourt, J. Labaki, L.C.M. Vieira Jr. and E. Mesquita (Editors), Campinas SP, Brazil, October 17th to 19th, 2022

MECSOL 2022 - TMD's optimization for dynamic analysis of footbridges excited by pedestrians.

Victor Casulli de Oliveira[1], Reyolando Manoel Lopes Rebello da Fonseca Brasil [1]

[1] University of Sao Paulo - Rua da Reitoria, 374 – University City, Butantã, São Paulo – SP, 05508-220

Abstract: In this work, optimization techniques for vibration absorbers (Tuned Mass Dumpers) will be presented for dynamic analysis of footbridges excited by pedestrians. Two software will be used to carry out this procedure, MIDAS CIVIL and MATLAB. First, a simply supported beam will be modeled by the Finite Element Method to represent a footbridge deck using MIDAS CIVIL. With such a model, it is possible to obtain its modal mass, modal stiffness and frequency. Next, a two-degree-of-freedom model will be presented that represents the critical vibration mode of the structure, close to the excitation frequency, together with the TMD. The system with 2 DOF will be implemented in MATLAB software to represent a vertical TMD and a horizontal TMD, modeled with a pendulum. Using MATLAB tools, the TMD mass ratio between the TMD mass and the mass of the footbridge is optimized. The optimization has the objective function of minimizing the difference between the maximum acceleration of the dynamic system and the limit acceleration recommended by standards for human comfort. With the TMD's dimensioned, they will be modeled in MIDAS CIVIL to verify if the optimization parameters reached the expected goal.

Keywords: Footbridge, Optimization, Tuned Mass Dumper, Dynamic Analysis, Vibration.

INTRODUCTION

This article has as main proposal the dynamic analysis and optimization of passive vibration control of large civil structures, with linear behavior, excited by loading of people walking. If components of these loads are in resonance with the structure, it is recommended the development of some type of damper or vibration absorber to mitigate the effects of displacements and accelerations.

Large-scale civil structures today include high-rise residential and commercial buildings, factories, monuments, ports, train stations and subways, among others. Footbridges, for example, are useful as a means for people to cross rivers and roads safely.

According to [1] dynamic loads on footbridges and bridges are, among others,

- people motions;
- rotation, oscillation and impact of machines;
- wind flow;
- vehicle traffic, trains and construction work.

As an example, for footbridges, the main dynamic loading is given by the motion of the human body, both in the vertical and horizontal directions, while walking in the former one.

If the walking frequency is close to the natural frequency of the structure, even with a small load this effect can be amplified. The worst case would be a crowd walking with the same stride on the footbridge, as, for example, the synchronized march of a group of soldiers.

Some examples of footbridge vibrations induced by people walking can be found in [7], [9], [10], [11], [12] e [13]. They present the dynamic behavior of the footbridges and how TMD's mitigate the effects of vibration.

With this, the optimization of tuned mass dampers, TMD, becomes important in order to arrive at viable projects of footbridges.

OBJECTIVE

Present the TMD optimization process for walkways in resonance with people's walking. No attempt is made to review other vibration control systems besides TMD’s, as this is outside the scope of this work.

DYNAMIC SYSTEM MODELING

The modeling of a dynamic system with 1 DOF is displayed in Fig. 1.

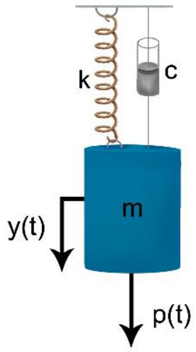

Figure 1 - Mass-spring-damper system

For a 1 DOF dynamic system, the equation of motion can be expressed by:

$$m\ddot{y} + c\dot{y} + ky = p(t) \tag{1}$$

To determine the frequency, it is necessary to consider the system in undamped free vibration being

$$m\ddot{y} + ky = 0 \tag{2}$$

where

$$\omega = \sqrt{\frac{k}{m}} \tag{3}$$

$$f = \frac{\omega}{2\pi} \tag{4}$$

where ω is the frequency in rad/s and f the frequency in Hz.

Performing the same process for 2 DOF systems, we obtain Eq. (5) for Fig. 2 model, and Eq. (6) for Fig. 3 model.

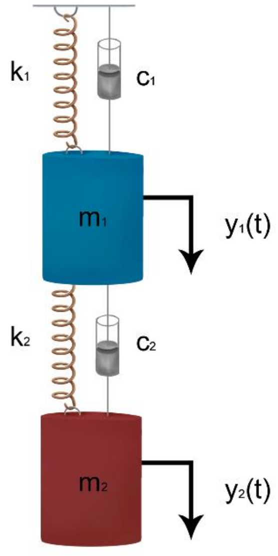

Figure 2 - Vertical mass-spring damper system with TMD

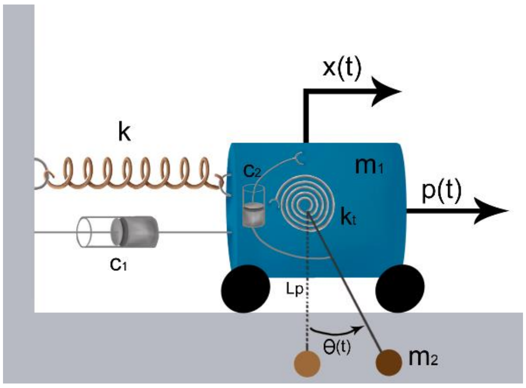

Figure 3 - Horizontal mass-spring-pendulum damper system with TMD

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} p(t) \\ 0 \end{Bmatrix} \quad (5)$$

and

$$\begin{bmatrix} m_1 + m_2 & m_2 Lp\cos(\theta) \\ m_2 Lp\cos(\theta) & m_2 Lp^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 & -m_2 Lp\dot{\theta}\,\mathrm{sen}(\theta) \\ -m_2 Lp\dot{\theta}\,\mathrm{sen}(\theta) & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k_t + m_2 g Lp \dfrac{\mathrm{sen}(\theta)}{\theta} \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} p(t) \\ 0 \end{Bmatrix} \quad (6)$$

where the natural frequency of a pendulum with rigid torsion is given by

$$\omega = \sqrt{\frac{k_t + m_2 g Lp}{m_2 Lp^2}} \quad (7)$$

PERIODIC DYNAMIC LOADING

In a civil structures, the human body can develop several different dynamic loads, such as:

- walk;
- run;
- jump;
- dance.

For footbridges, the main dynamic loading is due to people walking. This type of loading has a characteristic of not losing contact with the structure, unlike running and jumping. This force can be divided into three directions: vertical, horizontal transverse to the deck and horizontal longitudinal to the deck. The acceleration and deceleration of the person's mass generate such forces. Due to its magnitude, this loading has been studied more, compared to other forces.

In [3], Saul and Tuan reviwed the main work, until then, on forces produced by human activities. At the beginning of the 20th century, Moreland in 1905 and by Tilden in 1913 presented more relevant works on the subject. These authors already had the perception that the human load interacts with the structure dynamically. To circumvent a more complex analysis, many design standards recommended and still recommend the simplification of this force by an equivalent static force [4]. According to [4] the walking cycle of a person lasts around 0.54 second. This amount varies from person to person according to age, gender, body type, culture, personality and many other things.

According to [5], the periodic dynamic loading due to people walking can be expressed by:

$$p(t) = G\left(1 + \sum_{i=1}^{n} \alpha_k \ \mathrm{sen}\left(2i\pi f_p t - \varphi_k\right)\right) \quad (8)$$

where:

G: represents the person's weight [N/m²];

f_p: fundamental frequency of the person's walk [Hz];

α_k: Fourier coefficient of each harmonic;

k: is the number of the i-th harmonic considered n: is the considered harmonic number;

φ_k: represents the phase difference between the ith harmonic and the first [rad]

t: is the time intent [s].

For horizontal transverse and longitudinal loading, the static portion can be ignored, since the human body at rest causes only vertical efforts. With that, Eq. 35 can be written as:

$$p(t) = G\left(\sum_{i=1}^{n} \alpha_k \ \mathrm{sen}\left(2i\pi f_p t - \varphi_k\right)\right) \tag{9}$$

The following table brings together several authors who provide several different indexes and parameters for dynamic load formation.

Table 1 - Dynamic loading factors estimated by different authors [6].

Authors	Dynamic loading factors α_k	Comments	Activity-Direction
Blanchard (1977)	$\alpha_1 = 0.257$	Dynamic loading factor is reduced for frequencies from 4 to 5 Hz.	Walking - Vertical Transverse
Bachmann e Ammann (1987)	$\alpha_1 = 0.4 - 0.5$ $\alpha_2 = \alpha_3 = 0.1$	Between 2 and 2.4 Hz, Approximately 2 Hz	Walking - Vertical Transverse
Schulze (after Bachmann e Ammann (1987))	$\alpha_1 = 0.37; \alpha_2 = 0.10$ $\alpha_3 = 0.12; \alpha_4 = 0.04$ $\alpha_5 = 0.08$	At 2 Hz	Walking - Vertical Transverse
Schulze (after Bachmann e Ammann (1987))	$\alpha_1 = 0.039; \alpha_2 = 0.010$ $\alpha_3 = 0.043; \alpha_4 = 0.012$ $\alpha_5 = 0.015$	At 2 Hz	Walking - Horizontal Transversal
Schulze (after Bachmann e Ammann (1987))	$\alpha_{1/2} = 0.037; \alpha_1 = 0.204$ $\alpha_{3/2} = 0.026; \alpha_2 = 0.083$ $\alpha_{5/2} = 0.024$	At 2 Hz	Walking - Longitudinal
Bachmann et al. (1995)	$\alpha_1 = 0.4/0.5$ $\alpha_2 = \alpha_3 = 0.1/-$	At 2.0/2.4 Hz	Walking - Vertical Transverse
Bachmann et al. (1995)	$\alpha_1 = \alpha_3 = 0.1$	At 2 Hz	Walking - Horizontal Transversal
Bachmann et al. (1995)	$\alpha_{1/2} = 0.1; \alpha_1 = 0.2$ $\alpha_2 = 0.1$	At 2 Hz	Walking - Longitudinal
Young (2001)	$\alpha_1 = 0.37(f - 0.95) \leq 0.5$ $\alpha_2 = 0.054 + 0.0044f$ $\alpha_3 = 0.026 + 0.0050f$ $\alpha_4 = 0.010 + 0.0051f$	These are average values for dynamic loading factors.	Walking - Vertical Transverse

For [7] there are some factors that can be considered to mitigate the dynamic effects of loading. The first factor is the synchronization factor of people's walking, which can be at most s = 1. The second factor is a factor that, if the loading model chosen for verification does not clearly have parameters for horizontal motions, transforms the loading vertical to horizontal. Horizontal loading can vary this “d” factor from 10% to 4%, transforming Eq.9, disregarding the static part, into:

$$p(t) = s\ d\ G\left(\sum_{i=1}^{n} \alpha_k\ \text{sen}\left(2i\pi f_p t - \varphi_k\right)\right) \tag{10}$$

Varela, [4], in turn carried out experiments to try to obtain a more accurate loading profile, with a period of 0.6 seconds, representing heel impact for vertical loading. The loading is composed of 5 equations, as follows in Eq. (11).

$$p(t) \begin{cases} \left(\dfrac{\text{fmi Fm-G}}{0{,}04T_p}\right)t+G & \text{se } 0 \le t < 0{,}04T_p \\ \text{fmi Fm}\left[\dfrac{C_1\left(t-0{,}04T_p\right)}{0{,}02T_p}+1\right] & \text{se } 0{,}04T_p \le t < 0{,}06T_p \\ \text{Fm} & \text{se } 0{,}06T_p \le t < 0{,}15T_p \\ G\left(1+\sum_{i=1}^{n}\alpha_k\ \text{sen}\left(2i\pi f_p t - \varphi_k\right)\right) & \text{se } 0{,}15T_p \le t < 0{,}90T_p \\ 10(G-C_2)\left(\dfrac{t}{T_p}-1\right)+G & \text{se } 0{,}90T_p \le t < T_p \end{cases} \tag{11}$$

where:

Fm: maximum value of the Fourier series;

fmi: increase factor of heel impact, that is, it is the ratio between the maximum peak value referring to heel impact and Fm;

C_1 and C_2: are coefficients given by the following equations.

$$\text{Fm} = G\left(1+\sum_{i=1}^{n}\alpha_k\right) \tag{12}$$

$$C_1 = \left(\frac{1}{\text{fmi}}-1\right) \tag{13}$$

$$C_1 = \begin{cases} G(1-\alpha_2) & \text{if } k=3 \\ G(1-\alpha_2+\alpha_4) & \text{if } k=4 \end{cases} \tag{14}$$

The heel impact enhancement factor can be considered 1.12 [4]. The phase angles are given by $\varphi_1 = 0$, $\varphi_2 = \pi/2$, $\varphi_3 = \pi$ and $\varphi_4 = 3\pi/2$. The dynamic coefficients for k = 3 are given by α_1 = 159/608 = 0.262; α_2 = 61/608 = 0.100; α_3 = 20/608 = 0.033. If k = 4, we have the following dynamic coefficients given by:

$$\alpha_1 = -0.22169 f_p^3 + 1.11946 f_p^2 - 1.44748 f_p + 0.5967 \tag{15}$$

$$\alpha_2 = -0.0120374(2f_p)^3 + 0.1494(2f_p)^2 - 0.53146(2f_p) + 0.6285 \tag{16}$$

$$\alpha_3 = 0.00009068(3f_p)^5 - 0.0021066(3f_p)^4 + 0.018364(3f_p)^3 - 0.077278(3f_p)^2 + 0.17593(3f_p) - 0.1477 \tag{17}$$

$$\alpha_4 = 0.00051715(4f_p)^4 - 0.014388(4f_p)^3 + 0.14562 - 0.62994(4f_p) + 1.018469 \tag{18}$$

Modal loading is given by

$$P(t) = p(t)\int_0^L \Phi\ dz \tag{19}$$

MODELING THE SIMPLY SUPPORTED BEAM BY FINITE ELEMENTS

Using the MIDAS CIVIL software, a simply supported beam of transverse section 2.10 x 2.10 meters will be modeled, considering a C25 concrete with modulus of elasticity of 28.0 GPa, density of 2.5 t/m³ and gravity of 9.81 m/s². The span of the beam is 40 meters and a 3 meter wide deck (not considering the mass of the deck). The loading due to people walking is applied linearly across the entire beam. The beam was divided into 40 beam elements and, for numerical integration, MIDAS CIVIL processed in a time of 10 seconds at a step of 0.001 sec. and uses the Lanczos eigenvalue method to determine the frequencies and modes of vibrations. The modal mass, modal stiffness and frequency for the first mode for this model are:

$$K = 3.48 \times 10^7 \text{ N/m} \tag{20}$$

$$M = 220500\ \text{kg} \tag{21}$$

$$\omega = 1.99\text{Hz} \tag{22}$$

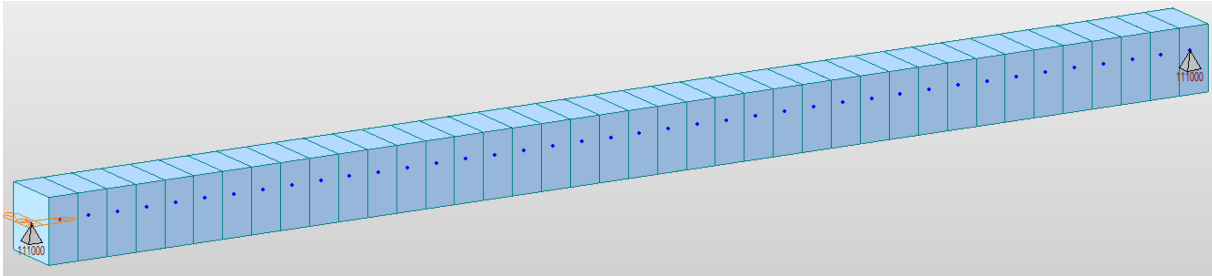

Figure 4 - Bi-supported beam model with 40m span.

Results for accelerations are shown in Fig. 5 and Fig. 6. Comparisons between authors is indicated in the legend: Red: Varela, Green: Brasil e Silva, Blue: Young, Yellow: Schulze, Ciano: Blanchard, Pink: Sine.

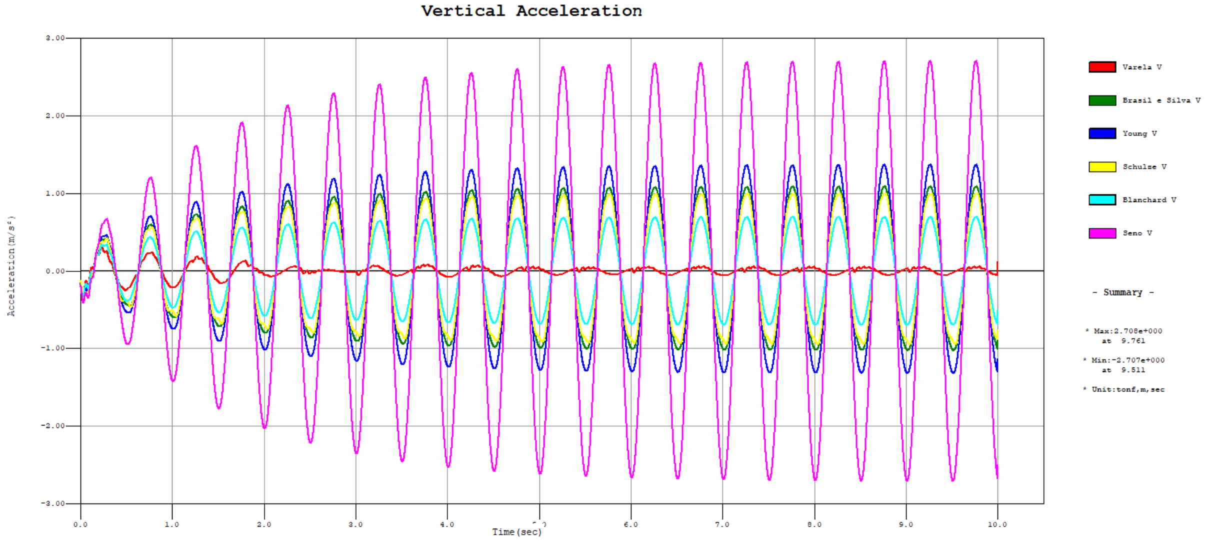

Figure 5 - Vertical acceleration of the beam

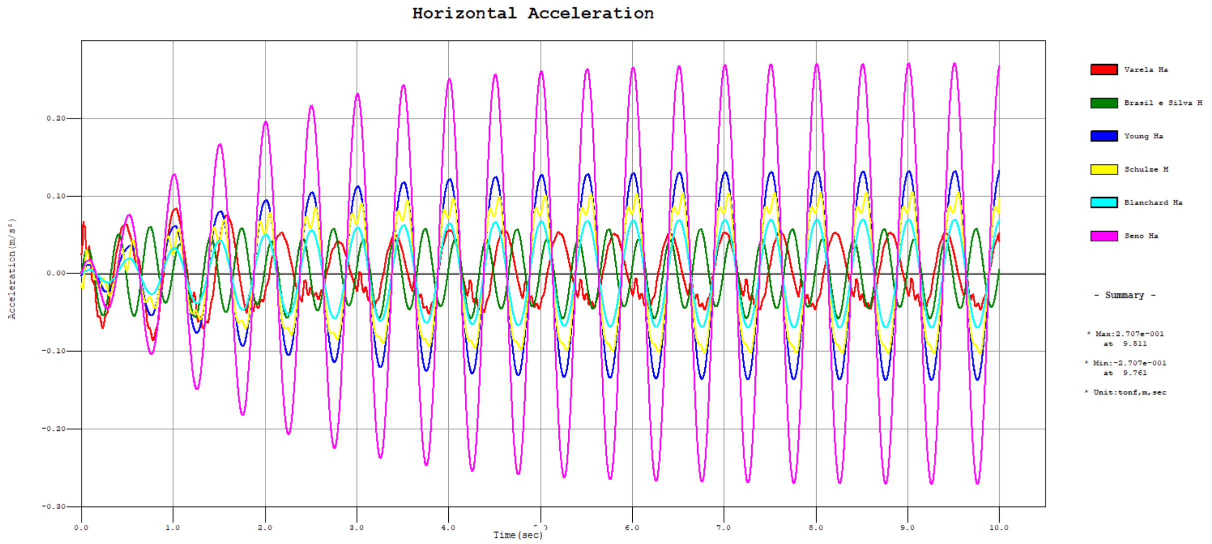

Figure 6 - Horizontal transverse acceleration of the beam

Note that the model due to the sine has much higher amplitudes, both for the transverse acceleration and for the horizontal transverse, due to the lack of the dynamic loading factor. For vertical motions, the Varela model presented much lower accelerations, since this specific loading was modeled for 1.67 Hz, which is not in resonance with this structure. For the other loads, the Brasil e Silva and Schulze model presented a similar behavior and with values closer to the average between the Young and Blanchard models. Table 2 is a summary of maximum accelerations for each model.

Table 2 - Maximum acceleration

	Maximum acceleration	
	Vertical (m/s²)	Horizontal (m/s²)
Sine	2.70	0.27
Blanchard	0.67	0.07
Schulze	1.00	0.10
Young	1.37	0.13
Brasil and Silva	1.09	0.06
Varela	0.38	0.01

For horizontal transverse acceleration, the Schulze model presented good results, with intermediate values between Brasil and Silva, Young and Blanchard. It is important to emphasize that the Varela model adapted for horizontal is not very interesting for the analysis because its loading was done exclusively for vertical loading, generating a loading profile that does not match the lateral motion of a person's walk.

Thus, the periodic dynamic loading used for the vertical and horizontal optimization process the follows will be the Schulze loading because it presents accelerations close to the average of the other models, thus being able to better represent the phenomenon of people's walking behavior.

OPTIMIZATION LIMITS

Our optimization scheme is an iterative process, where we try to minimize the difference between the acceleration of the system and acceleration limits of international stadards. Upper and lower limits are also used for the desired variables. With this, one can find local or global minima, within the chosen range.

For the presented dynamic problem of the simply supported beam, optimization will be applied with the addition of TMD's, in order to mitigate the dynamic responses. As the objective function, acceleration, is a nonlinear function of the design variables, the optimization problem is characterized as a nonlinear programming problem. Once the objective function of the problem is defined and the lateral constraints are imposed, the mass optimization problem of the vertical TMD can be defined as follows.

Minimize:

$$f(x) = \left(\max_{i=1,\ldots,n}|\{\ddot{u}\}| - \ddot{u}_{lim}\right)^2 \tag{23}$$

Subject to:

$$g_1(X) = -X \leq 0 \tag{24}$$

$$g_2(X) = X - 0.02 \leq 0 \tag{25}$$

where,

$\{\ddot{u}\}$ is the acceleration vector;

$\ddot{u}_{lim}$ is the maximum allowable acceleration;

X is the ratio between the mass of the vertical TMD and the total mass of the structure.

So, the ratio between the masses must be greater than 0 and less than 2%.

Similarly, for the optimization problem of the vertical pendulum TMD, after defining the objective function and imposing the necessary lateral constraints, the mathematical optimization problem is posed as follows.

Minimize:

$$f(x) = \left(\max_{i=1,\ldots,n}|\{\ddot{u}\}| - \ddot{u}_{lim}\right)^2 \tag{26}$$

Subject to:

$$g_1(X) = -X_1 \leq 0 \tag{27}$$

$$g_2(X) = X_1 - 0.05 \leq 0 \tag{28}$$

$$g_3(X) = - X_2 + 0.5 \leq 0 \tag{29}$$

$$g_4(X) = X_2 - 2 \leq 0 \tag{30}$$

where,

X_1 is the ratio between the mass of the horizontal TMD and the total mass of the structure;

X_2 is the length of the pendulum.

In other words, the ratio between the masses must be greater than 0 and less than 5% and the length of the pendulum must vary between 0.5 and 2 meters.

With Eq. 5 and Eq. 6 it is possible to determine which TMDs are necessary to achieve the desired acceleration.

RESULTS

Using MATLAB software, for the same time interval and time step as MIDAS CIVIL, and using the 4th order Runge-Kutta method, the optimization of the dynamic systems was performed to achieve the desired acceleration. For vertical acceleration, an optimization was performed to reduce the acceleration from 1 m/s² to 0.5 m/s². In the horizontal model, the optimization was made to reduce the acceleration from 0.1 m/s² to 0.05 m/s². With the factors of mass ratio between TMD and footbridge, spring stiffness and pendulum length, it was possible to model the damping system in the MIDAS CIVIL software.

For the vertical TMD, a mass ratio of 1.07% was obtained, that is, a TMD of 4.72 tons with a spring of 7.46×10^5 N/m. For the horizontal TMD, the mass ratio was 1.08% with a pendulum length of 2 meters and a torsion spring of 2.92×10^6 N/rad.

Figure 7 presents results for the vertical acceleration of beam obtained using the MATLAB software, after optimization. In Fig.8, the same result is obtained, for the vertical acceleration, by modeling the designed TMD in the MIDAS CIVIL software.

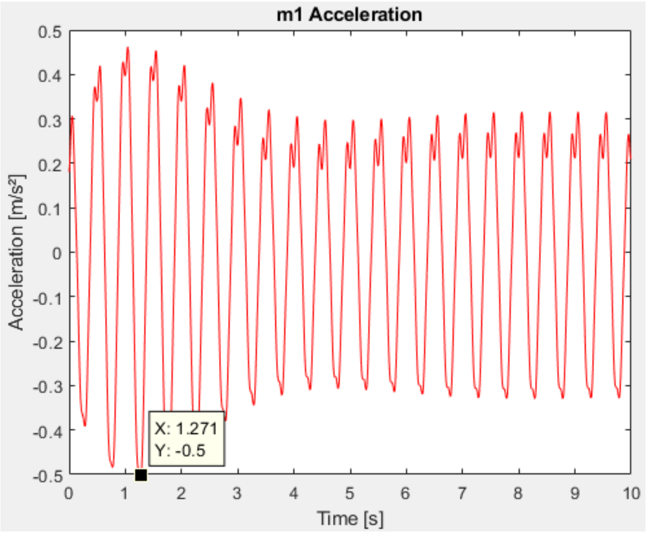

Figure 7 - Maximum vertical acceleration with optimized TMD (MATLAB).

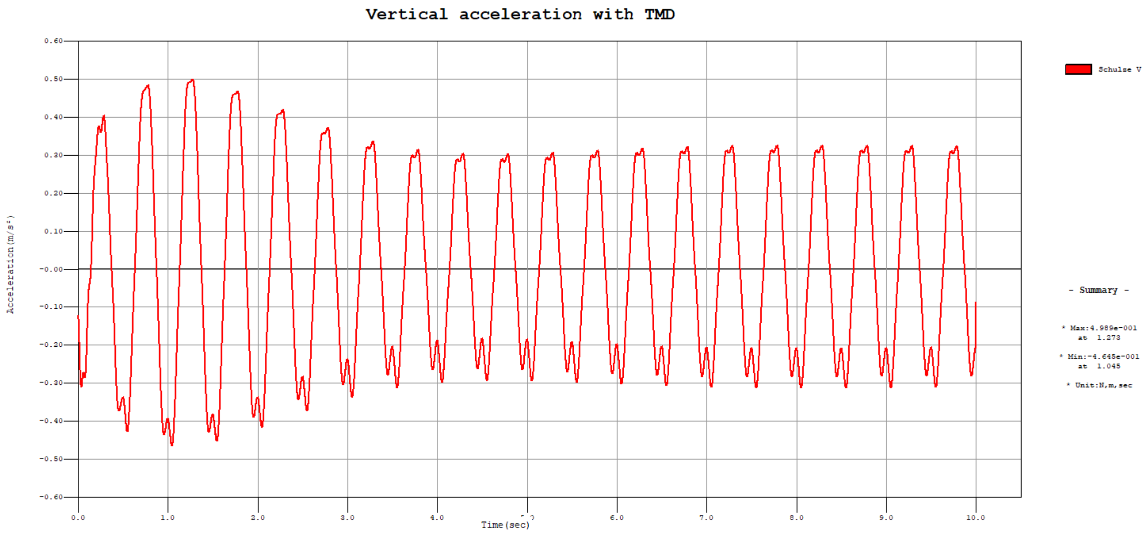

Figure 8 - Maximum vertical acceleration (MIDAS CIVIL).

In Fig. 9 we present results for the horizontal acceleration of beam, obtained using the MATLAB software. In Fig.10, the same result is obtained for the horizontal acceleration, modeling the designed TMD in the MIDAS CIVIL software.

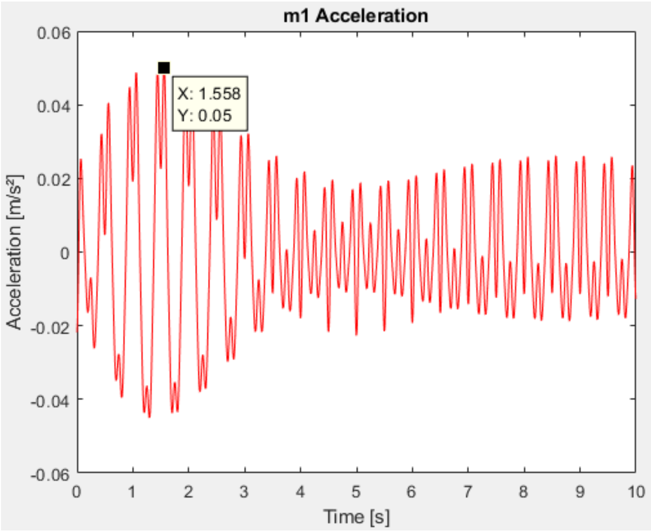

Figure 9 - Maximum horizontal transverse acceleration with optimized TMD (MATLAB).

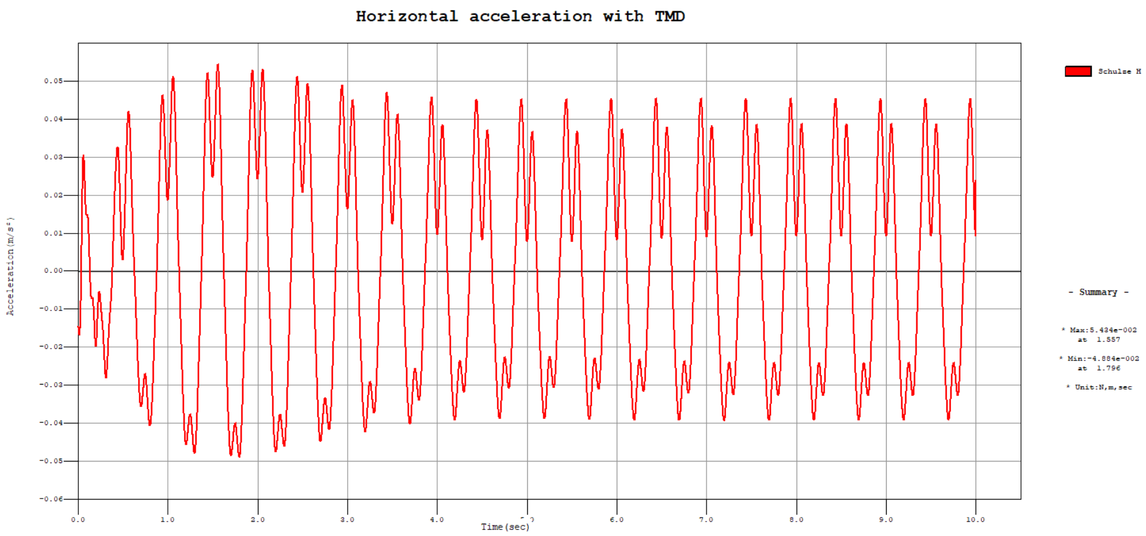

Figure 10 - Maximum horizontal transverse acceleration (MIDAS CIVIL).

In both cases it is noted that the acceleration graphs converged to the same desired limit acceleration.

CONCLUSION

Throughout this article, the concepts of deterministic periodic dynamic loads as a function of time were presented.

With the application of optimization procedures, it was shown how to optimize the TMD's in order to find the smallest relationship between the mass of the TMD and the footbridge. The results obtained using two different software were presented, one commercial, MIDAS CIVIL, and another academic, MATLAB.

The results were compared, and it was concluded that it is possible to optimize the TMD's to mitigate the dynamic effects in order to reach comfortable accelerations for human use. With the modeling in MIDAS CIVIL and the dynamic system with 2 DOF in MATLAB, it is possible to carry out the optimization of more complex footbridge structures and obtain good results.

No attempt was made to review other vibration control systems besides TMD's, as this is outside the scope of this work.

ACKNOWLEDGMENTS

The authors acknowledge support by CAPES and CNPq, both Brazilian research funding agencies.

REFERENCES

Bachmann, H., Ammann, W. Vibrations in Structures - Induced by Man and Machines, Structural Engineering Documents, Vol. 3e, International Association of Bridge and Structural Engineering (IABSE), Zürich, 1987.

Brasil, R. M. L. R. F, Silva, M. A. Introdução à Dinâmica das Estruturas para a Engenharia Civil, Segunda Edição. Editora Edgard Blucher, São Paulo, 2015.

Saul, W. E. Tuan, C. Y. Review of live loads due to human movements. Journal of Structural Engineering, v. 112, n. 5, p. 995-1004, mai. 1986.

Varela, W. D. Modelo teórico-experimental para análise de vibrações induzidas por pessoas caminhando sobre lajes de edifícios. 2004. Tese de doutorado (Engenharia Civil) – COPPE/UFRJ, Rio de Janeiro.

Živanović, S., Pavić, A., Reynolds, P. Vibration serviceability of footbridges under human-induced excitation: a literature review. Journal of Sound and Vibration, Vol. 279, No. 1-2, pp. 1-74. 2005.

Segundinho, P. G. A.; Dias, A. A.; Carreira, M. R. Carregamentos Dinâmicos Induzidos Pelos Movimentos de Pedestres em Estruturas. Seminário: Ciências Exatas e tecnológicas, v. 38, n.2, p 3-16, ago./dez. 2017.

Hauksson F. Dynamic Behaviour of Footbridges Subjected to Pedestrian-Induced Vibrations. Dissertação de mestrado. Lund University. Suécia. Nov. 2005.

Brasil, R. M. L. R. F, Silva, M. A. Otimização de Projetos de Engenharia. (2019).

Queiroz, M. M., Lima, S. S. Técnicas de Análise Dinâmica de Passarelas. Dissertação de Mestrado, Escola Politécnica da Universidade Federal do Rio de Janeiro, Rio de Janeiro 2014.

Votsis, R. A.; Stratford, T. J.; Chryssanthopoulos, M. K.; Tantele, E. A. Dynamic Assessment of a FRP Suspension Footbridge Though Field Testing and Finite Element Modelling. Steel and Composite Structure. Volume 23. No 2 205-215. 2017.

Bruno, L.; Venuti, F., Nascé, V. Pedestrian-induced Torsinao Vibrations of Suspended Footbridges: Proposal and Evaluation of Vibration Countermeasure. ElSevier Journal: Engineering Structures 36, 228-238. Jan. 2012.

Carpineto, N., Lacarbinara, W., Vertroni, F. Mitigation of Pedestrian-induced Vibrations in Suspension Footbridges via Multiple Tuned Mass Dampers. Departamento de engenharia estrutura e geotecnia. University of Rome La Sapienza. Jun. 2009.

Oliveira V. C., Brasil, R. M. L. R. F. Análise Dinâmica de Passarelas Excitadas por Pedestres. Dissertação de Mestrado, Universidade Federal do ABC, Santo André 2020.

RESPONSIBILITY NOTICE