

# NUMERICAL MODELING OF SOLID ELEMENTS OF CONVENTIONAL STEEL, STAINLESS STEEL AND INCONEL ALLOY 718 THROUGH THE FINITE ELEMENT METHOD

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*The work will present the development and implementation of linear numerical models, with the objective of performing mechanical analyzes on solid elements of conventional steel (MS250 and HS350), stainless steel (SS304) and inconel alloy (IA718). For this, a mathematical formulation based on the Finite Element Method (FEM) was implemented computationally, with the aid of the FORTRAN programming language, whose purpose will be to obtain the values of deformations and nodal displacements at different points of the parts, in addition to the von Mises stresses in each finite element that makes up the structure. For the discretization of the solid elements, the 4-node tetrahedral finite element (T4) and the 8-node hexahedral finite element (H8) were used. For a greater scope of validation of the implemented module and to prove its efficiency, solids with different geometric and physical characteristics were analyzed. In order to verify the responses obtained from the implemented computer program, comparisons were made with results found in the literature.*

**Keywords** *Finite Element Method, Numerical Analysis, Mechanical Analysis, FORTRAN.*

## INTRODUCTION

The use of computational tools for solving structural analysis problems has become increasingly frequent and is responsible for optimizing the calculation and design processes of structures in most large companies that carry out structural projects of mechanical systems around the world. This optimization is basically due to the advancement of calculation methodologies based on efficient computational methods, such as: the finite element method, the finite difference method and the boundary element method.

According to Bath (1996), the finite element method in engineering was initially developed on a physical basis for the analysis of problems in structural mechanics. However, it was soon recognized that the technique could be applied equally well to the solution of many other classes of problems. Finite element procedures are currently widely used in engineering analysis, where the procedures are widely employed in the analysis of solids and structures, and, in fact, finite element methods are useful in virtually all fields of engineering analysis.

From the research developed in the area, we can cite those of the authors Almeida (2005) and Maciel (2013), in which they approached the development of a computer program for the numerical analysis of three-dimensional problems using the finite element method, since the first used from the java programming language and the second from the c++ programming language and both obtained satisfactory results with the implementations made.

Therefore, the present research aims to perform numerical (mechanical) analyzes of solid elements referring to conventional steels (MS250 and HS350) and stainless steel (SS304), in addition to the inconel alloy (IA718). Such analyzes will be carried out from the numerical modeling of solid elements based on a computer program implemented in FORTRAN language (Chapman), based on the Finite Element Method, in which two finite elements will be used to model the problems, namely: the 4-node tetrahedral finite element (T4) and the 8-node hexahedral finite element (H8).

Thus, the aim of this research is to obtain the values of stresses, deformations and displacements along the solids under study, in which they will be subjected to certain types of loads. In order to validate the numerical results obtained with the aid of the developed computer program, the answers will be compared with results found in the literature.

## FORMULATIONS

The Fig. 1, shows the master elements of the 4-node tetrahedral (T4), 8-node hexahedral (H8) and 20-node hexahedral (H20) finite elements, respectively.

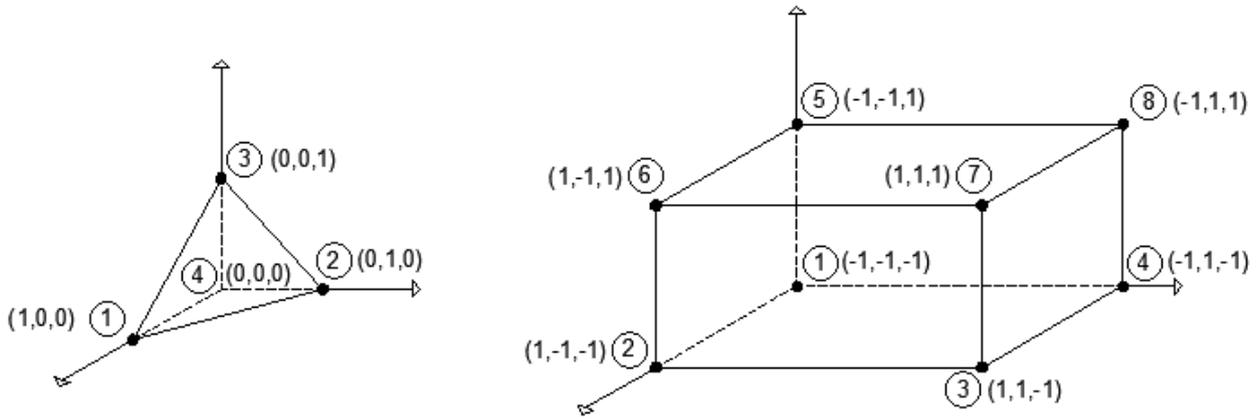


Figure 1 – Master elements

Next, the formulations of the solid finite elements studied in this research are presented.

#### 4-node Tetrahedral Finite Element

The shape functions corresponding to this element are:

$$N_1 = \xi \quad N_2 = \eta \quad N_3 = \varphi \quad (1)$$

$$N_1 + N_2 + N_3 + N_4 = 1 \quad (2)$$

The displacement vector will be described as:

$$q = [q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6 \ q_7 \ q_8 \ q_9 \ q_{10} \ q_{11} \ q_{12}] \quad (3)$$

The relationship between the displacement field vector and the nodal displacement vector being given by:

$$u = Nq \quad (4)$$

Where **N** is the matrix that represents the shape functions, given by:

$$N = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 \end{bmatrix} \quad (5)$$

Then, with the help of Eq. (4) and Eq. (5), it is possible to conclude that:

$$u = N_1q_1 + N_2q_4 + N_3q_7 + N_4q_{10} \quad (6a)$$

$$v = N_1q_2 + N_2q_5 + N_3q_8 + N_4q_{11} \quad (6b)$$

$$w = N_1q_3 + N_2q_6 + N_3q_9 + N_4q_{12} \quad (6c)$$

Since the function **u** depends on x, y and z, and these depend on the natural coordinate's  $\xi$ ,  $\eta$  and  $\varphi$ , then the function **u** is also dependent on  $\xi$ ,  $\eta$  and  $\varphi$ . However, there has been:

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{bmatrix} \quad (7)$$

Since the Jacobian matrix is given by:

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{bmatrix} \quad (8)$$

Considering that matrix  $\mathbf{A}$  is the inverse matrix of the Jacobian matrix, we get:

$$\mathbf{A} = \mathbf{J}^{-1} \quad (9)$$

Getting to:

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} = \mathbf{A} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \end{Bmatrix} \quad (10a)$$

$$\begin{Bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial z} \end{Bmatrix} = \mathbf{A} \begin{Bmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial v}{\partial \varphi} \end{Bmatrix} \quad (10b)$$

$$\begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{Bmatrix} = \mathbf{A} \begin{Bmatrix} \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial \varphi} \end{Bmatrix} \quad (10c)$$

We have that the relationship between the strain vector and the displacement vector is:

$$\boldsymbol{\varepsilon} = \mathbf{B} \mathbf{q} \quad (11)$$

Knowing that the strain vector is defined by:

$$\boldsymbol{\varepsilon} = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{zy} \quad \gamma_{zx} \quad \gamma_{yx}]^T \quad (12)$$

After some mathematical manipulations, it is possible to conclude that matrix  $\mathbf{B}$  is equal to:

$$\mathbf{B} = \begin{bmatrix} A_{11} & 0 & 0 & A_{12} & 0 & 0 & A_{13} & 0 & 0 & -\tilde{A}_1 & 0 & 0 \\ 0 & A_{21} & 0 & 0 & A_{22} & 0 & 0 & A_{23} & 0 & 0 & -\tilde{A}_2 & 0 \\ 0 & 0 & A_{31} & 0 & 0 & A_{32} & 0 & 0 & A_{33} & 0 & 0 & -\tilde{A}_3 \\ 0 & A_{31} & A_{21} & 0 & A_{12} & A_{22} & 0 & A_{33} & A_{23} & 0 & -\tilde{A}_3 & -\tilde{A}_2 \\ A_{31} & 0 & A_{11} & A_{32} & 0 & A_{12} & A_{33} & 0 & A_{13} & -\tilde{A}_3 & 0 & -\tilde{A}_1 \\ A_{21} & A_{11} & 0 & A_{22} & A_{12} & 0 & A_{23} & A_{13} & 0 & -\tilde{A}_2 & -\tilde{A}_1 & 0 \end{bmatrix} \quad (13)$$

Given that:

$$-\tilde{A}_1 = [A_{11} + A_{12} + A_{13}] \quad (14a)$$

$$-\tilde{A}_2 = [A_{21} + A_{22} + A_{23}] \quad (14b)$$

$$-\tilde{A}_3 = [A_{31} + A_{32} + A_{33}] \quad (14c)$$

The stiffness of the element can be obtained based on the equation of the internal energy of deformation, given by:

$$U_e = \frac{1}{2} \mathbf{q}^T \mathbf{B}^T \mathbf{D} \mathbf{B} \mathbf{q} \int_e dV \quad (15)$$

where the element stiffness matrix will be defined by:

$$\mathbf{k}_e = V_e \mathbf{B}^T \mathbf{D} \mathbf{B} \quad (16)$$

In turn, the body force (corresponding to the self-weight) will be given by:

$$\mathbf{f}_e = \int_0^1 \int_0^{1-\xi} \int_0^{1-\xi-\eta} \mathbf{N}^T \mathbf{f} |det J| d\varphi d\eta d\xi \quad (17)$$

The calculation of the volume of a tetrahedral, via FEM, is defined by Eq. (18):

$$V_e = (det J) \int_0^1 \int_0^{1-\xi} \int_0^{1-\xi-\eta} d\varphi d\eta d\xi \quad (18)$$

### 8-node Hexahedral Finite Element

The Lagrangian shape functions are represented as:

$$N_i = \frac{1}{8} (1 + \xi_i \xi)(1 + \eta_i \eta)(1 + \varphi_i \varphi) \quad (19)$$

In turn, the nodal displacements will be represented by the vector:

$$\mathbf{q} = [q_1, q_2, q_3, \dots, q_{23}, q_{24}]^T \quad (20)$$

Hence, the relationship between the vector representing the displacement field and the nodal displacement vector is equal to:

$$\mathbf{u} = \mathbf{N} \mathbf{q} \quad (21)$$

The relationship between the strain vector and the displacement vector will be:

$$\boldsymbol{\varepsilon} = \mathbf{B} \mathbf{q} \quad (22)$$

The element stiffness matrix corresponding to the 8-node hexahedral finite element is defined as:

$$\mathbf{k}_e = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \mathbf{B}^T \mathbf{D} \mathbf{B} |det J| d\varphi d\eta d\xi \quad (23)$$

Being  $\mathbf{J}$  the Jacobian matrix with dimension (3x3) and remembering that the integrals will be solved numerically with the aid of the Gauss-Legendre Method (Gauss Quadrature).

The relationships between the partial derivatives of the displacements can be represented in a matrix form, as:

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} \quad (24)$$

Considering that the gamma matrix is an inverse matrix of the Jacobian matrix, we have:

$$\boldsymbol{\Gamma} = \mathbf{J}^{-1} \quad (25)$$

Hence, substituting Eq. (25) in Eq. (24), one arrives at:

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} = \boldsymbol{\Gamma} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \end{Bmatrix} \quad (26)$$

The matrix B corresponding to the hexahedral finite element is represented as:

$$\mathbf{B} = \mathbf{H} \boldsymbol{\Gamma}_u \mathbf{D} \mathbf{N} \quad (27)$$

From Eq. (28) it is possible to conclude that:

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial v}{\partial \varphi} \\ \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial \varphi} \end{bmatrix} = DN q \quad (28)$$

since the DN matrix has 9 lines and 24 columns and will be organized with the use of sub-matrices proposed in the present work and presented as:

$$DN1 = \begin{bmatrix} \frac{\partial N1}{\partial \xi} & 0 & 0 & \frac{\partial N2}{\partial \xi} & 0 & 0 \\ \frac{\partial N1}{\partial \eta} & 0 & 0 & \frac{\partial N2}{\partial \eta} & 0 & 0 \\ \frac{\partial N1}{\partial \varphi} & 0 & 0 & \frac{\partial N2}{\partial \varphi} & 0 & 0 \end{bmatrix} \quad DN2 = \begin{bmatrix} \frac{\partial N3}{\partial \xi} & 0 & 0 & \frac{\partial N4}{\partial \xi} & 0 & 0 \\ \frac{\partial N3}{\partial \eta} & 0 & 0 & \frac{\partial N4}{\partial \eta} & 0 & 0 \\ \frac{\partial N3}{\partial \varphi} & 0 & 0 & \frac{\partial N4}{\partial \varphi} & 0 & 0 \end{bmatrix} \quad DN3 = \begin{bmatrix} \frac{\partial N5}{\partial \xi} & 0 & 0 & \frac{\partial N6}{\partial \xi} & 0 & 0 \\ \frac{\partial N5}{\partial \eta} & 0 & 0 & \frac{\partial N6}{\partial \eta} & 0 & 0 \\ \frac{\partial N5}{\partial \varphi} & 0 & 0 & \frac{\partial N6}{\partial \varphi} & 0 & 0 \end{bmatrix} \quad (29.a)$$

$$DN4 = \begin{bmatrix} \frac{\partial N7}{\partial \xi} & 0 & 0 & \frac{\partial N8}{\partial \xi} & 0 & 0 \\ \frac{\partial N7}{\partial \eta} & 0 & 0 & \frac{\partial N8}{\partial \eta} & 0 & 0 \\ \frac{\partial N7}{\partial \varphi} & 0 & 0 & \frac{\partial N8}{\partial \varphi} & 0 & 0 \end{bmatrix} \quad DN5 = \begin{bmatrix} 0 & \frac{\partial N1}{\partial \xi} & 0 & 0 & \frac{\partial N2}{\partial \xi} & 0 \\ 0 & \frac{\partial N1}{\partial \eta} & 0 & 0 & \frac{\partial N2}{\partial \eta} & 0 \\ 0 & \frac{\partial N1}{\partial \varphi} & 0 & 0 & \frac{\partial N2}{\partial \varphi} & 0 \end{bmatrix} \quad DN6 = \begin{bmatrix} 0 & \frac{\partial N3}{\partial \xi} & 0 & 0 & \frac{\partial N4}{\partial \xi} & 0 \\ 0 & \frac{\partial N3}{\partial \eta} & 0 & 0 & \frac{\partial N4}{\partial \eta} & 0 \\ 0 & \frac{\partial N3}{\partial \varphi} & 0 & 0 & \frac{\partial N4}{\partial \varphi} & 0 \end{bmatrix} \quad (29.b)$$

$$DN7 = \begin{bmatrix} 0 & \frac{\partial N5}{\partial \xi} & 0 & 0 & \frac{\partial N6}{\partial \xi} & 0 \\ 0 & \frac{\partial N5}{\partial \eta} & 0 & 0 & \frac{\partial N6}{\partial \eta} & 0 \\ 0 & \frac{\partial N5}{\partial \varphi} & 0 & 0 & \frac{\partial N6}{\partial \varphi} & 0 \end{bmatrix} \quad DN8 = \begin{bmatrix} 0 & \frac{\partial N7}{\partial \xi} & 0 & 0 & \frac{\partial N8}{\partial \xi} & 0 \\ 0 & \frac{\partial N7}{\partial \eta} & 0 & 0 & \frac{\partial N8}{\partial \eta} & 0 \\ 0 & \frac{\partial N7}{\partial \varphi} & 0 & 0 & \frac{\partial N8}{\partial \varphi} & 0 \end{bmatrix} \quad DN9 = \begin{bmatrix} 0 & 0 & \frac{\partial N1}{\partial \xi} & 0 & 0 & \frac{\partial N2}{\partial \xi} \\ 0 & 0 & \frac{\partial N1}{\partial \eta} & 0 & 0 & \frac{\partial N2}{\partial \eta} \\ 0 & 0 & \frac{\partial N1}{\partial \varphi} & 0 & 0 & \frac{\partial N2}{\partial \varphi} \end{bmatrix} \quad (29.c)$$

$$DN10 = \begin{bmatrix} 0 & 0 & \frac{\partial N3}{\partial \xi} & 0 & 0 & \frac{\partial N4}{\partial \xi} \\ 0 & 0 & \frac{\partial N3}{\partial \eta} & 0 & 0 & \frac{\partial N4}{\partial \eta} \\ 0 & 0 & \frac{\partial N3}{\partial \varphi} & 0 & 0 & \frac{\partial N4}{\partial \varphi} \end{bmatrix} \quad DN11 = \begin{bmatrix} 0 & 0 & \frac{\partial N5}{\partial \xi} & 0 & 0 & \frac{\partial N6}{\partial \xi} \\ 0 & 0 & \frac{\partial N5}{\partial \eta} & 0 & 0 & \frac{\partial N6}{\partial \eta} \\ 0 & 0 & \frac{\partial N5}{\partial \varphi} & 0 & 0 & \frac{\partial N6}{\partial \varphi} \end{bmatrix} \quad DN12 = \begin{bmatrix} 0 & 0 & \frac{\partial N7}{\partial \xi} & 0 & 0 & \frac{\partial N8}{\partial \xi} \\ 0 & 0 & \frac{\partial N7}{\partial \eta} & 0 & 0 & \frac{\partial N8}{\partial \eta} \\ 0 & 0 & \frac{\partial N7}{\partial \varphi} & 0 & 0 & \frac{\partial N8}{\partial \varphi} \end{bmatrix} \quad (29.d)$$

or even in a compact form like:

$$DN = \begin{bmatrix} DN1 & DN2 & DN3 & DN4 \\ DN5 & DN6 & DN7 & DN8 \\ DN9 & DN10 & DN11 & DN12 \end{bmatrix} \quad (30)$$

Based on the equation referring to the strain vector, it is possible to conclude that:

$$\boldsymbol{\varepsilon} = \mathbf{H} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{bmatrix} \quad (31)$$

where the matrix  $\mathbf{H}$  will be expressed by:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (32)$$

Hence, from Eq. (33):

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{bmatrix} = \boldsymbol{\Gamma}_u \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \varphi} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial v}{\partial \varphi} \\ \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial \varphi} \end{bmatrix} \quad (33)$$

It is possible to arrive at  $\boldsymbol{\Gamma}_u$ , defined by the relation that follows:

$$\boldsymbol{\Gamma}_u = \begin{bmatrix} \Gamma(\xi, \eta, \varphi) & 0 & 0 \\ 0 & \Gamma(\xi, \eta, \varphi) & 0 \\ 0 & 0 & \Gamma(\xi, \eta, \varphi) \end{bmatrix} \quad (34)$$

Since the matrix  $\Gamma(\xi, \eta, \varphi)$  has 3 rows and 3 columns.

### Calculation of von Mises Stress

For elements that are subject to the plane stress state, it is possible to inform that the von Mises stress is represented by the following relation:

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \quad (35)$$

and

$$\sigma_{VM} \leq \sigma_y \quad (36)$$

where  $\sigma_y$  is the yield stress and  $\sigma_{VM}$  is the von Mises stress.

## RESULTS AND DISCUSSIONS

Two examples will be presented for the validation of the implemented code. For the first example, the solid modeling was carried out by the four-node tetrahedral finite element, aiming to validate the code implemented for such finite element, using the mechanical and physical properties found in an example in the literature, making its results were compared, and then performed numerical analyzes using the mechanical properties of the materials under study. For the second example, the solid was modeled with the eight-node hexahedral finite element, the results were compared, using the mechanical properties of an example found in the literature, and then, the mechanical analysis of the materials used in this research.

As for the analysis of the solids under study, the mechanical properties of the following steels and alloys were used: for the conventional steels of medium and high mechanical strength (MS250 and HS350, respectively), the Modulus of Elasticity (E) of 200 GPa was used and Poisson's ratio ( $\nu$ ) of 0.3; for stainless steel (SS304), Modulus of Elasticity (E) of 193 GPa and Poisson's Ratio ( $\nu$ ) of 0.27 and for inconel alloy (IA718), Modulus of Elasticity (E) of 206 GPa and the Poisson ratio ( $\nu$ ) of 0.28.

For the yield stress of the materials, the following values were used: 250 MPa and 350 MPa for conventional steels of medium and high mechanical strength, respectively; 215 MPa for stainless steel and 820 MPa for inconel alloy.

### Exemplo 1

The example shown in Fig. 2, refers to modeling the solid that has been discretized into just a single 4-node tetrahedral finite element.

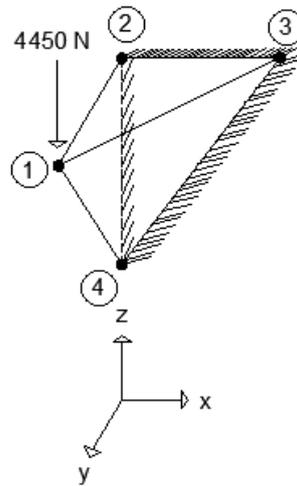


Figure 2 – Exemplo 1

For the analysis of the nodal displacement in the solid above, the application of a nodal load of 4450 N on node 1 was considered, taking into account that nodes 2, 3 and 4 were fixed. The coordinates of the nodes, given in millimeters, were defined as:

$$1 (0,25,25) \quad 2 (0,0,25) \quad 3 (25,0,25) \quad 4 (0,0,0)$$

The Tab. 1 shows the comparison of the result obtained by the research and that found in the literature. For the mechanical properties, the Modulus of Elasticity (E) of 207 GPa and the Poisson's Ratio ( $\nu$ ) of 0.3 were applied:

Table 1 – Displacement of nodes

Node	Displacement in z (mm)		
	Present work	Chandrupatla e Belegundu (2012)	Present work / Literature
1	-0.01341	-0.0134	1.0007463
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0

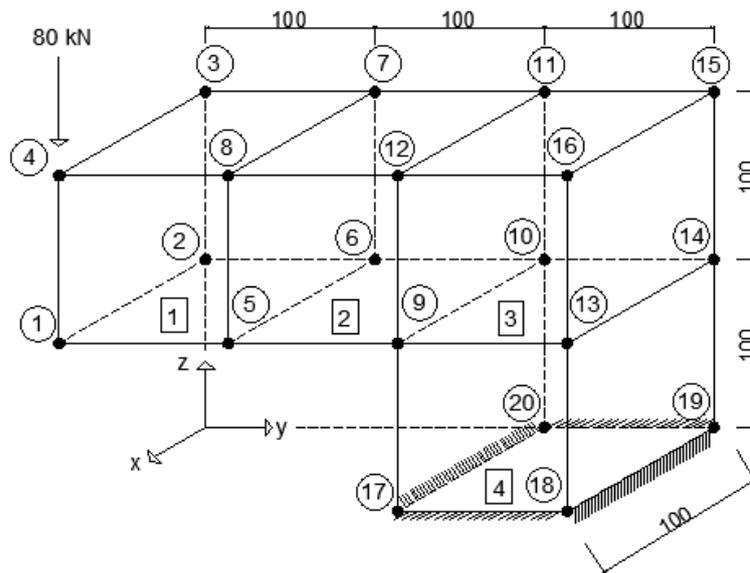
Therefore, given the convergence of the values compared in Tab. 1, are presented in Tab. 2 the results obtained by the code implemented for the materials used in this research.

**Table 2 – Displacement of nodes T4**

Nodes	Displacement in z (mm)			
	MS250	HS350	SS304	IA718
1	-0.01388	-0.01388	-0.01406	-0.01327
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

**Exemplo 2**

The example shown in Fig. 3, this is the modeling of the solid that was discretized into four 8-node hexahedral finite elements, with a total number of 20 nodes.



**Figure 3 – Exemplo 2**

As for the implemented code, for the example above, the application of a nodal load of 80 kN in node 4 was considered, the measurements were given in millimeters (mm), the Modulus of Elasticity (E) used was 200 GPa and the coefficient Poisson ( $\nu$ ) of 0.3, in addition to considering that nodes 17, 18, 19 and 20 were fixed.

In this way, Tab. 3 shows the values obtained by the implemented code for the displacements of nodes 1, 2, 3, 4, 15, 16 and 20, in the direction of the z axis, and Tab. 4 the displacements in the direction of the x axis are shown, for the same nodes, and these values are compared with those found in the literature.

**Table 3 – Displacement of the comparative z-axis nodes**

Nodes	Displacement in z (m)		
	Present work	Chandrupatla e Belegundu (2012)	Present work / Literature
1	- 0.4098	- 0.409828806	0.9999297
2	- 0.3323	- 0.33229407	1.0000178
3	- 0.3268	- 0.326759294	1.0001245
4	- 0.4278	- 0.427797766	1.0000052
15	- 0.0558	- 0.055798691	1.0000234
16	- 0.03958	- 0.03958274	0.9999308
20	0.0	0.0	1.0

**Table 4 – Displacement of the comparative x-axis nodes**

Nodes	Displacement in x (m)		
	Present work	Chandrupatla e Belegundu (2012)	Present work / Literatura
1	- 0.02157	- 0.021568703	1.0000601
2	- 0.02531	- 0.025306207	1.0001499
3	0.05735	0.057350356	0.9999934
4	0.05776	0.057756218	1.0000655
15	0.02663	0.026633192	0.9998801
16	0.02061	0.02060568	1.0002096
20	0.0	0.0	1.0

Therefore, they are found in Tab. 5 the values of the displacements, of nodes 1, 2, 3, 4, 15, 16 and 20, in the direction of the z axis for the materials under analysis.

**Table 5 – Displacement of nodes H8**

Nodes	Displacement in z (m)			
	MS250	HS350	SS304	IA718
1	- 0.4098	- 0.4098	- 0.4317	- 0.4025
2	- 0.3323	- 0.3323	- 0.3527	- 0.3281
3	- 0.3268	- 0.3268	- 0.3467	- 0.3225
4	- 0.4278	- 0.4278	- 0.4507	- 0.4202
15	- 0.0558	- 0.0558	0.05946	- 0.05524
16	- 0.03958	- 0.03958	0.04200	- 0.03907
20	0.0	0.0	0.0	0.0

For the von Mises stress calculated for each element, they are presented in Tab. 6 the comparison of the values obtained by the code with the values found in the literature for elements 1 and 2, since the numerical integration occurred with 8 points.

**Table 6 – Comparative von Mises Stress**

Element	von Mises Stress (MPa)		
	Present work	Chandrupatla e Belegundu (2012)	Present work / Literature
1	23.36	23.359	1.0000428
	29.93	29.929	1.0000334
	40.58	40.582	0.9999507
	43.4	43.396	1.0000922
	18.54	18.545	0.9997304
	14.85	14.849	1.0000673
	15.98	15.984	0.9997497
	17.91	17.91	1.0000000
2	31.42	31.416	1.0001273
	61.17	61.174	0.9999346
	28.27	28.272	0.9999293
	51.61	51.608	1.0000388
	35.72	35.722	0.9999440
	35.95	35.948	1.0000556
	26.19	26.193	0.9998855
	38.61	38.615	0.9998705

Therefore, we have in Tab. 7 the values of von Mises stresses, for element 2, of the materials under study in this research, since for this it obtained the highest values.

**Table 7 – von Mises Stress**

Element	von Mises Stress (MPa)			
	MS250	HS350	SS304	IA718
2	31.42	31.42	31.50	31.47
	61.17	61.17	61.57	61.44
	28.27	28.27	28.52	28.44
	51.61	51.61	52.56	52.25
	35.72	35.72	36.31	36.12
	35.95	35.95	36.79	36.51
	26.19	26.19	26.94	26.69
	38.61	38.61	38.61	38.61

## CONCLUSIONS

It is noteworthy that the code implemented for the 4-node tetrahedral finite element, when demonstrated in example 1, presented the values of the nodal displacements with a percentage difference of 0.07463%, when compared with the values found in the literature. As for the materials analyzed in this work, the smallest nodal displacement occurred for the inconel alloy, thus demonstrating the influence that the Modulus of Elasticity and Poisson's ratio have on the ability of a solid to resist, when it is subjected to a certain load.

For example 2, the integrals were numerically solved with the aid of the Gauss-Legendre Method (Gauss Quadrature). The largest percentage difference between the von Mises stress results obtained via implemented computer code and the literature results was 0.02697%. Regarding the nodal displacements analyzed, when compared with the responses in the literature, the greatest difference was 0.02086%. The inconel alloy had the lowest displacement value and stainless steel the highest displacement value, thus demonstrating that the Modulus of Elasticity is inversely proportional to the resistance capacity of a given material.

As for the analysis of the von Mises stresses for example 2, none of the materials analyzed failed, due to this, the fact that the values of the von Mises stresses obtained were below the values of the stresses of material flow.

In both examples presented, there was a convergence between the results obtained between the implemented code, both for the 4-node tetrahedral finite element and for the 8-node hexahedral finite element, and the responses obtained in the literature. It is concluded, therefore, that the implementation developed was satisfactory, contributing with precise values of stress, deformation and displacement in solids submitted to a certain type of loading.

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