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Effect of rigid versus flexible supports on the design of a bridge through topology optimization

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Abstract. This article investigates the consequences of considering rigid or flexible supports in the design of a bridge through topology optimization. Flexible supports consist of elastic piles embedded in a homogeneous soil layer, which are modeled via a coupled finite- and boundary-element scheme. This represents the support case that actual bridges can experience in civil engineering practice. The rigid support case is obtained by simply imposing zero-displacement at the supported nodes of the finite element mesh of the bridge. The bridge is modeled with a non-design deck, under uniformly distributed vertical loads, which is a reasonable approximation to its working condition. Topology optimization in this analysis is performed with the classical Bi-directional Evolutionary Structural Optimization (BESO) method. The objective function is the minimization of the compliance of the bridge, subject to a given volume reduction. The results show that considering rigid or pile supports has a significant effect on the resulting optimal topology, as well as on the achievable compliance of the bridge.
Keywords: soil-structure interaction, topology optimization, BESO, piles, bridge design

INTRODUCTION

Topology optimization is a design paradigm that uses the computer to generate structural layouts with maximized performance. The performance to be maximized can range from classical stiffness, to more intricate objective functions such as the modal response of the structure. Search spaces are typically confined within constraints such as a prescribed volume reduction. Search algorithms consist in finding local minima under these constraints by reallocating material between different parts of the design domain.

It is generally agreed that topology optimization began with the paper by Bendsøe and Kikuchi (1988), who used numerical homogenization to evaluate the mechanical properties of infinitely small cells with rectangular holes. Historically, a substantial portion of topology optimization analyses consider the optimization of distribution of a homogeneous isotropic material (Bendsøe 1989; Zhou and Rozvany 1991; Mlejnek 1992; Bendsøe and Sigmund 1999), but more recent ones have considered more sophisticated material models as well (Zhang et al., 2017; Kick and Junker, 2021; Huang et al., 2015; Jia et al., 2016). A variety of different methods have been proposed to obtain these optimized topologies, which can be based on density (Sigmund, 2001) level sets (Wang et al. 2003; Allaire et al. 2004), evolutionary approaches (Xie and Steven 1993), linearized integer programming (Sivapuram and Picelli, 2018), and floating projection optimization (Huang, 2021). Topology optimization as a design tool has largely transcended the realm of academic interest and has been applied to the engineering practice in fields such as the aerospace, automotive, architecture, and healthcare industries (Zhu et al. 2016; Yang and Chahande 1995; Beghini et al. 2014; Wang et al. 2016).

On the other hand, when it comes to topology optimization of soil-structure interaction problems, there is still much to be learned. In these problems, the flexibility of the soil as an unbounded, wave-propagating medium is known to significantly affect the response of structures resting on it and buried in it, and this is a fair indicator that structural design via topology optimization should take into consideration these effects as well. The few results in the literature that looked into the effect of soil flexibility in topology optimization are those by Seitz and Grabe (2016) and Cavalcante et al. (2022). The first used the Solid Isotropic Material with Penalization (SIMP) method (Bendsøe, 1989) to obtain optimal topologies for the buried foundation of a plate under centered and eccentric loads. The soil was considered to be a granular medium. The results showed that optimal foundation topologies depend on the type of loading, and result in organic geometries. The latter used the Bi-directional Evolutionary Structural Optimization (BESO) method (Huang and Xie, 2007) to obtain optimal topologies for structures supported by embedded elastic pile foundations. The results showed that disregarding the flexibility of the foundations may result in severely overestimated structural stiffness of the system.

This paper considers the problem of the topology optimization of a bridge, as a representative case of soil-structure interaction problem that in engineering practice is strongly affected by the flexibility of its foundations. Foundations for the bridge are considered to be embedded elastic piles, the response of which is obtained via a boundary–finite element coupled method. Topology optimization is obtained via the classical BESO method. The paper discusses how a first approximation that the bridge is over rigid supports affects its optimal topology and optimization objective.

PROBLEM DEFINITION

Consider the bridge problem shown in Fig. 1. The $40\,\mathrm{m} \times 20.6\,\mathrm{m}$ prismatic geometry shown in Fig. 1 is the initial design domain, within which material can be removed and reallocated towards the optimization goal. The only portion of the bridge that is not subject to optimization is a $0.6\,\mathrm{m}-$thick vehicle deck, and the transitable void space on top of it. The bridge is built with an isotropic material with Young's modulus E_b and Poisson ratio ν_b, and is supported by four piles of length $L_p = 10\,\mathrm{m}$ and diameter $d_p = 0.6\,\mathrm{m}$, with Young's modulus E_p and Poisson ratio ν_p. In this analysis, $E_p = E_b$ and $\nu_p = \nu_b$, considering that in engineering practice both the bridge and the piles are made of concrete. The piles are embedded in a $20\,m-$deep soil layer of Young's modulus $E_h = 10^{-3}E_s$ and Poisson ratio $\nu_h = \nu_b$. The problem consists in finding the topology for the bridge that maximizes its stiffness for a prescribed reduction in the volume of material used.

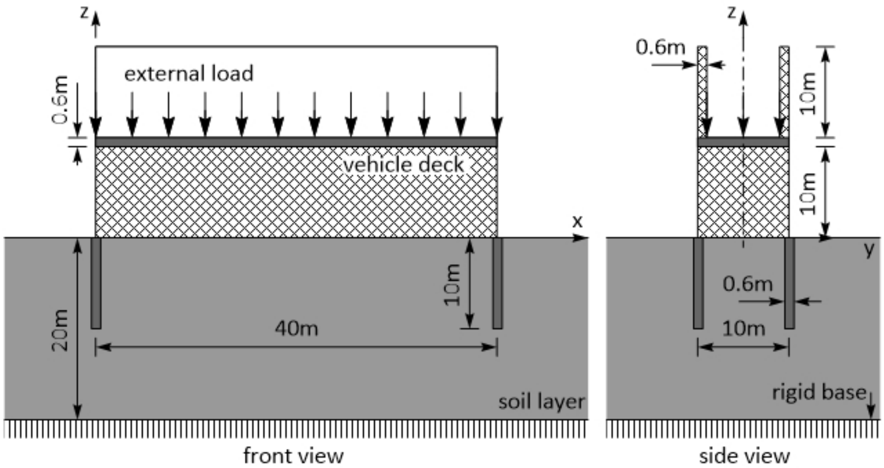

Figure 1 – Front and side views of the initial design domain of the bridge considered in this paper.

PILED BRIDGE MODEL

The model used in this work to model the piled bridge consists in the classical Finite Element Method (FEM) for the body of the bridge, and the stiffness matrix method for the embedded piles, which uses superposition of soil influence functions to describe the soil–pile interaction.

Pile model

In this work, the embedded piles are modeled by approximating the response of the piles as an assembly of discrete, linear-elastic, one-dimensional finite beam elements. The loads that they exchange with the surrounding soil are modeled via the superposition of axisymmetric influence function, in the framework of a boundary element scheme. Imposing equilibrium and continuity conditions at the pile–soil interface results in the equilibrium equation for the embedded piles. Kaynia and Kausel (1992) have used this strategy to obtain the following equilibrium equation for the pile group:

$$P_e = \left(K_p + \Psi^T \left(F_s + F_p \right)^{-1} \Psi \right) U_e = K_e U_e, \tag{1}$$

in which P_e is the vector of loads at the pile ends, K_p is the stiffness matrix of the beam elements, Ψ is the stiffness matrix of beam elements considering clamped-end conditions, F_p and F_s are the flexibility matrices of the pile and soil coupling, U_e is the vector of displacements and rotations at the pile ends, and K_e can be thought of as the stiffness matrix of the pile group. The reader may refer to Kaynia and Kausel (1992) for a detailed description of these terms.

Bridge model

The bridge is modeled with classical hexahedral finite elements, with three displacement degrees of freedom per node, the elemental stiffness matrices of which are given by

$$k_s = \int_{V_e} B^T D B dV_e = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T D B \det(J)\, d\xi d\eta d\zeta, \tag{2}$$

in which V_e is the volume of the element, D is the constitutive matrix, N and B are the matrices of shape functions and their derivatives, and J is the Jacobian relating (ξ,η,ζ) to (x,y,z) (Bathe, 2006). After the classical FEM assembly procedure, an equilibrium equation for the entire bridge can be assembled from k_s as

$$P_s = K_s U_s, \tag{3}$$

in which P_s and U_s are the vector of nodal forces displacements, and K_s is the global stiffness matrix of the bridge.

Coupling procedure

In order to obtain a model of the bridge–pile group system, continuity and equilibrium conditions are imposed at the bridge–pile group interface. Given the dimensions of the pile compared to those of the bridge, the interface is considered to be the head of the pile and the node of the bridge with which it is connected. Following this procedure, the equilibrium equation for the piled bridge can be written as

$$P = KU, \tag{4}$$

in which P and U are the vectors of nodal forces and displacements of the piled bridge, and K is the stiffness matrix given by

$$K = \begin{bmatrix} K^s_{11} & \cdots & K^s_{1n} & \cdots & K^s_{1m} & \cdots & K^s_{1N} \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ K^s_{n1} & \cdots & K^s_{nn}+K^e_{ii} & \cdots & K^s_{nm}+K^e_{ij} & \cdots & K^s_{nN} \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ K^s_{m1} & \cdots & K^s_{mn}+K^e_{ji} & \cdots & K^s_{mm}+K^e_{jj} & \cdots & K^s_{mN} \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ K^s_{N1} & \cdots & K^s_{Nn} & \cdots & K^s_{Nm} & \cdots & K^s_{NN} \end{bmatrix}. \tag{5}$$

The terms denoted by K^e_{ab} and K^s_{ab} in Eq. 5 are sections of the stiffness matrices of the pile group and bridge, respectively. The sections of K comprising the superposition of these matrices correspond to the nodes of the mesh of the bridge that are in contact with pile heads. A full description of this coupling scheme is given in Tavares and Labaki (2019).

OPTIMIZATION SCHEME

In view of the equilibrium equation of the piled bridge system (Eq. 5), the problem of finding the bridge topology with maximum stiffness with a final volume of at most V^* can be posed as

$$\begin{aligned} &\underset{x_i}{\text{Minimize}}\ C(x_i) = \frac{1}{2}P^T U, \\ &\text{Subject to}\ \frac{V(x_i)}{V_0} \leq V^*, \\ &\qquad P{=}KU, \\ &\qquad x_i \in \{0,1\},\ i \in [1, N_d], \end{aligned} \tag{6}$$

in which x_i is the vector of design variables, N_d is the number of elements of the bridge, $C(x)$ is a scalar measure of the compliance of the bridge, $V(x)$ is the volume of the bridge, and V_0 is the volume of the initial design domain (Fig. 1). In this paper, this optimization problem is solved iteratively, in the framework of the Bi-directional Evolutionary Structural Optimization (BESO) algorithm. In this algorithm, Eq. 5 is solved multiple times, in which time the elements are ranked according to their contribution α_i^e (called sensitivity number) to the stiffness $C(x)$ of the system, in which

$$\alpha_i^e = \Delta C_i = \frac{1}{2} u_{si}^T k_s u_{si}, \tag{7}$$

in which u_{si} is the vector of nodal displacements of element i. Nodal sensitivity numbers are computed for nodes j as

$$\alpha_j^n = \frac{\sum_{j=1}^{N_j} V_i \alpha_i^e}{\sum_{j=1}^{N_j} V_i}, \tag{8}$$

in which N_j is the number of elements connected to node j, which are then incorporated into elemental sensitivities as

$$\alpha_i = \frac{\sum_{j=1}^{N_s} w(r_{ij})\, \alpha_j^n}{\sum_{j=1}^{N_s} w(r_{ij})}, \tag{9}$$

in which N_s is the number of nodes of the mesh within a spherical domain of radius r_{min} centered at the centroid of element i, r_{ij} is the distance between node j and the centroid of element i, and $w(r_{ij}) = r_{min} - r_{ij}$ ($j = 1 : N_s$) is a weight factor. This modification of the sensitivity number is a classical strategy to avoid mesh-dependency problems in the optimization scheme (Huang and Xie, 2007). In order to enforce the stability of the algorithm through the iterative process, the average sensitivity

$$\bar{\alpha}_i = \frac{1}{2}\left(\alpha_i^k - \alpha_i^{k-1}\right) \tag{10}$$

is also controlled through the iterations. An iterative target volume $V_{k+1} = V_k (1 \pm \vartheta)$ is set for each iteration, in which ϑ is the ratio by which the volume is allowed to change between interations. In each iteration, elements with $x_i = 1$ and $\alpha_i \leq \alpha_{th}$, corresponding to solid elements with too low of a sensitivity number are switched to void ($x_i = 0$). At the same time, elements with $x_i = 0$ are switched to $x_i = 1$ if $\alpha_i > \alpha_{th}$.

In this analysis, a ratio AR of the volume that is allowed to be added at a given iteration is also controlled. If AR is larger than a prescribed, arbitrary AR_{max}, elements with the highest sensitivity number are removed in order to set the target volume at the prescribed V_{k+1}.

The final criterion imposed in the algorithm is the convergence criterion

$$\frac{\sum_{m=1}^{M} C_{k-1+m} - \sum_{m=1}^{M} C_{k-M-m+1}}{\sum_{m=1}^{M} C_{k-m+1}} \leq \tau, \tag{11}$$

in which M is an arbitrary integer, and τ is small tolerance for the variation of the mean compliance.

NUMERICAL RESULTS

In view of the symmetry of the bridge problem (Fig. 1), in this paper only one quarter of the domain has been analyzed. Appropriate boundary conditions to represent the remainder of the domain have been imposed. The quarter of the domain that has been analyzed was discretized into 147,500 elements. The optimization parameters chosen were $V^* = 0.1V_0$, $\vartheta = 0.05$, $r_{min} = 0.6\,\text{m}$, and $\tau = 0.001$.

Figures 2 to 4 show the final topology obtained for the piled bridge under the prescribed conditions. These are the topologies that result in the stiffest bridge when only 10% of the initial volume of material is allowed to be used in its construction. Figures 2 to 4 also show side by side the optimal topology obtained when rigid supports are considered. These are obtained by imposing zero displacement conditions at the nodes that are supported by piles. The rigid support case had previously been addressed by Huand and Xie (2007). These results show that the flexibility of the foundation has a significant impact on the optimal topology that is obtained for the bridge. In the case of piled supports, the optimization algorithm resorts to allocating more material near the legs of the bridge, indicating that the sensitivity analysis was able to capture the increased flexibility of that region, in comparison with the rigid supports case. In comparison with the rigid supports case, the piled bridge topology presents additional reinforcements across the width and length of the bridge. It is strictly inaccurate to refer to these cluster of elements as "reinforcements", since the optimization algorithm is only able

Figure 2 – Front view of the bridge over a) piles and b) rigid supports.

Figure 3 – Side view of the bridge over a) piles and b) rigid supports.

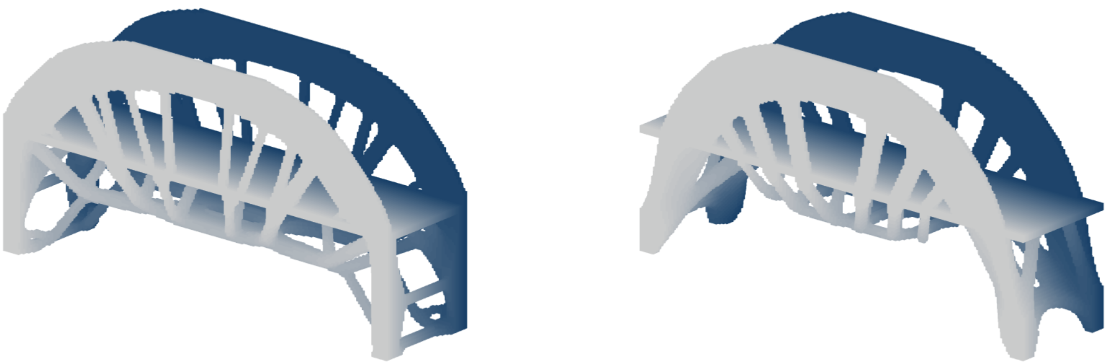

Figure 4 – Perspective view of the bridge over a) piles and b) rigid supports.

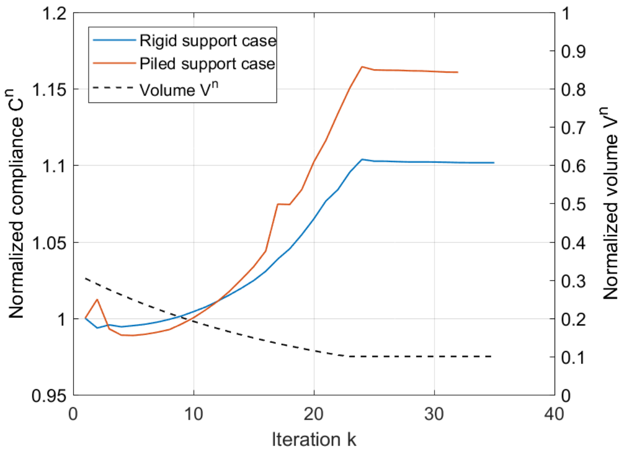

Figure 5 – Evolution of the normalized compliance and volume along iterations.

to decide which element is removed or remains in the structure, not their relation to each other as structural members. Nevertheless, the resulting solution is consistent with engineering design practices, and the function of clusters of elements can be interpreted as to their structural function.

Figure 5 shows quantitative results of this analysis. These results show the normalized compliance $C_k^n = C_k/C_1$ ($k = 1 : N$), in which N is the number of iterations necessary for the algorithm to find the optimal topology, and the normalized volume $V_k^n = V_k/V_0$. This normalization is introduced in order to eliminate the effect of the flexibility of the soil from the analysis. Without this normalization, the compliance of the piled bridge would be higher than that of the bridge over rigid supports simply because its supports are flexible. At the optimized topology, the algorithm found $C_N^n = 1.1608$ for the piled supports case and $C_N^n = 1.1017$ for the rigid supports approximation, after $N = 35$ and $N = 32$ iterations for the piled and rigid support cases, respectively. The 5% difference between the final compliances show that not only the optimized topology is significantly different for each case, but also that the achievable optimization goal is considerably affected by the flexibility of the supports.

CONCLUSIONS

This paper considered the problem of finding a topology that maximizes the stiffness of a piled bridge under a prescribed volume restriction. A model of the piled bridge was obtained by coupling a finite element model of the bridge with a stiffness matrix model of embedded piles. The optimization of the topology was found via the BESO method. The solution for the piled bridge were compared to that of a previous case in which rigid supports were considered for the bridge. The results showed that the rigid supports approximation yields a significantly different optimal topology and appreciable overestimation of the achievable optimization goal.

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